Chapter 10. Matrices

Formulae

An $m \times n$ matrix usually written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots a_{1j} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2j} & \dots a_{2n} \\ a_{i1} & a_{i2} & \dots a_{ij} & \dots a_{in} \\ a_{m1} & a_{m2} & \dots a_{mj} & \dots a_{mn} \end{bmatrix} \begin{array}{l} \to 1^{st} \text{ row} \\ \to 2^{nd} \text{ row} \\ \\ \to m^{th} \text{ row} \end{array}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 1^{st} & 2^{nd} & n^{th} \\ \text{column} & \text{column} & \text{column} \end{array}$$

Generally the matrix is represented by

$$A = [a_{ij}]_{m \times n} \text{ or } A = [a_{ij}].$$

The numbers $a_{11}, a_{12}, \dots\dots, a_{mn}$ are called the elements of matrix A.

Order of Matrix = Numbers of Row × Numbers of Column

Equality of matrices. Two matrices $A = [a_{ij}]_{p \times q}$ $B = [b_{ij}]_{r \times s}$ are equal *i.e.*, A = B if and only if

(i) A and B are in same order *i.e.*, $p = r$ and $q = s$

(ii) Each element of A is equal to corresponding element of the other *i.e.*, $a_{ij} = b_{ij}$.

Addition of Matrices: Let A and B be two matrices each of order m × n. Then their sum A + B is a matrix of order m × n and is obtained by adding the corresponding elements of A and B.

Example : Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 5 \\ 1 & 2 \end{bmatrix}$$

then $$A + B = \begin{bmatrix} 1+0 & 2+5 \\ 3+1 & 4+2 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix}$$

Properties of Matrix Addition:

1. Matrix addition is commutative
   i.e., A + B = B + A
2. Matrix addition is associative for any three matrices A, B and C.
   A + (B + C) = (A + B) + C.
3. Existence of identity.
   A null matrix is identity element for addition.
   i.e., A + 0 = A = 0 + A.

4. Cancelation laws hold good in case of matrices.
   A + B = A + C ⇒ B = C.

Subtraction of Matrices:
For two matrices A and B of the same order, we define
A – B = A + (- B).

Example : If $A = \begin{bmatrix} 2 & 9 \\ 6 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}$.

Properties of Matrices Multiplication

1. Matrix multiplication is not commutative in general for any two matrices AB ≠ BA.
2. Matrix multiplication is associative
   i.e., (AB) C = A (BC) when both sides are defined.
3. Matrix multiplication is distributed over matrix addition
   i.e., A (B + C) = AB + AC
   (A + B) C = AC + BC.
4. If A is an n × n matrix then
   $I_nA = A = AI_n$
5. The product of two matrices can be the null matrix while neither of them is the null matrix.

Determine the Following

Question 1. Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$.

Write :
(i) the order of the matrix X.
(ii) the matrix X.

Solution : (i) $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

The order of matrix X = 2 × 1

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

so $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a + b \\ -3a + 4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow 2a + b = 7$ ...(1)

$-3a + 4b = 6$ ...(2)

From (1) and (2) $a = 2, b = 3$

$\Rightarrow X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Ans.

Question 2. If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible ? Give a reason. If yes, find AB.

Solution : $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}_{2\times 1}$

the product AB is possible as the number of columns in A are equal to the number of rows in B.

$$\text{Now} \quad AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 2 + 5\times 4 \\ 4\times 2 + (-2)\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 26 \\ 0 \end{bmatrix} \qquad \text{Ans.}$$

Question 3. Find the value of p and q if :

$$\begin{bmatrix} 2p+1 & q^2-2 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} p+3 & 3q-4 \\ 5q-q^2 & 0 \end{bmatrix}$$

Solution : $2p + 1 = p + 3;$

$$2p - p = 3 - 1$$

$$p = 2 \qquad \ldots(1)$$

$$q^2 - 2 = 3q - 4$$

$$q^2 - 3q + 2 = 0$$

$$q^2 - 2q - q + 2 = 0$$

$$q(q-2) - (q-2) = 0$$

$$(q-2)(q-1) = 0 \qquad \ldots(2)$$

$$5q - q^2 = 6$$

$$q^2 - 5q + 6 = 0$$

$$(q-3)(q-2) = 0 \qquad \ldots(3)$$

By equation (2) and (3)

$$q = 2$$

$$\Rightarrow \quad p = 2, q = 2 \qquad \text{Ans.}$$

Question 4. Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$.

Find the values of p and q.

Solution :

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, \ B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2$$

$$\Rightarrow \quad -2q = -8 \Rightarrow q = 4$$

$$p = 8 \Rightarrow p = 8.$$

Ans.

Question 5. Find x, y if

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}.$$

Solution :

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \quad 2y = -4, \ 2x = 6$$

$$\Rightarrow \quad y = -2, x = 3$$

Thus required values is $x = 3, y = -2$.

Question 6. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

find

(i) AB, (ii) BA.

Solution : (i) $AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2.1 + 4.(-2) & 2.3 + 4.5 \\ 3.1 + 2.(-2) & 3.3 + 2.5 \end{bmatrix}$$

$$AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix} \quad \text{Ans.}$$

(ii) $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1.2 + 3.3 & 1.4 + 3.2 \\ -2.2 + 5.3 & -2.4 + 5.2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix} \quad \text{Ans.}$$

Question 7. Find x and y, if

$$\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

Solution : $\begin{bmatrix} -3x + 4 \\ -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$$-3x + 4 = -5; \quad y = -10$$

$$x = 3 \;, \quad y = -10$$

Question 8. Given that $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ and B $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and that AB = A + B, find the values of a, b and c.

Solution : AB = A + B

$$\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$$\begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix} = \begin{bmatrix} 3 + a & b \\ 0 & 4 + c \end{bmatrix}$$

$3a = 3 + a \qquad 3b = b$

$2a = 3 \qquad 2b = 0$

$a = \frac{3}{2} \qquad b = 0$

$4c = 4 + c$

$3c = 4$

$c = \frac{4}{3}$. Ans.

Question 9. Find X and Y, if

$$\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

Solution :

$$\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$8x = 16, x = 2$$

$$9y = 9, y = 1$$

Question 10. $\begin{bmatrix} 2\sin 30^\circ & -2\cos 60^\circ \\ -\cot 45^\circ & \sin 90^\circ \end{bmatrix}$

$\begin{bmatrix} \tan 45^\circ & \sec 60^\circ \\ \operatorname{cosec} 30^\circ & \cos 0^\circ \end{bmatrix}$

Solution : $\begin{bmatrix} 2\sin 30^\circ & -2\cos 60^\circ \\ -\cot 45^\circ & \sin 90^\circ \end{bmatrix}$

$\begin{bmatrix} \tan 45^\circ & \sec 60^\circ \\ \operatorname{cosec} 30^\circ & \cos 0^\circ \end{bmatrix}$

$$= \begin{bmatrix} 2\times\frac{1}{2} & -2\times\frac{1}{2} \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$ Ans.

Question 11. Find the value of x given that $A^2 = B$

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Solution :

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

$$\because \quad A^2 = B \text{ (given)}$$

$$\therefore \quad \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad x = 36$$ Ans.

Question 12. Find x and y if

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}.$$

Solution : $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \quad 5x = 5$

$\Rightarrow \quad x = 1$

and $\quad 6y = 12$

$\Rightarrow \quad y = 2.$

Question 13. Find x and y, if

$$\begin{pmatrix} x & 3x \\ y & 4y \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}.$$

Solution : On multiplying L.H.S., matrices, we get $\begin{pmatrix} 5x \\ 6y \end{pmatrix}$, which is equal to R.H.S., matrix, evaluate their corresponding elements to get the values of x and y.

Hence, $x = 1, y = 2.$ Ans.

Question 14. Find x and y if :

$$\begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$$

Solution : $\begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$

$\Rightarrow \quad \begin{pmatrix} -3x + 4 \\ 0 - 10 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$

$\Rightarrow \quad -3x + 4 = -5$

$\Rightarrow \quad -3x = -5 - 4$

$\Rightarrow \quad -3x = -9$

$\Rightarrow \quad x = 3$

and $\quad y = -10$

Question 15. Construct *a* 2×2 matrix whose elements a_{ij} are given by

(i) $a_{ij} = 2i - j$ (ii) $\frac{(i+2j)^2}{2}$.

Solution : (i) We have $a_{ij} = 2i - j$

$$\text{Now} \quad \begin{aligned} a_{11} &= 2 \times 1 - 1 = 1 \\ a_{12} &= 2 \times 1 - 2 = 0 \\ a_{21} &= 2 \times 2 - 1 = 3 \\ a_{22} &= 2 \times 2 - 2 = 2 \end{aligned}$$

So the required matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \qquad \text{Ans.}$$

(ii) We have $a_{ij} = \frac{(i+2j)^2}{2}$

$$a_{11} = \frac{(1+2\times 1)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+2\times 2)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2\times 1)^2}{2} = \frac{16}{2} = 8$$

$$a_{22} = \frac{(2+2\times 2)^2}{2} = 18.$$

The required matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

Question 16. Given

$$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}, C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

Find the matrix X such that $A + 2X = 2B + C$.

Solution. $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix},$

$$C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A + 2X = 2B + C$$

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$2X = 2\begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}, X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix} \qquad \text{Ans.}$$

Question 17. Find matrices X and Y, if

$$X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix} \text{ and } X-Y=\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}.$$

Solution : We have

$$X+Y = \begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$$

and

$$X-Y = \begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

Now $(X+Y)+(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$

$$2X = \begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

Also $(X+Y)-(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}-3 & -6\\ 0 & 1\end{bmatrix}$

$$\Rightarrow \quad 2Y = \begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}$$

$$Y = \frac{1}{2}\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

Thus $X = \begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$ and $Y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$ Ans.

Question 18. If $A =\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ and $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

find $A^2 - 5A + 7I$.

Solution :

$$\begin{aligned}A^2 &= A\cdot A\\
&= \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\\
&= \begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}\\
&= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\Rightarrow A^2-5A+7I\\
&= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\
&= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}\\
&= \begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}\\
&= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\\
&= 0\end{aligned}$$

Ans.

Question 19. If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find matrix C such that 5A + 5B + 2C is a null matrix.

Solution : Let $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ and B $\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

Now $5A + 3B + 2C = 0$

$$\Rightarrow 5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45+3+2a & 5+15+2b \\ 35+21+2c & 40+36+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \quad 48 + 2a &= 0 \Rightarrow 2a = -48 \Rightarrow a = -24 \\ 20 + 2b &= 0 \Rightarrow 2b = -20 \Rightarrow b = -10 \\ 56 + 2c &= 0 \Rightarrow 2c = -56 \Rightarrow c = -28 \\ 76 + 2d &= 0 \Rightarrow 2d = -76 \Rightarrow d = -38 \end{aligned}$$

Thus $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$. **Ans.**

Question 20. Find x and y, if :

$$\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 2x \\ 1 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 5 \end{pmatrix} = 4\begin{pmatrix} 2 \\ y \end{pmatrix}$$

Solution : $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 2x \\ 1 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 5 \end{pmatrix} = 4\begin{pmatrix} 2 \\ y \end{pmatrix}$

$$\begin{pmatrix} 6x - 2 \\ -2x + 4 \end{pmatrix} + \begin{pmatrix} -8 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \\ 4y \end{pmatrix}$$

$$\begin{pmatrix} 6x - 2 - 8 \\ -2x + 4 + 10 \end{pmatrix} = \begin{pmatrix} 8 \\ 4y \end{pmatrix}$$

$$\begin{aligned} \text{Now} \quad 6x - 10 &= 8 \\ \therefore \quad 6x &= 18 \quad \therefore \; x = \frac{18}{6} = 3 \\ \text{and} \quad -2x + 14 &= 4y \\ -2 \times 3 + 14 &= 4y \\ \text{or} \quad 4y &= 14 - 6 = 8 \\ \therefore \quad y &= \frac{8}{4} = 2 \end{aligned}$$

$\therefore x = 3, y = 2.$ **Ans.**

Question 21. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 = 8A + kI$.

Soution : We have

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

And $$8A + kI = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$= \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

Thus $A^2 = 8A + kI$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$\Rightarrow \quad 1 = 8 + k$

$\Rightarrow \quad k = -7$

Also $\quad 56 + k = 49$

$\Rightarrow \quad k = -7.$

Question 22. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that

$$A^2 - 5A + 7I_2 = 0.$$

Solution : We have

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$-5A = \begin{bmatrix} (-5)\cdot 3 & (-5)\cdot 1 \\ (-5)\cdot(-1) & (-5)\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$$

$$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

So $A^2 - 5A + 7I_2$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So $A^2 - 5A + 7I_2 = 0$. Hence proved.

Question 23. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$, find matrix C if AC = B.

Solution. Let $C = \begin{bmatrix} a \\ b \end{bmatrix}$ then

$$AC = B$$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3a+b \\ -a+2b \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$$\Rightarrow 3a + b = 7 \quad \ldots(1)$$

$$-a + 2b = 0 \quad \ldots(2)$$

From equation (1),

$$6a + 2b = 14 \quad \ldots(3)$$

From (3) – (2) given

$$7a = 14$$

$$\Rightarrow a = 2$$

Put $a = 2$ in (1), we get

$$6 + b = 7$$

$$\Rightarrow b = 7 - 6 = 1$$

$$\therefore C = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Question 24. If $X = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that

$6X - X^2 = 9I$, where I is unit matrix.

Solution : Here

$$X^2 = X \cdot X$$

$$= \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$\text{L.H.S.} = 6X - X^2$$

$$= 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24-15 & 6-6 \\ -6+6 & 12-3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 9I = \text{R.H.S.}$ Hence proved.

Question 25. Evaluate x, y if

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

Solution :

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 3 \times 2x + (-2) \times 1 \\ -1 \times 2x + 4 \times 1 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2-8 \\ -2x+4+10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$6x - 10 = 8$$
$$\Rightarrow \quad 6x = 18, \; x = 3$$
$$-2x + 14 = 4y$$
$$\Rightarrow \quad -2 \times 3 + 14 = 4y$$
$$y = \frac{14-6}{4}$$
$$= \frac{8}{4} = 2$$
$$x = 3, y = 2. \quad \text{Ans.}$$

Question 26. Given

$$A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \text{evaluate } A^2 - 4A.$$

Solution : $A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$$
$$4A = 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$
$$A^2 - 4A = \begin{bmatrix} 9 & 14 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$
$$= \begin{bmatrix} 9-4 & 14-4 \\ 32-32 & 17-12 \end{bmatrix}$$
$$A^2 - 4A = \begin{bmatrix} 5 & 10 \\ 0 & 5 \end{bmatrix} \quad \text{Ans.}$$

Question 27. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$,

$B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

Find $A^2 + AC - 5B$.

Solution :

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 4+0 & 2-2 \\ 0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$5B = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$\therefore A^2 + AC - 5B$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}.$$

Question 28. Find the 2×2 matrix X which satisfies the equation.

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Solution : $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} - \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} \frac{-34}{2} & \frac{-32}{2} \\ \frac{-24}{2} & \frac{-10}{2} \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix} \text{ Ans.}$$

Question 29. If $A = \begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix}$, find matrix X such that $3A + 5B - 2X = 0$.

Soution: Let $X = \begin{bmatrix} x & y \\ z & u \end{bmatrix}$

We have $A = \begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix}$

$$3A = 3\begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 27 & 3 \\ 15 & 9 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix} = \begin{bmatrix} 5 & 25 \\ 35 & -55 \end{bmatrix}$$

Now $3A + 5B - 2X = 0$

$$\Rightarrow \begin{bmatrix} 27 & 3 \\ 15 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 25 \\ 35 & -55 \end{bmatrix} + \begin{bmatrix} -2x & -2y \\ -2z & -2u \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 27 + 5 - 2x & 3 + 25 - 2y \\ 15 + 35 - 2z & 9 - 55 - 2u \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 32 - 2x & 28 - 2y \\ 50 - 2z & -46 - 2u \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \quad 32 - 2x &= 0 \Rightarrow 2x = 32 \Rightarrow x = 16 \\ 28 - 2y &= 0 \Rightarrow 2y = 28 \Rightarrow y = 14 \\ 50 - 2z &= 0 \Rightarrow 2z = 50 \Rightarrow z = 25 \\ -46 - 2u &= 0 \Rightarrow 2u = -46 \Rightarrow u = -23 \end{aligned}$$

Question 30. Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$. Find $A^2 - A + BC$

Solution : $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$

and $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$.

$$\therefore \quad A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 12 & -8 + 6 \\ 24 - 18 & -12 + 9 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 + 2 & 0 - 2 \\ -2 - 1 & 3 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

Now $A^2 - A + BC$

$$= \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}.$$

Question 31. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$.
Find $A^2 + AB + B^2$.

Solution :

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times2 & 1\times0+0\times1 \\ 2\times1+1\times2 & 2\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$AB = A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+0\times-1 & 1\times3+0\times0 \\ 2\times2+1\times-1 & 2\times3+1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times-1 & 2\times3+3\times0 \\ -1\times2+0\times-1 & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$\therefore A^2 + AB + B^2$

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}.$$ Ans.

Prove the Following

Question 1. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that $A^2 - (a+d)A = (bc - ad)I$.

Solution : Here $A^2 - (a+d)A$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & cb+d^2 \end{bmatrix} - \begin{bmatrix} a^2+ad & ab+bd \\ ac+dc & ad+d^2 \end{bmatrix}$$

$$= \begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} = (bc-ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= (bc - ad)I.$ Hence proved

Question 2. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ verify that

(i) (AB)C = A(BC), (ii) A(B + C) = AB + AC.

Solution : (i)

$$AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -18+14 & 6+0 \\ -6+14 & 2+0 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ 8 & 2 \end{bmatrix}$$

Now, $$BC = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6+2 & 2+0 \\ -6+6 & 2+0 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 0 & 2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4+0 & 2+4 \\ 8+0 & -4+6 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ 8 & 2 \end{bmatrix}$$

Hence, (AB)C = A(BC).

(ii) $$B + C = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 1+1 \\ 2+2 & 3+0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+1 & 7+1 \\ 2+12 & 7-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$$

Hence $A(B+C) = AB + AC$.

Question 3. If $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$, then show that $(A - B)^2 \neq A^2 - 2AB + B^2$.

Solution. $A - B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3-1 & 1+2 \\ 2-5 & 1-3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

$$(A - B)^2 = (A - B)(A - B)$$

$$\Rightarrow \quad (A - B)^2 = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-9 & 6-6 \\ -6+6 & -9+4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$$

and $\quad A^2 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 9+2 & 3+1 \\ 6+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}$$

and $\quad B^2 = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1-10 & -2-6 \\ 5+15 & -10+9 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -8 \\ 20 & -1 \end{bmatrix}$$

and $\quad AB = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3+5 & -6+3 \\ 2+5 & -4+3 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ 7 & -1 \end{bmatrix}$$

Now $\quad A^2 - 2AB + B^2$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - 2\begin{bmatrix} 8 & -3 \\ 7 & -1 \end{bmatrix} + \begin{bmatrix} -9 & -8 \\ 20 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - \begin{bmatrix} 16 & -6 \\ 14 & -2 \end{bmatrix} + \begin{bmatrix} -9 & -8 \\ 20 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 11-16-9 & 4+6-8 \\ 8-14+20 & 3+2-1 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 2 \\ 14 & 4 \end{bmatrix}$$

Hence, from above calculations, we get

$(A - B)^2 \neq A^2 - 2AB + B^2$.