

Sphere

Q.1. If A (- 1, 4 , - 3) is one end of the diameter AB of the sphere $x^2 + y^2 + z^2 - 2y + 2z - 15 = 0$ then find the coordinates of the other end point B.

Solution : 1

The given sphere is , $x^2 + y^2 + z^2 - 2y + 2z - 15 = 0$

The centre is (0, 1 , - 1) .

Point A is (- 1, 4 , - 3) .

Let point B is (α , β , γ) .Then

$$(\alpha - 1) / 2 = 0 \Rightarrow \alpha = 1.$$

$$(\beta + 4) / 2 = 1 \Rightarrow \beta = - 2$$

$$(\gamma - 3) / 2 = - 1 \Rightarrow \gamma = 1.$$

Therefore required point is (1 , - 2 , 1) .

Q.2. Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle whose radius is unity.

Solution : 2



We have equation of sphere as : $x^2 + y^2 + z^2 - x + z - 2 = 0$

Then centre of sphere is $(1/2 , 0 , -1/2) = (- u, - v, - w)$

And radius = $R = \sqrt{(u^2 + v^2 + w^2 - d)} = \sqrt{(1/4 + 1/4 + 2)} = \sqrt{(5/2)}$

Distance of centre from plane $x + 2y - z = 4$ is

$$d = [(1/2 + 1/2 - 4)/\sqrt{(1 + 4 + 1)}] = 3/\sqrt{6}.$$

Let radius of the circle be r then $r = \sqrt{(R^2 - d^2)} = \sqrt{(5/2 - 9/6)} = \sqrt{(2/2)} = 1$. [**Proved.**]

Q.3. Show that the equation to a sphere passing through three points $(2, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$ and having its centre on the plane $2x + 3y + 4z = 27$ is

$$x^2 + y^2 + z^2 - 6x - 6y - 6z + 8 = 0 .$$

Solution : 3

The general equation of sphere is : $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + c = 0$

$(2, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$ lie on it

$$\text{Therefore, } 2^2 + 0 + 0 + 4u + 0 + 0 + c = 0$$

$$\text{Or, } 4 + 4u + c = 0 \text{ ----- (1)}$$

$$4 + 4v + c = 0 \text{ ----- (2)}$$

$$\text{and } 4 + 4w + c = 0 \text{ ----- (3)}$$

Centre $(-u, -v, -w)$ lie on plane $2x + 3y + 4z = 27$

$$\text{Hence, } -3u - 3v - 4w = 27 \text{ ----- (4)}$$

Putting value of u, v, w from (1), (2) and (3) in (4), we get

$$-2(-c-4)/4 - 3(-c-4)/4 - 4(-c-4)/4 = 27$$

$$\text{Or, } (2c + 8 = 3c + 12 + 4c + 16)/4 = 27$$

$$\text{Or, } 9c + 36 = 108 \text{ Or, } 9c = 72 \Rightarrow c = 8.$$

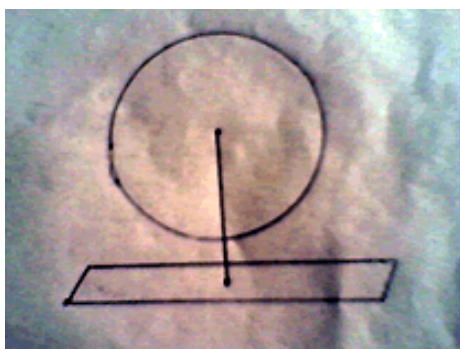
$$\text{Therefore, } 4 + 4u + 8 = 0 \Rightarrow u = -3 \text{ [from (1)]}$$

$$\text{Similarly using (2) and (3) } v = -3 \text{ and } w = -3 ,$$

Putting u, v and w in general equation, we get $x^2 + y^2 + z^2 - 6x - 6y - 6z + 8 = 0$. [**Proved.**]

Q.4. Find the least distance of the plane $12x + 4y + 3z = 327$ from the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$.

Solution : 4



The given equation of sphere is , $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$

Therefore, $u = 2, v = -1, w = -3, d = -155,$

The centre of the sphere is $(-u, -v, -w) = (-2, 3, 3).$

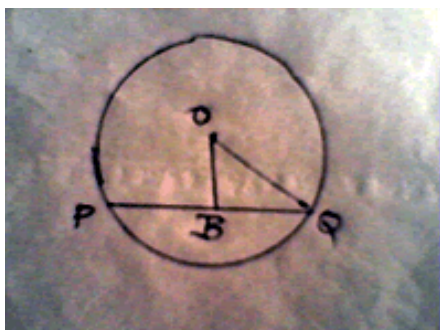
Distance of centre from the plane = $[12(-2) + 4 \times 1 + 3 \times 3 - 327]/\sqrt{(144 + 16 + 9)} = |(-24 + 4 + 9 - 327)/13| = 26$

Radius of the sphere = $\sqrt{(4 + 1 + 9 + 155)} = 13,$

Hence the least distance of plane from sphere = $26 - 13 = 13.$

Q.5. Find the radius of the circular section of the sphere $x^2 + y^2 + z^2 = 49$ cut by a plane $2x + 3y - z - 5\sqrt{14} = 0$.

Solution : 5



Equation of sphere is $x^2 + y^2 + z^2 = 49$ and plane is $2x + 3y - z - 5\sqrt{14} = 0$

In fig. above, O is the centre (0, 0, 0) of the sphere and PQ represents a plane.

Length of perpendicular (OB) = $|2 \times 0 + 3 \times 0 - 1 \times 0 - 5\sqrt{14}/\sqrt{14}| = |-5| = 5$ units.

BQ is the radius of the circle formed by cutting the sphere by plane.

ΔOBQ is a right angled triangle.

Therefore, $OB^2 + BQ^2 = OQ^2$

Or, $5^2 + BQ^2 = 49 \Rightarrow BQ^2 = 49 - 25 = 24$

Therefore, $BQ = \sqrt{24}$ units.

Hence, radius of the circle = $\sqrt{24}$ units.