

Chapter 7. Reflection

Formulae

1. Rule to find the reflection of a point in the x-axis:
 - (i) Retain the abscissa i.e. x-coordinate.
 - (ii) Change the sign of ordinate i.e. y-coordinate.
2. Rule to find the reflection of a point in the y-axis:
 - (i) Change the sign of abscissa i.e., x-coordinate.
 - (ii) Retain the ordinate i.e., y-coordinate.
3. Reflection of a point in a line parallel to x-axis. The reflection of the point $P(x, y)$ in the line $y = a$ is the point $P(x, -y+2a)$.
4. Reflection of a point in a line parallel to y-axis. The reflection of the point $P(x, y)$ in the line $x = a$ is the point $P'(-x+2a, y)$.
5. Reflection of a point in the origin:
 - (i) Change the sign of abscissa i.e., x-coordinate.
 - (ii) Change the sign of ordinate i.e., y-coordinate.
6. A point is called an Invariant point with respect to a given line if and only if lies on the line.

Determine the Following

Question 1. The triangle $A(1, 2)$, $B(4, 4)$ and $C(3, 7)$ is first reflected in the line $y = 0$ onto triangle $A'B'C'$ and then triangle $A'B'C'$ is reflected in the origin onto triangle $A''B''C''$. Write down the co-ordinates of:

- (i) A', B', C' , (ii) A'', B'', C'' .

Solution. (i) $A' \rightarrow (1, -2)$,
 $B' \rightarrow (4, -4)$,
 $C' \rightarrow (3, -7)$.

- (ii) $A'' \rightarrow (-1, 2)$,
 $B'' \rightarrow (-4, 4)$,
 $C'' \rightarrow (-3, 7)$.

Question 2. The point $P(a, b)$ is first reflected in the origin and then reflected on the Y-axis to p' . If P' has co-ordinates $(3, -4)$, evaluate a, b .

Solution. $P(a, b)$ after reflection at the origin =
 $(-a, -b)$ $(-a, -b)$ after reflection on the Y-axis =
 $p'(a-b)$

According to question,

$$\Rightarrow (a, -b) = (3, -4)$$

$$\Rightarrow a = 3, -b = -4$$

$$\Rightarrow a = 3, b = 4.$$

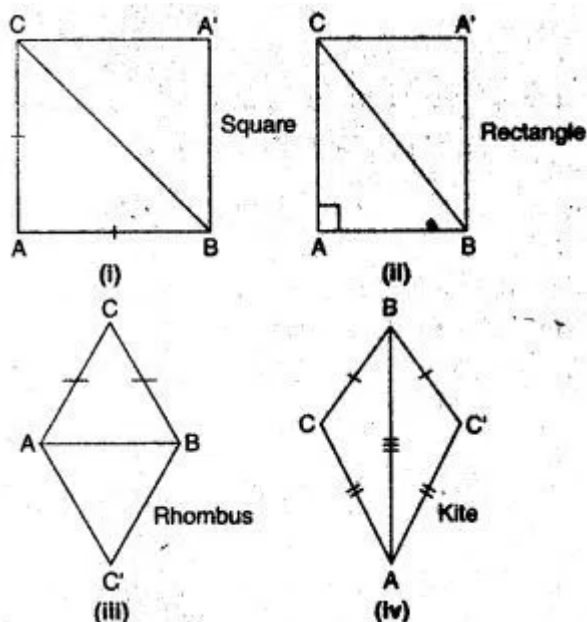
Ans.

Figure Based Questions

Question 1. Name the figure formed by a triangle and its reflection, when:

- (i) An isosceles right-angled triangle is reflected in its hypotenuse.
- (ii) A right-angled triangle is reflected in its hypotenuse.
- (iii) An isosceles triangle is reflected in its unequal side.
- (iv) A scalene triangle is reflected in its greatest side.

Solution:



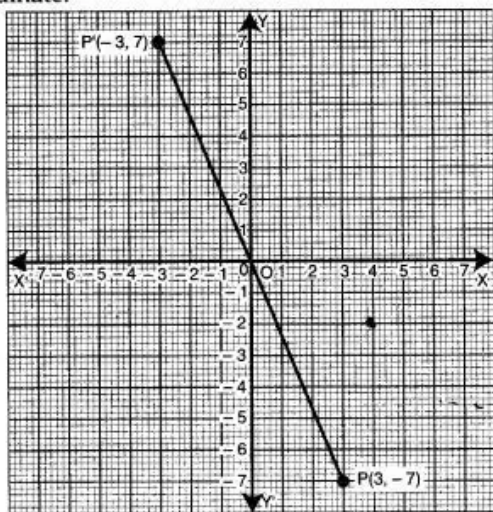
Graphical Depiction

Question 1. Find the co-ordinates of the images of the following under reflection in the origin:

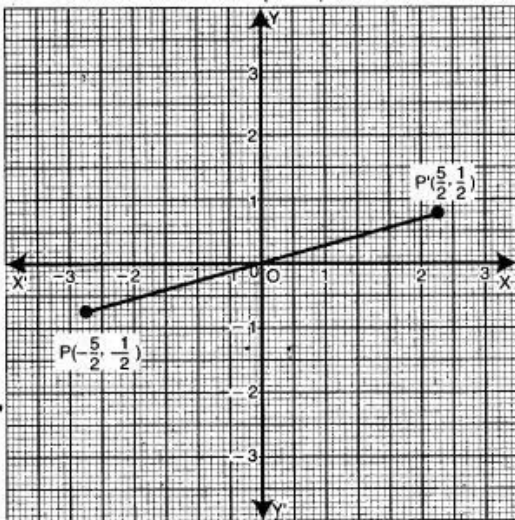
- (i) $(3, -7)$ (ii) $\left(\frac{-5}{2}, \frac{-1}{2}\right)$ (iii) $(0, 0)$.

Solution : (i) The reflection (image) of the point $P(3, -7)$ at the origin is the point $P'(-3, 7)$.

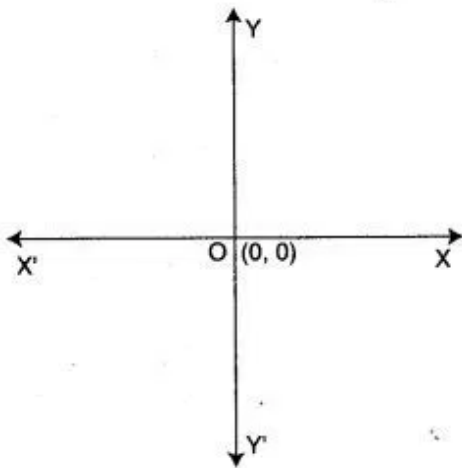
Note : To find the reflection of a point in the origin change : (a) The sign of abscissa; i.e., X co-ordinate. (b) The sign of ordinate; i.e., Y co-ordinate.



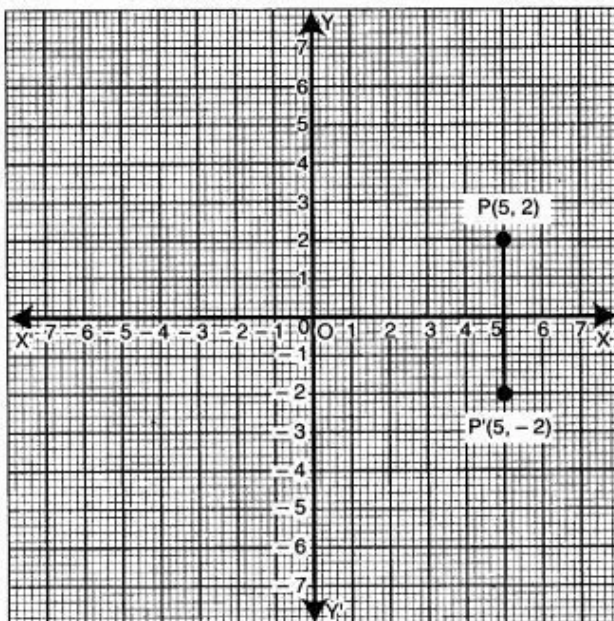
- (ii) The reflection of the point $P\left(\frac{-5}{2}, \frac{-1}{2}\right)$ at the origin is the point $P'\left(\frac{5}{2}, \frac{1}{2}\right)$.



(iii) The reflection (image) of the point $(0, 0)$ at the origin is the point $(0, 0)$ itself.



Question 2. The image of a point P under reflection on the X -axis is $(5, -2)$. Write down the co-ordinates of P .



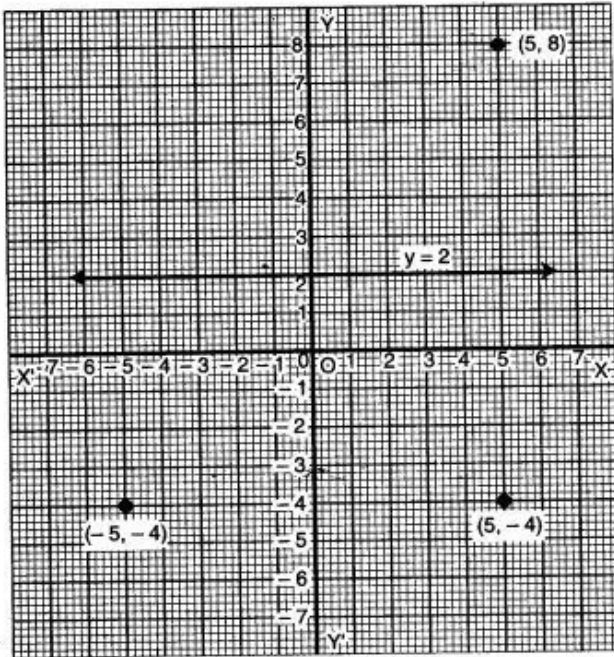
Solution : The image of a point P under reflection on the X -axis is $P'(5, -2)$.

So, co-ordinates of $P = (5, 2)$.

Question 3. Write down the co-ordinates of the image of $(5, -4)$.

- (i) Reflection in $x = 0$;
 (ii) Reflection in $y = 2$.

Solution :



- (i) Reflection in $x = 0$ is $(-5, -4)$. Ans.
 (ii) Reflection in $y = 2$ is $(5, 8)$. Ans.

Question 4. Use a graph paper for this question.

- (i) The point $P(2, -4)$ is reflected about the line $x = 0$ to get the image Q . Find the coordinates of Q .
 (ii) Point Q is reflected about the line $y = 0$ to get the image R . Find the co-ordinates of R .
 (iii) Name the figure PQR .
 (iv) Find the area of figure PQR .

Solution :

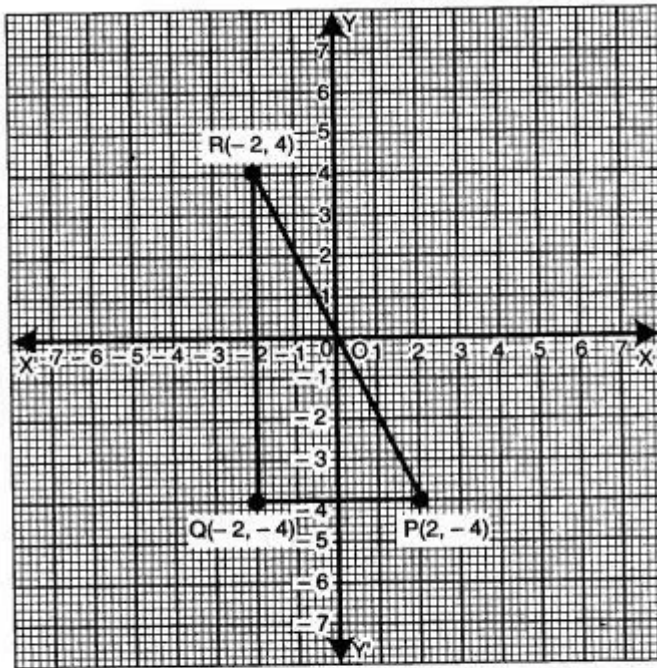
(i) $P(2, -4)$ is reflected in $(x = 0)$ y -axis and Q -image.

$$P(2, -4) \xrightarrow{My} Q(-2, -4) \quad \text{Ans.}$$

(ii) $Q(-2, -4)$ is reflected in $(y = 0)$ x -axis to get R .

$$Q(-2, -4) \xrightarrow{Mx} R(-2, 4) \quad \text{Ans.}$$

(iii) The figure PQR is right angle triangle as shown ahead :



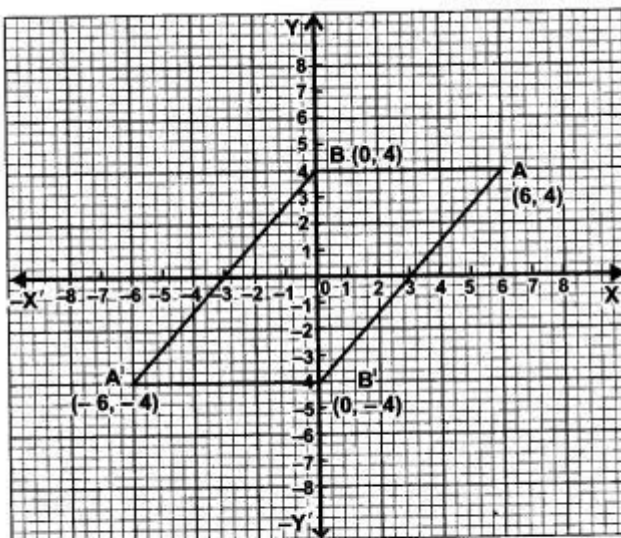
Ans.

$$\begin{aligned} \text{(iv) Area of } \triangle PQR &= \frac{1}{2} \times PQ \times QR \\ &= \frac{1}{2} \times 4 \times 8 = 16 \text{ sq. unit. } \text{Ans.} \end{aligned}$$

Question 5. Using a graph paper, plot the points A (6,4) and B (0,4).

- Reflect A and B in the origin to get the images A' and B'.
- Write the co-ordinates of A' and B'.
- State the geometrical name for the figure ABA'B'.
- Find its perimeter.

Solution : (i)



(ii) A' (-6, -4) and B' (0, -4) Ans.

(iii) ABA'B' is a parallelogram. Ans.

(iv) From the figure AB = 6, BB' = 8, A'B' = 6

In $\triangle ABB'$, $(AB')^2 = AB^2 + (BB')^2$

$$6^2 + 8^2 = 100$$

$$\therefore AB' = 10 = A'B$$

{ABA'B' is a parallelogram}

$\therefore$ Perimeter of ABA'B'

$$= AB + BA' + A'B' + B'A$$

$$= 6 + 10 + 6 + 10 = 32 \text{ units}$$

Question 6. (i) Find the reflection of the point (3, 5) on X-axis.

(ii) Find the reflection of the point (-3, 5) on X-axis.

(iii) Find the reflection of the point (-3, -5) on X-axis.

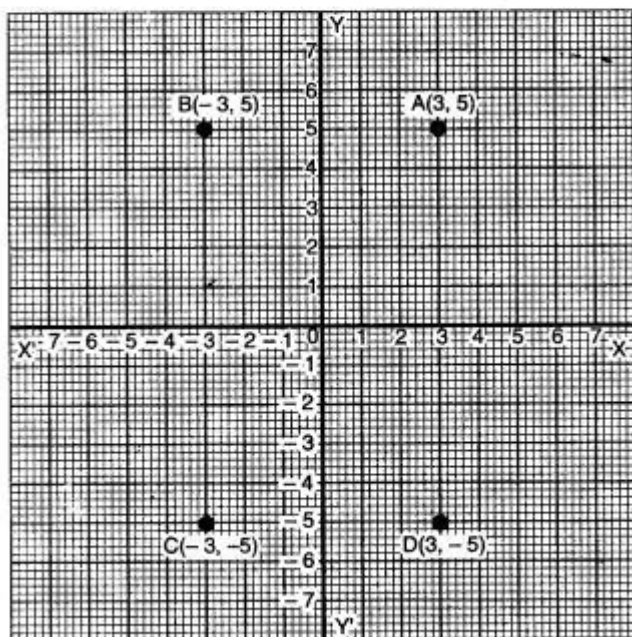
(iv) Find the reflection of the point (3, -5) on X-axis.

Solution : (i) $A(3, 5) \xrightarrow{M_x} D(3, -5)$

(ii) $B(-3, 5) \xrightarrow{M_x} C(-3, -5)$

(iii) $C(-3, -5) \xrightarrow{M_x} B(-3, 5)$

(iv) $D(3, -5) \xrightarrow{M_x} A(3, 5)$



Question 7. P, Q have co-ordinates (-1, 2) and (6, 3) respectively. Reflect P on the X-axis to P'. Find:

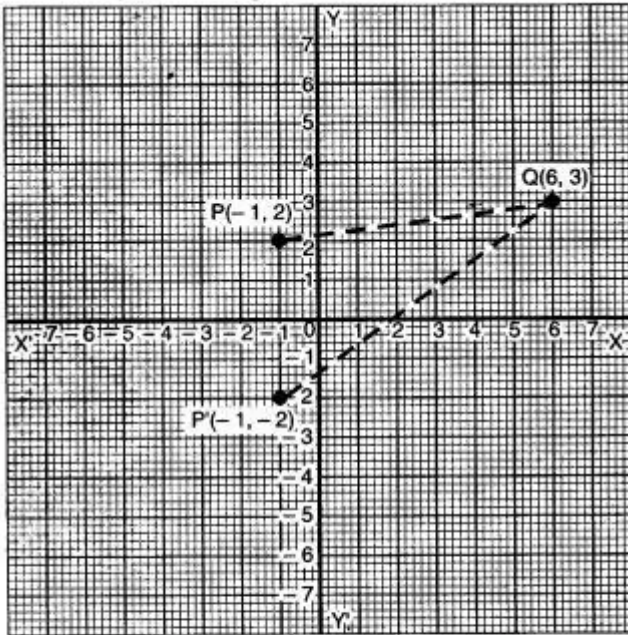
(i) The co-ordinate of P'

(ii) Length of P'Q.

(iii) Length of PQ.

(iv) Is $P'Q = PQ$?

Solution :



(i) $P' \rightarrow (-1, -2)$. Ans.

(ii) $P'Q = \sqrt{(6+1)^2 + (3+2)^2}$
 $= \sqrt{49 + 25} = \sqrt{74}$. Ans.

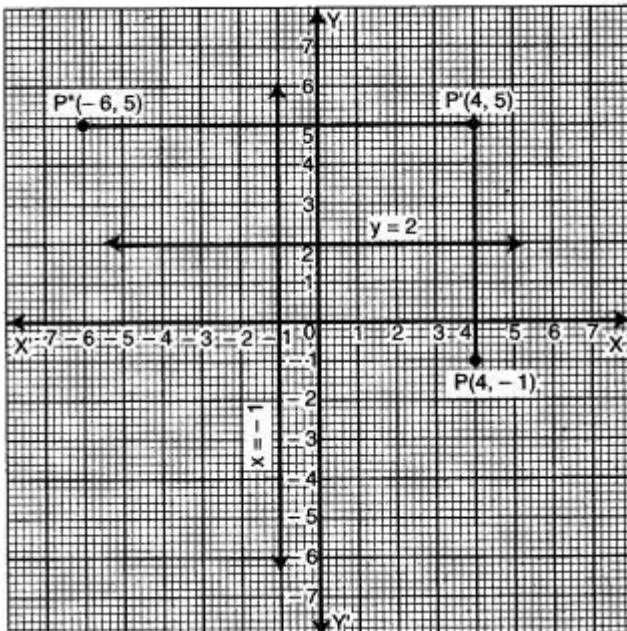
(iii) $PQ = \sqrt{(6+1)^2 + (3-2)^2}$
 $= \sqrt{49 + 1} = \sqrt{50}$. Ans.

(iv) No. ($P'Q \neq PQ$) Ans.

Question 8. A point $P(4, -1)$ is reflected to P' in the line $y = 2$ followed by the reflection to P'' in the line $x = -1$. Find :

- (i) The co-ordinates of P' .
- (h) The co-ordinates of P'' .
- (iii) The length of PP' .
- (iv) The length of $P'P''$.

Solution :



- (i) $P' \rightarrow (4, 5)$. Ans.
- (ii) $P'' \rightarrow (-6, 5)$. Ans.
- (iii) $PP' = 6$ units Ans.
- (iv) $P'P'' = 10$ units Ans.

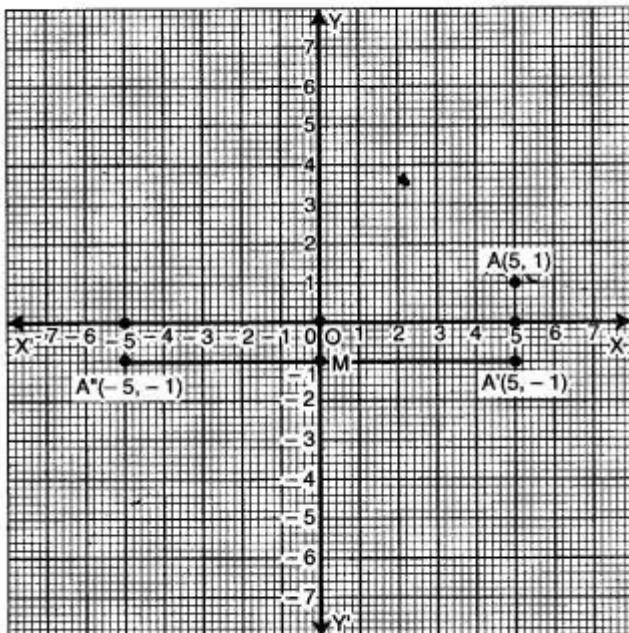
Question 9. Point A (5, 1) on reflection on X- axis is mapped as A'. Also A on reflection on Y- axis is mapped as A''.

- (i) Write the co-ordinates of A'.
- (ii) Write the co-ordinates of A''.
- (iii) Calculate the distance A' A''.

(iv) On which coordinate axis does the middle point M of A''A' lie ?

Solution : See graph,

- (i) $A' \rightarrow (5, -1)$. Ans.
- (ii) $A'' \rightarrow (-5, -1)$. Ans.



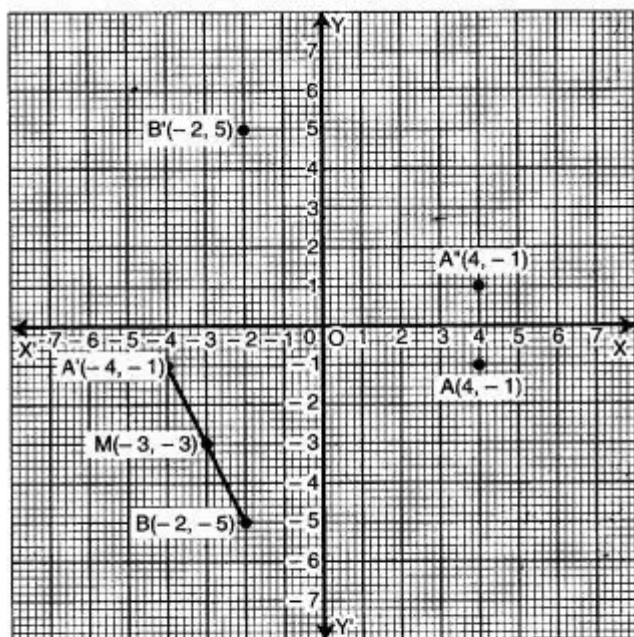
- (iii) Distance $A'A'' = 5 + 5 = 10$. Ans.
- (iv) M lies on Y-axis. Ans.

Question 10. Point A(4, - 1) is reflected as A' on Y-axis. Point B on reflection on X-axis is mapped as B' (- 2, 5).

- (i) Write the co-ordinates of A'.
- (ii) Write the co-ordinates of B.
- (iii) Write the co-ordinates of the middle point M of the segment A'B.

(iv) Write the co-ordinates of the point of reflection A'' of A on X-axis.

Solution :



- | | | |
|-------|----------------------------|------|
| (i) | $A' \rightarrow (-4, -1).$ | Ans. |
| (ii) | $B \rightarrow (-2, -5).$ | Ans. |
| (iii) | $M \rightarrow (-3, -3).$ | Ans. |
| (iv) | $A'' \rightarrow (4, 1).$ | Ans. |

Question 11. Point $A(2, -4)$ is reflected in origin as A' . Point $B(-3, 2)$ is reflected on X-axis as B' .

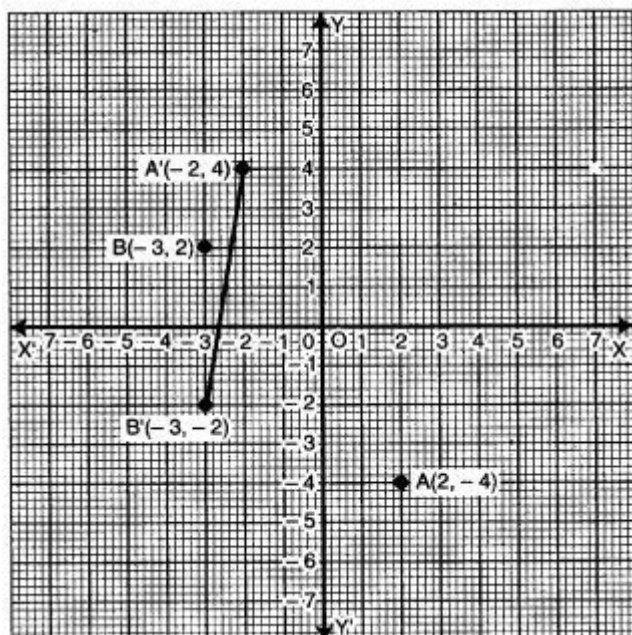
(i) Write the co-ordinates of A' .

(ii) Write the co-ordinates of B' .

(iii) Calculate the distance $A'B'$.

Give your answer correct to 1 decimal place, (do not consult tables).

Solution :



- (i) $A' \rightarrow (-2, 4)$. Ans.
- (ii) $B' \rightarrow (-3, -2)$. Ans.
- (iii) Distance $A'B' = \sqrt{(-3+2)^2 + (-2-4)^2}$
 $= \sqrt{1+36}$
 $= \sqrt{37}$
 $= 6.1$ Ans.

Question 12. (i) Point $P(a, b)$ reflected on the X-axis to $P'(5, 2)$. Write down the value of a and b .

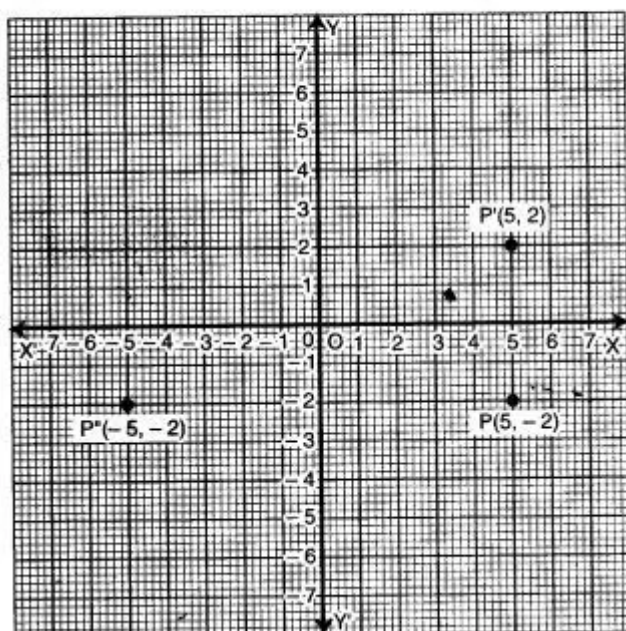
(ii) P'' is the image of P when reflected on the Y-axis. Write down the co-ordinates of P'' .

(iii) Name a single transformation that maps P' to P'' .

Solution : (i) The value of $a = 5$ and $b = -2$. Ans.

(ii) Co-ordinates of $P'' = (-5, -2)$. Ans.

(iii) $(x, y) \rightarrow (-x, -y)$. Ans.



Question 13. Points $(3, 0)$ and $(-1, 0)$ are invariant points under reflection in the line L_1 ; point $(0, -3)$ and $(0, 1)$ are invariant points on reflection in line L_2 .

(i) Write the equation of the line L_1 and L_2 .

(ii) Write down the images of points $P(3, 4)$ and $Q(-5, -2)$ on reflection in L_1 . Name the images as P' and Q' respectively.

(iii) Write down the images of P and Q on reflection in L_2 . Name the images as P'' and Q'' respectively.

Solution : (i) $(3, 0)$ and $(-1, 0)$ lies on X-axis, so these are invariant under reflection on the X-axis. Hence, L_1 lies on X-axis. So, equation of line L_1 , is $y = 0$.

$(0, -3)$ and $(0, 1)$ lies on Y-axis, so these are invariant under reflection on the Y-axis. So, equation of line L_2 is $y = 0$. Ans.

- (ii) $P' \Rightarrow (3, -4)$
 $Q' \Rightarrow (-5, 2)$ Ans.
- (iii) $P'' \Rightarrow (-3, 4)$
 $Q'' \Rightarrow (5, -2)$ Ans.

Question 14. A point $P(a, b)$ is reflected in the X-axis to $P'(2, -3)$. Write down the value of a & b . P'' is the image of P , when reflected on the Y-axis. Write down the co-ordinates of P'' when

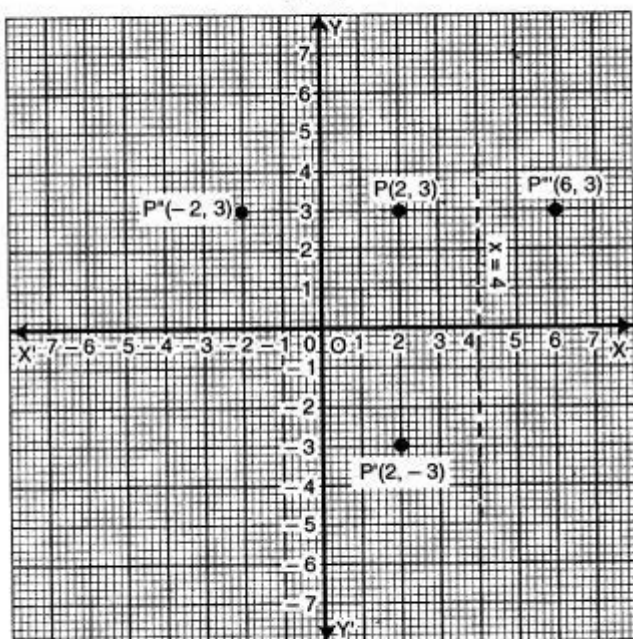
P is reflected in the line parallel to the Y-axis, such that $x = 4$.

Solution : Reflection of $P(a, b)$ on the X-axis

$P'(a, -b)$

$$\begin{aligned} \text{(i) } \therefore P'(a, -b) &= P'(2, -3) \\ +a &= +2, \therefore a = 2 \\ -b &= -3, \therefore b = 3 \\ P(2, 3) \end{aligned}$$

Ans.



(ii) P'' is the image of $P(2, 3)$ under reflection
 Y-axis = $P''(-2, 3)$. Ans.

(iii) P''' is the image of $P(2, 3)$ under reflection
 $x = 4 \Rightarrow P'''(6, 3)$. Ans.

Question 15. Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axis):

- Plot $A(4, 4)$, $B(4, -6)$ and $C(8, 0)$, the vertices of a triangle ABC .
- Reflect ABC on the y-axis and name it as $A'B'C'$.
- Write the coordinates of the images A' , B' and C' .
- Give a geometrical name for the figure $AA' C'B' BC$.

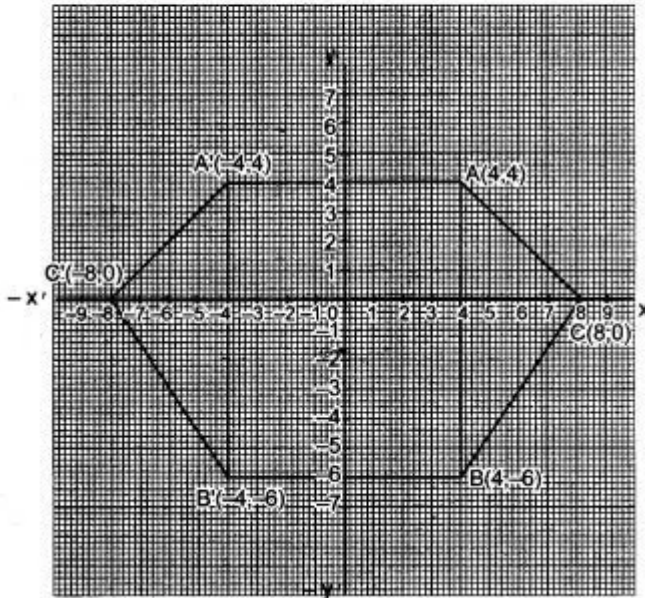
(v) Identify the line of symmetry of AA' C' B' BC.

Solution : (i) and (ii) see the given graph.

(iii) $A'(-4, 4)$, $B'(-4, -6)$, $C'(-8, 0)$ Ans.

(iv) AA' C' B' BC is a Hexagon. Ans.

(v) y -axis is the line of symmetry. Ans.



Question 16. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axis). P and Q have co-ordinates (0, 5) (- 2, 4).

(i) P is invariant when reflected in an axis. Name the axis.

(ii) Find the image of Q on reflection in the axis found in (i).

(iii) (0, k) on reflection in the origin is invariant. Write the value of k.

(iv) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin following by reflection in x-axis.

Solution : (i) The axis is y -axis or $x = 0$.

(ii) Image of 'Q'

$$\begin{aligned} Q' &= M_{x=0}(-2, 4) \\ &= (2, 4) \end{aligned}$$

(iii) $M_0(a, b) = (-a, -b)$

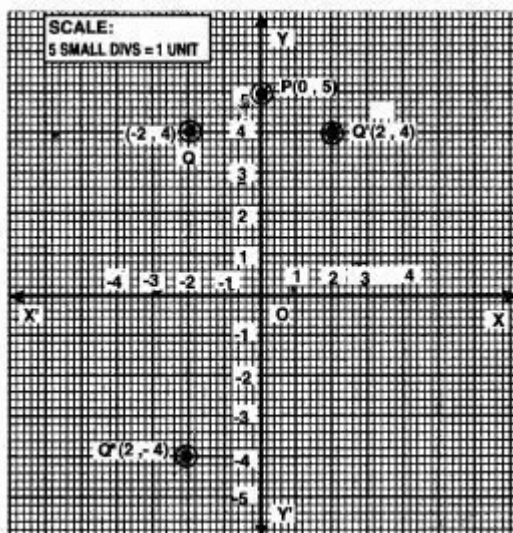
$$\therefore M_0(0, k) = (0, -k)$$

$$\therefore k = -k$$

$$\therefore 2k = 0$$

$$\therefore k = 0$$

$$\begin{aligned} \text{(iv)} \quad Q'' &= M_x M_0 Q \\ &= M_x M_0(-2, 4) \\ &= M_x(2, -4) \\ &= (-2, -4) \end{aligned}$$



Question 17. The image of triangle OXY under reflection in the origin, O is the triangle OX_1Y_1 , where $X_1(-3, -4)$ is the image of X and $Y_1(0, -5)$ is the image of Y.

(i) Draw a diagram to represent this information and write down the co-ordinates of X and Y.

(ii) What kind of figure is the quadrilateral XYX_1Y_1 ? Give reason for your answer. State, with a reason, whether the figure XYX_1Y_1 has any lines of symmetry.

(iii) Find the co-ordinates of X_2 , the image of X under reflection in the origin followed by reflection on the Y-axis.

(iv) Find the co-ordinates of Y_2 , the image of Y under reflection on the X-axis followed by reflection in the origin.

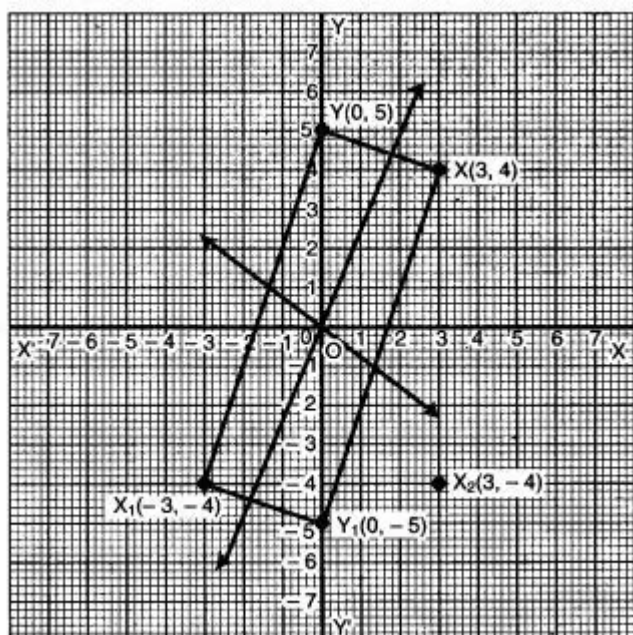
Solution : (i) $X_1 \rightarrow (-3, -4)$

$X \rightarrow (3, 4)$

$Y_1 \rightarrow (0, -5)$

$Y \rightarrow (0, 5).$

Ans.



(ii) Rectangle.

Reason : $X_1Y_1 = YX$

and $X_1Y = Y_1X$

Also each angle of the quadrilateral XX_1Y_1Y is 90° .

It has two lines of symmetry. These are the perpendicular bisectors of each pair of opposite sides.

Ans.

(iii) $X_2 = (3, -4)$.

Ans.

(iv) $Y_2 = (0, 5)$.

Ans.

Question 18. The point $P(3, 4)$ is reflected to P' in the x-axis and O' is the image of O (the Origin) in the line PP' . Find :

(i) The coordinates of P' and O' .

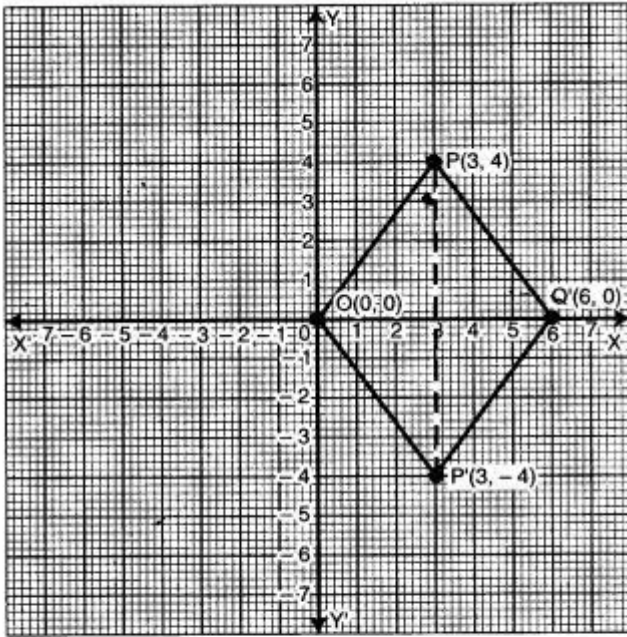
(ii) The length of segment PP' and OO' .

(iii) The perimeter of the quadrilateral $POP'O'$

(iv) What is the special name of the quadrilateral POP'O'.

Solution : (i) P' (3, -4), O'(6, 0)

Ans.



(ii), $PP' = \sqrt{(3-3)^2 + (-4-4)^2} = \sqrt{64} = 8$ units

$$OO' = \sqrt{(0-6)^2 + (0-0)^2}$$

$$= \sqrt{36} = 6 \text{ units.}$$

Ans.

(iii) Perimeter = 5 + 5 + 5 + 5 = 20 units. Ans.

(iv) Rhombus. Ans.

Question 19. Use graph paper for this question.

The points A (2, 3), B (4, 5) and C (7, 2) are the vertices of ΔABC .

(i) Write down the coordinates of A', B', C' if $\Delta A'B'C'$ is the image of ΔABC , when reflected in the origin.

(ii) Write down the co-ordinates of A'', B'', C'' if $\Delta A''B''C''$ is the image of ΔABC , when reflected in the x-axis.

(iii) Mention the special name of the quadrilateral BCC''B'' and find its area.

The points A (2, 3), B (4, 5) and C (7, 2) are the vertices of ΔABC .

(i) Write down the coordinates of A', B', C' if $\Delta A'B'C'$ is the image of ΔABC , when reflected in the origin.

(ii) Write down the co-ordinates of A'' , B'' , C'' if $A''B''C''$ is the image of ΔABC , when reflected in the x-axis.

(iii) Mention the special name of the quadrilateral $BCC''B''$ and find its area.

Solution : The point $A(2, 3)$, $B(4, 5)$ and $C(7, 2)$.

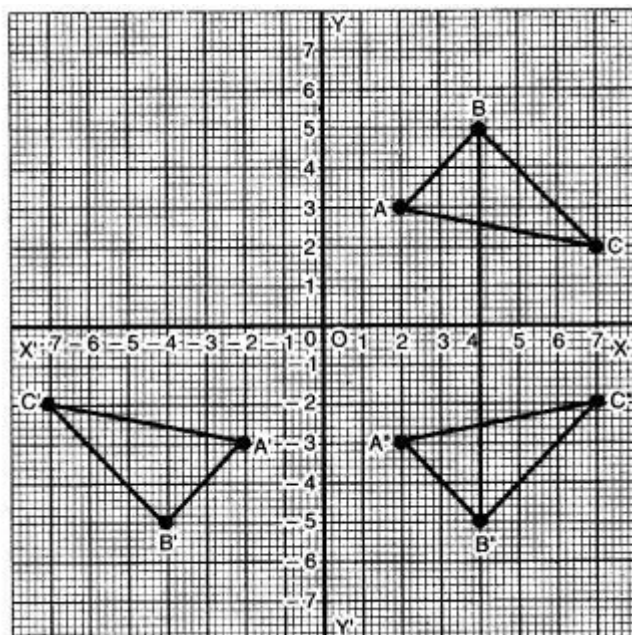
(i) Reflection in origin

$$(x, y) \xrightarrow{M_0} = (-x, -y)$$

$$\therefore A(2, 3) \xrightarrow{M_0} = A'(-2, -3)$$

$$B(4, 5) \xrightarrow{M_0} = B'(-4, -5)$$

$$C(7, 2) \xrightarrow{M_0} = C'(-7, -2)$$



(ii) Now A, B, C is reflected in X axis. Ans.

Reflection in X axis

$$(x, y) \xrightarrow{M_x} = (x, -y)$$

$$\therefore A(2, 3) \xrightarrow{M_x} = A''(2, -3)$$

$$B(4, 5) \xrightarrow{M_x} = B''(4, -5)$$

$$C(7, 2) \xrightarrow{M_x} = C''(7, -2) \text{ Ans.}$$

(iii) $BCC''B''$ is an isosceles trapezium.

$$CD = 7 - 4 = 3$$

$$CC'' = 2 + 2 = 4 \text{ and}$$

$$BB'' = 5 + 5 = 10$$

$$\text{Area of Trapezium} = \frac{1}{2} (CC'' + BB'') \times CD$$

$$= \frac{1}{2} (4 + 10) \times 3$$

$$= \frac{1}{2} \times 14 \times 3 = 21 \text{ sq. unit}$$

Question 20. Use a graph paper for this question (take 10 small divisions = 1 unit on both axis).

Plot the points P (3, 2) and Q (-3, -2), from P and Q draw perpendicular PM and QN on the X-axis.

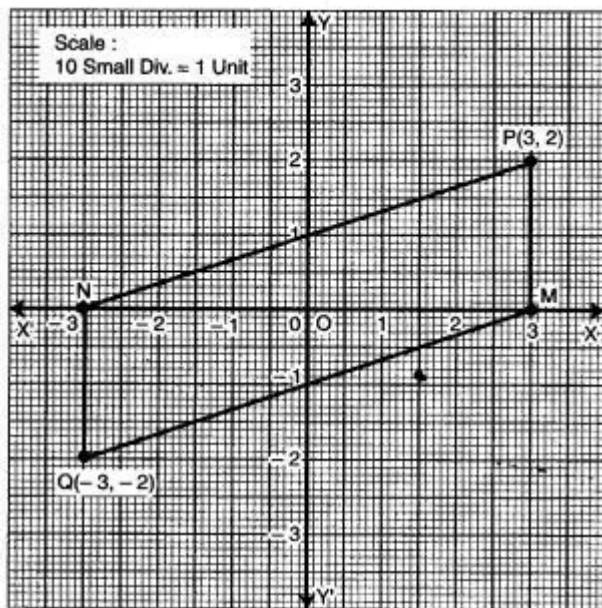
(i) Name the image of P on reflection at the origin.

(ii) Assign, the . special name to the geometrical figure. PMQN and find its area.

(iii) Write the co-ordinates of the point to which M is mapped on reflection in (i) X- axis,

(ii) Y-axis, (iii) origin.

Solution :



In the graph paper

(i) Q (-3, -2)

Ans.

(ii) Parallelogram;

$$\text{Area of } \Delta PMN = \frac{1}{2} PM \times MN$$

$$= \frac{1}{2} \times 2 \times 6$$

$$\therefore \text{Area of PMQN} = 2 \times \Delta PMN$$

$$= 2 \times 6$$

$$= 12 \text{ square unit}$$

Ans.

(iii) Co-ordinates of M (3, 0)

(i) (3, 0), (ii) (-3, 0), (iii) (-3, 0)

Ans.

Question 21. Using graph paper and taking 1 cm = 1 unit along both x-axis and y-axis.

(i) Plot the points A (-4, 4) and B (2, 2).

(ii) Reflect A and B in the origin to get the images A' and B' respectively.

(iii) Write down the co-ordinates of A' and B'.

(iv) Give the geometrical name for the figure ABA'B'.

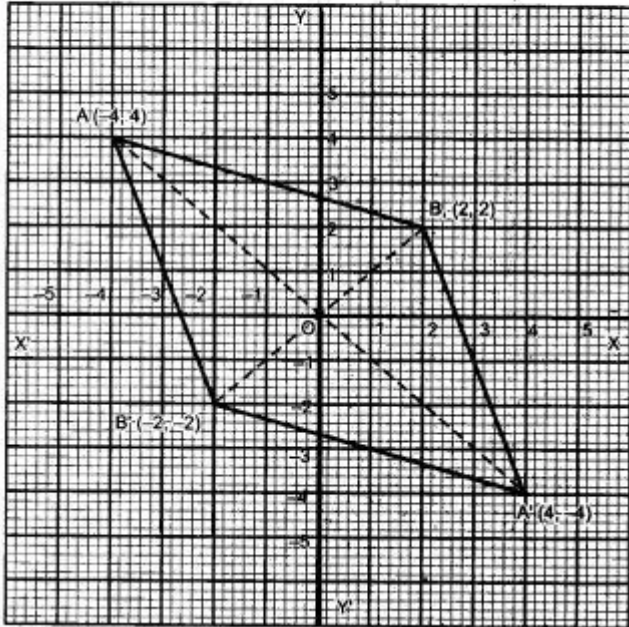
(v) Draw and name its lines of symmetry.

Solution : (i), (ii) (In the graph paper)

(iii) $A' (4, -4)$ $B' (-2, -2)$

(iv) Rhombus

(v) AA' , and BB'



Question 22. Use graph paper for this question.

The point $P (5, 3)$ was reflected in the origin to get the image P' .

(i) Write down the co-ordinates of P' .

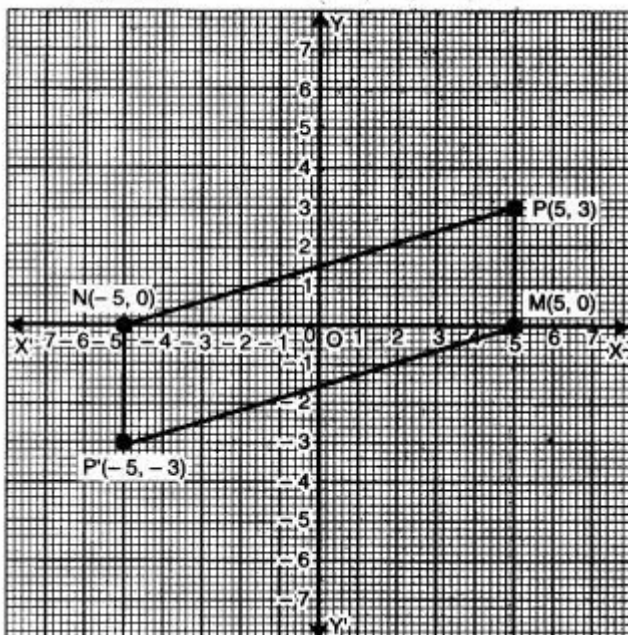
(ii) If M is the foot of the perpendicular from P to the X -axis, find the co-ordinates of M .

(iii) If N is the foot of the perpendicular from P' to the X -axis, find the co-ordinates of N .

(iv) Name the figure $PMP'N$.

(v) Find the area of the figure $PMP'N$.

Solution :

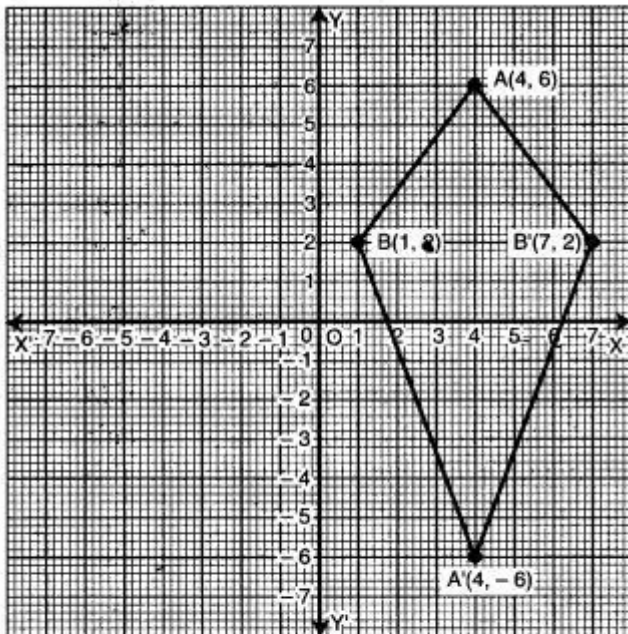


- (i) $P'(-5, -3)$.
- (ii) $M(5, 0)$.
- (iii) $N(-5, 0)$.
- (iv) The figure $PMP'N$ is a parallelogram.
- (v) The area of figure $PMP'N$

$$\begin{aligned}
 &= \frac{1}{2} \times 10 \times 3 + \frac{1}{2} \times 10 \times 3 \\
 &= 15 + 15 \\
 &= 30 \text{ sq. units.}
 \end{aligned}$$

Question 23. Use graph paper to answer this question:

- (i) Plot the points $A(4, 6)$ and $B(1, 2)$.
- (ii) A' is the image of A when reflected in X -axis,
- (iii) B' is the image of B when B is reflected in the line AA' .
- (iv) Give the geometrical name for the figure $ABA'B'$.



Solution : (i) In the graph paper.

(ii) $A(4, 6) \rightarrow A'(4, -6)$

(iii) $B(1, 2) \rightarrow B'(7, 2)$

(iv) $ABA'B'$ is a kite.

Ans.

Question 24. Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axis).

(i) Plot the points A (- 4, 2) and B (2, 4).

(ii) A' is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A'.

(iii) B' is the image of B when reflected in the line AA'. Write the coordinates of B'.

(iv) Write the geometric name of the figure ABA'B'.

(v) Name a line of symmetry of the figure formed.

Solution :

(i) On the graph.

(ii) Co-ordinates of A' = (4, 2).

(iii) Co-ordinates of B' = (2, 0).

(iv) Geometric name of Fig. ABA'B' is Kite

(v) Line of symmetry = AA'.

