

Chapter 13. Similarity

Formulae

Similarities of triangles: When two triangles are similar, their corresponding angles are equal and corresponding sides are proportional.

For example :

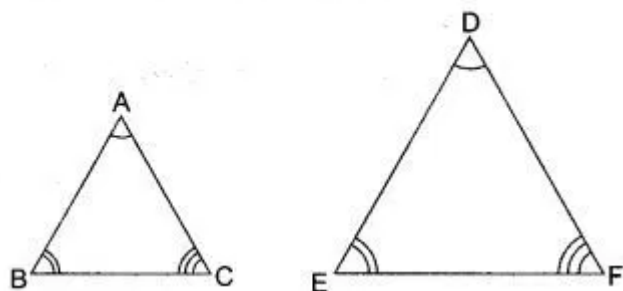
If ΔABC is similar to ΔDEF ,

i.e., $\Delta ABC \sim \Delta DEF$;

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$,

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

The sign ' $\sim$ ' is read as, 'is similar to'.



Axioms of similarity of triangles: (i.e., three similarity postulates for triangle)

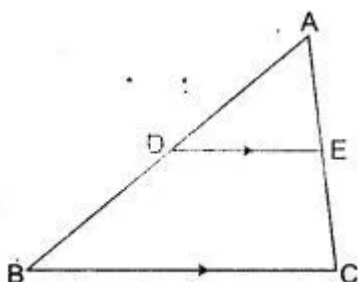
1. If two triangles have a pair of corresponding angles equal and the sides including them proportional; then the triangles are similar (SAS postulate).
2. If two triangles have two pairs of corresponding angles equal; the triangles are similar (AA or AAA postulate).
3. If two triangles have their three pairs of corresponding sides proportional, the triangles are similar (SSS postulate).

Basic Theorem of Proportionality:

1. A line drawn parallel to any side of a triangle, divides the other two sides proportionally. (Basic proportionality theorem).

In the given figure, $DE \parallel BC$

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{CE}$$

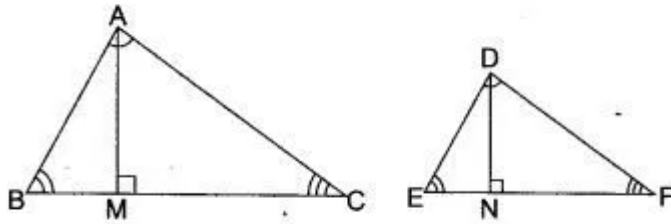


Conversely: If a line divides two sides of a triangle proportionally, the line is parallel to the third side.

$$\text{i.e., if } \frac{AD}{BD} = \frac{AE}{CE} \Rightarrow DE \parallel BC.$$

2. **Relation between the areas of two triangles: Theorem:** The areas of two similar triangles are proportional to the squares of their corresponding sides.

If $\triangle ABC \sim \triangle DEF$
 such that $\angle BAC = \angle EDF$,
 $\angle B = \angle E$ and $\angle C = \angle F$.



Then :

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Determine the Following

Question 1. The model of a building is constructed with scale factor 1:30.

- (i) If the height of the model is 80 cm, find the actual height of the building in metres.
- (ii) If the actual volume of a tank at the top of the building is 27 m^3 , find the volume of the tank on the top of the model.

Solution : (i) $\frac{\text{Height of model}}{\text{Height of actual building}} = \frac{1}{30}$

$$\frac{80}{H} = \frac{1}{30}$$

$$\Rightarrow H = 2,400 \text{ cm} = 24 \text{ m. Ans.}$$

(ii) $\frac{\text{Volume of model}}{\text{Volume of tank}} = \left(\frac{1}{30}\right)^3$

$$\frac{V}{27} = \frac{1}{27,000}$$

$$V = \frac{1}{1,000} \text{ m}^3$$

$$= 1,000 \text{ cm}^3. \quad \text{Ans.}$$

Question 2. Triangles ABC and DEF are similar.

(i) If area (ΔABC) = 16 cm^2 , area (ΔDEF) = 25 cm^2 and $BC = 2.3 \text{ cm}$ find EF .

(ii) If area (ΔABC) = 9 cm^2 , area (ΔDEF) = 64 cm^2 and $DE = 5.1 \text{ cm}$, find AB .

(iii) If $AC = 19 \text{ cm}$ and $DF = 8 \text{ cm}$, find the ratio between the areas of two triangles.

(iv) If area (ΔABC) = 36 cm^2 , area (ΔDEF) = 64 cm^2 and $DE = 6.2 \text{ cm}$, find AB .

Solution : (i) We have

$$\text{area } (\Delta ABC) = 16 \text{ cm}^2$$

$$\text{area } (\Delta DEF) = 25 \text{ cm}^2$$

and $BC = 2.3 \text{ cm}$

Since, $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{BC^2}{EF^2}$

$$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$$

$$\Rightarrow \frac{2.3}{EF} = \frac{4}{5}$$

$$\Rightarrow 4 EF = 5 \times 2.3$$

$$\Rightarrow EF = \frac{11.5}{4}$$

$$\Rightarrow EF = 2.875 \text{ cm.} \quad \text{Ans.}$$

(ii) We have

$$\text{area } (\Delta ABC) = 9 \text{ cm}^2$$

$$\text{area } (\Delta DEF) = 64 \text{ cm}^2$$

and $DE = 5.1 \text{ cm}$

Since, $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AB^2}{DE^2}$

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{AB}{DE} = \frac{3}{8}$$

$$\Rightarrow \frac{AB}{5.1} = \frac{3}{8}$$

$$\Rightarrow AB = \frac{3}{8} \times 5.1 = \frac{15.3}{8}$$

$$\Rightarrow AB = 1.9125 \text{ cm.} \quad \text{Ans.}$$

(iii) In ΔABC and ΔDEF , $AC = 19 \text{ cm}$, $DF = 8 \text{ cm}$.

Since, $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{(8)^2} = \frac{361}{64}$

Hence, the required ratio is $361 : 64$. Ans.

(iv) Area (ΔABC) = 36 cm^2

Area (ΔDEF) = 64 cm^2 .

$DE = 6.2 \text{ cm}$

$AB = ?$

We have

$$\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2}$$

$$\Rightarrow \frac{AB}{6.2} = \frac{6}{8}$$

$$\Rightarrow AB = \frac{6 \times 6.2}{8}$$

$$\Rightarrow AB = 4.65 \text{ cm.}$$

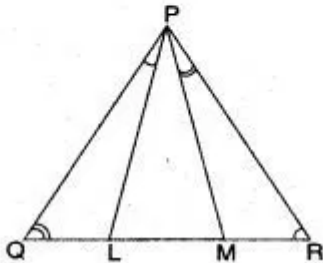
Prove the Following

Question 1. In $\triangle PQR$, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$.

Prove that : (i) $\triangle PQL \sim \triangle RPM$

(ii) $QL \cdot RM = PL \cdot PM$

(iii) $PQ^2 = QR \cdot QL$.



Solution : (i) Consider

$\triangle PQL$ and $\triangle RPM$

Since $\angle PQL = \angle RPM$

and $\angle QPL = \angle RPM$ and

$\angle QPL = \angle PRM$

By A. A. Criterion

$\triangle PQL \sim \triangle RPM$.

Hence proved.

(ii) In $\triangle PQL \sim \triangle RPM$

$$\frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{MR}$$

then
$$\frac{QL}{PM} = \frac{PL}{MR}$$

$$QL \cdot MR = PL \cdot PM. \quad \text{Hence Proved.}$$

(iii) In $\triangle PQR$ and $\triangle LQP$

$$\angle PQR = \angle LQP$$

$$PQ = PQ$$

Hence $\triangle PQR \sim \triangle LQP$

$$\frac{QR}{PQ} = \frac{PQ}{LQ}$$

$$PQ^2 = QR \cdot QL.$$

Hence proved.

Question 2. D and E are points on the sides AB and AC respectively of a ΔABC such that $DE \parallel BC$ and divides ΔABC into two parts, equal in area. Find $\frac{BD}{AB}$.

Solution: We have

$$\text{area}(\Delta ADE) = \text{area}(\text{trapezium } BCED)$$

$$\Rightarrow \text{area}(\Delta ADE) + \text{area}(\Delta ADE)$$

$$= \text{area}(\text{trapezium } BCED)$$

$$+ \text{area}(\Delta ADE)$$

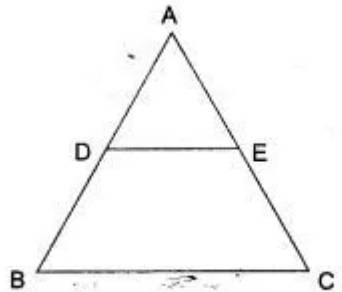
$$\Rightarrow 2 \text{ area}(\Delta ADE) = \text{area}(\Delta ABC) \quad \dots(i)$$

In ΔADE and ΔABC , we have

$$\angle ADE = \angle B, \quad [\because DE \parallel BC]$$

$$\therefore \angle AED = \angle C \text{ (corresponding angles)}$$

$$\text{and } \angle A = \angle A, \quad [\text{Common}]$$



$$\therefore \Delta ADE \sim \Delta ABC$$

$$\Rightarrow \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{\text{area}(\Delta ADE)}{2 \text{ area}(\Delta ADE)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{AD}{AB} \right)^2$$

$$\Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow AB = \sqrt{2} AD$$

$$\Rightarrow AB = \sqrt{2} (AB - BD)$$

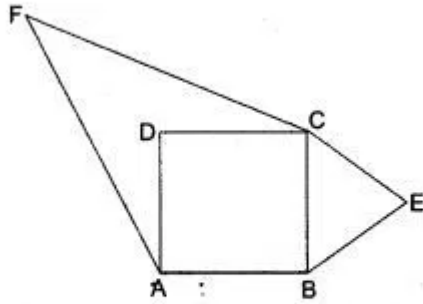
$$\Rightarrow (\sqrt{2} - 1) AB = \sqrt{2} BD$$

$$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$= \left(\frac{2 - \sqrt{2}}{2} \right)$$

Question 3. Prove that the area of the triangle BCE described on one side BC of a square ABCD as base is one half of the area of similar triangle ACF described on the diagonal AC as base.

Solution : ABCD is a square ΔBCE is described on side BC is similar to ΔACF described on diagonal AC.



Since, ABCD is a square. Therefore

$$AB = BC = CD = DA$$

and $AC = \sqrt{2} BC$

$$[\because \text{Diagonal} = \sqrt{2} (\text{side})]$$

Now, $\Delta BCE \sim \Delta ACF$

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$$

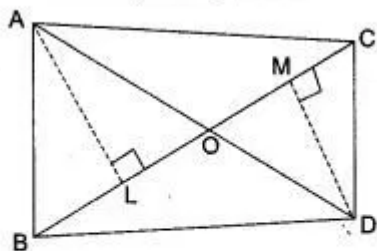
$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{(\sqrt{2} BC)^2} = \frac{1}{2}$$

$$\Rightarrow \text{Area}(\Delta BCE) = \frac{1}{2} \text{area}(\Delta ACF).$$

Hence proved.

Question 4. In figure ABC and DBC are two triangles on the same base BC. Prove that

$$\frac{\text{Area } (\triangle ABC)}{\text{Area } (\triangle DBC)} = \frac{AO}{DO}$$



Solution : In $\triangle AOL$ and $\triangle DOM$

$$\angle ALO = \angle DMO, \quad (90^\circ \text{ each})$$

$$\angle AOL = \angle DOM,$$

(Vertically opposite 2 sides)

(Vert. opp-angles)

$$\therefore \triangle AOL \sim \triangle DOM$$

$$\therefore \frac{AL}{DM} = \frac{AO}{DO} \quad \dots(1)$$

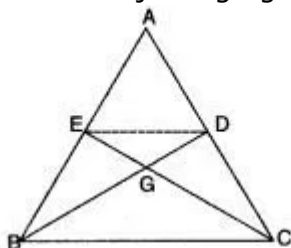
If two Δ 's are similar the ratio between their corresponding sides is the same.

$$\text{Now, } \frac{\text{area } (\triangle ABC)}{\text{area } (\triangle DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM} = \frac{AL}{DM}$$

From (1), we get

$$\frac{\text{area } (\triangle ABC)}{\text{area } (\triangle DBC)} = \frac{AO}{DO} \quad \text{Hence proved.}$$

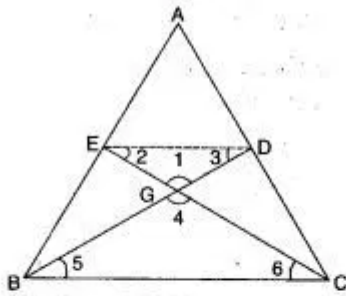
Question 5. In the adjoining figure, the medians BD and CE of a $\triangle ABC$ meet at G. Prove that



(i) $\triangle EGD \sim \triangle CGB$ and

(ii) $BG = 2GD$ for (i) above.

Solution : Since D and E are mid-point of AC and AB respectively in $\triangle ABC$, ED is parallel to BC.



(i) In Δ s EGD and CGB,

$$\angle EGD = \angle CGB$$

(Vertically opp., angles)

$$\angle EDG = \angle CBG \quad (\text{Alternate angles})$$

So, Δ EGD $\sim$ Δ CGB. Hence proved.

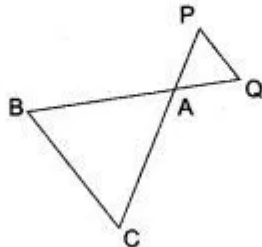
$$(ii) \therefore \frac{BG}{GD} = \frac{BC}{DE}$$

$$\text{But } \frac{BC}{DE} = 2, \text{ So } \frac{BG}{GD} = 2$$

$$\Rightarrow BG = 2GD. \quad \text{Hence proved.}$$

Figure Based Questions

Question 1. In the adjoining figure, Δ ACB $\sim$ Δ APQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm find the area (Δ ACB) : area (Δ APQ).



Solution : Δ ACB $\sim$ Δ APQ.

$$\begin{aligned} \text{Then, } \frac{\text{area } (\Delta \text{ ACB})}{\text{area } (\Delta \text{ APQ})} &= \frac{BC^2}{PQ^2} \\ &= \frac{(10)^2}{(5)^2} \\ &= \frac{100}{25} \\ &= \frac{4}{1} \end{aligned}$$

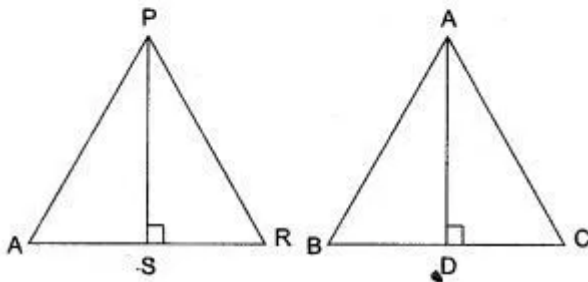
Required ratio is 4 : 1.

Question 2. Two isosceles triangle have equal vertical angles and their areas are in the ratio of 36 : 25. Find the ratio between their corresponding heights.

Solution : $\triangle ABC$ and $\triangle PQR$ be the two isosceles triangles such that

$$\angle A = \angle P.$$

Then $\triangle ABC \sim \triangle PQR$.



Let AD and PS be their heights then

$$\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{36}{25} = \frac{AD^2}{PS^2}$$

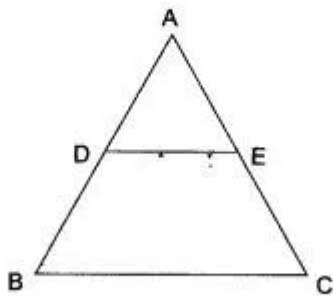
$$\Rightarrow \frac{AD}{PS} = \frac{6}{5}$$

$$\Rightarrow AD : PS = 6 : 5. \quad \text{Ans.}$$

Question 3. In $\triangle ABC$, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of $\triangle ADE$ and $\triangle ABC$.

Solution : Since, D and E are the mid-points of AB and AC respectively.

Therefore, $DE \parallel BC$.



Consequently,

$$\triangle ADE \sim \triangle ABC$$

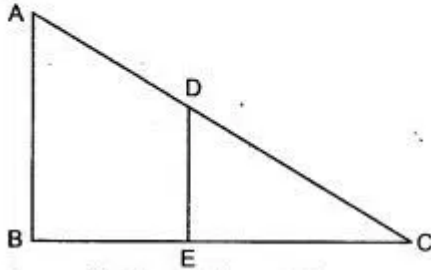
$$\begin{aligned} \Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle ABC)} &= \frac{AD^2}{AB^2} \\ &= \frac{AD^2}{(2AD)^2} \\ &= \frac{1}{4} \quad (\because AB = 2AD) \end{aligned}$$

Question 4. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If $AB = 6$ cm; $DE = 4$ cm and $AC = 15$ cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC .



Solution : (i) Given $AB \perp BC$

$DE \perp BC$

To prove $\Delta ABC \sim \Delta DEC$

Proof : In ΔABC and ΔDEC

$$\angle ABC = \angle DEC = 90^\circ$$

each (given)

$$\angle C = \angle C \quad \text{{common}}$$

$$\therefore \Delta ABC \sim \Delta DEC$$

{A.A criteria}

Hence proved.

(ii) $AB = 6$ cm, $DE = 4$ cm

$AC = 15$ cm, $CD = ?$

Since $\Delta ABC \sim \Delta DEC$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{CD}$$

{Corresponding sides of similar Δ 's are proportional}

$$\therefore \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow CD = \frac{15 \times 4}{6} = 10. \quad \text{Ans.}$$

$$(iii) \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2} \quad \text{{Area theorem}}$$

$$= \frac{36}{16} = \frac{9}{4} \text{ or } 9 : 4. \quad \text{Ans.}$$

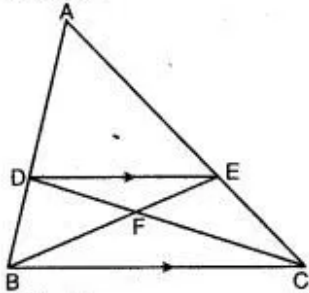
Question 5. In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

(i) Determine the ratios $\frac{AD}{AB}$, $\frac{DE}{BC}$.

(ii) Prove that $\triangle DEF$ is similar to $\triangle CBF$.

Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of $\triangle DEF$ and $\triangle BFC$?



Solution : (i) Given

$$DE \parallel BC$$

and

$$\frac{AD}{DB} = \frac{3}{2}$$

In $\triangle ADE$ and $\triangle ABC$,

$$\angle A = \angle A, \text{ (Common Angles)}$$

$$\angle D = \angle B$$

(Corresponding Angles)

$$\therefore \triangle ADE \sim \triangle ABC$$

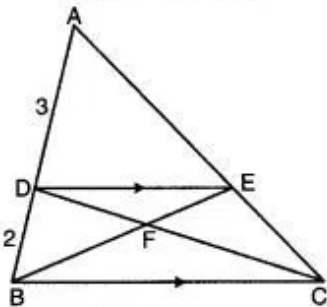
(by A.A. criterion)

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\text{Now, } \frac{AD}{AB} = \frac{AD}{AD + DB}$$

$$= \frac{3}{3 + 2} = \frac{3}{5}$$

$$\therefore \frac{AD}{AB} = \frac{3}{5} = \frac{DE}{BC} \quad \text{Ans.}$$



(ii) In $\triangle DEF$ and $\triangle CBF$,

$$\angle FDE = \angle FCB \text{ (Alternate Angle)}$$

$$\angle DFE = \angle BFC$$

(Vertically Opposite Angle)

$$\therefore \triangle DEF \sim \triangle CBF$$

(by A.A. criterion)

Hence proved.

$$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$$

$$\therefore \frac{EF}{FB} = \frac{3}{5} \quad \text{Ans.}$$

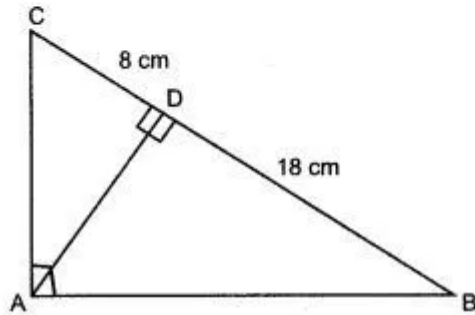
$$\text{(iii) } \frac{\text{Area of } \triangle DFE}{\text{Area of } \triangle CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25} \quad \text{Ans.}$$

Question 6. In the adjoining figure ABC is a right angled triangle with $\angle BAC = 90^\circ$, and $AD \perp BC$.

(i) Prove $\triangle ADB \sim \triangle CDA$.

(ii) If $BD = 18$ cm, $CD = 8$ cm find AD .

(iii) Find the ratio of the area of $\triangle ADB$ is to area of $\triangle CDA$.



Solution : (i) In $\triangle ADB$ and $\triangle ADC$

$$AD = AD \quad (\text{self})$$

$$\angle ADC = 90^\circ \quad (AD \perp BC)$$

$$\angle ADB = 90^\circ \quad (AD \perp BC)$$

$$\text{then } \angle ADC = \angle ADB$$

$$\text{so, } \triangle ADB \sim \triangle ADC$$

(By AAA similarity)

$$\text{or } \triangle ADB \sim \triangle CDA \quad \text{Hence proved.}$$

$$(ii) \because \triangle ADB \sim \triangle CDA$$

$$\therefore \frac{AD}{BD} = \frac{CD}{AD}$$

(corresponding parts of similar
 Δ 's are proportional)

$$\text{or } AD^2 = BD \times CD$$

$$\Rightarrow AD^2 = 18 \times 8 \left(\begin{array}{l} BD = 18 \\ CD = 8 \end{array} \text{ (given)} \right)$$

$$\Rightarrow AD^2 = 144$$

$$AD = 12 \text{ cm}$$

$$(iii) \frac{\text{Area of } \triangle ADB}{\text{Area of } \triangle CDA} = \frac{BD^2}{AD^2}$$

(Area theorem of
similar triangles)

$$= \frac{18^2}{12^2} = \frac{18 \times 18}{12 \times 12}$$

$$= \frac{3 \times 3}{2 \times 2} = \frac{9}{4}$$

$$\Rightarrow \text{area } (\triangle ADB) : \text{area } (\triangle CDA) = 9 : 4. \quad \text{Ans.}$$

Question 7. Equilateral triangles are drawn on the sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

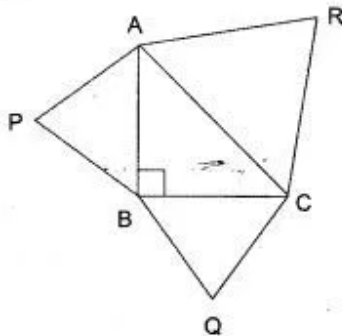
Solution :

Given. A right angled triangle ABC with right angle at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

To Prove.

Area (Δ PAB) + Area (Δ QBC) = Area (Δ RAC).

Proof. Since, triangles PAB, QBC and RAC are equilateral. Therefore they are equiangular and hence similar.



$$\begin{aligned} \therefore \frac{\text{area } (\Delta \text{ PAB})}{\text{area } (\Delta \text{ RAC})} + \frac{\text{area } (\Delta \text{ QBC})}{\text{area } (\Delta \text{ RAC})} &= \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} \\ &= \frac{AB^2 + BC^2}{AC^2} \\ &= \frac{AC^2}{AC^2} = 1 \end{aligned}$$

$$\left[\begin{array}{l} \because \Delta \text{ ABC is a right angled triangle with } \angle B = 90^\circ \\ \therefore AC^2 = AB^2 + BC^2 \end{array} \right]$$

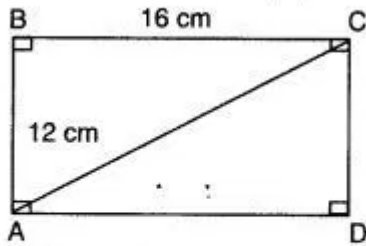
$$\Rightarrow \frac{\text{area } (\Delta \text{ PAB}) + \text{area } (\Delta \text{ QBC})}{\text{area } (\Delta \text{ RAC})} = 1$$

$$\Rightarrow \text{area } (\Delta \text{ PAB}) + \text{area } (\Delta \text{ QBC}) = \text{area } (\Delta \text{ RAC}).$$

Question 8: On a map drawn to scale of 1 : 2,50,000 a rectangular plot of land ABCD has the following measurement AB = 12 cm, BC = 16 cm angles A, B, C, and D are 90° each. Calculate:

- (i) The diagonal distance of the plot of land in
(ii) Actual length of diagonal.

Solution : (i) Here $k = \frac{1}{2,50,000}$



Length of diagonal (on map)

$$\begin{aligned}
 &= \sqrt{AB^2 + BC^2} \\
 &= \sqrt{12^2 + 16^2} \\
 &= \sqrt{400} = 20 \text{ cm}
 \end{aligned}$$

(ii) Length of diagonal on map.

$$= k \times \text{Actual length of the diagonal}$$

$$\Rightarrow 20 = \frac{1}{2,50,000} \times \text{Actual length of the diagonal}$$

$\Rightarrow$ Actual length of diagonal

$$= 20 (2,50,000) \text{ cm}$$

$$= 5000000 \text{ cm}$$

$$= 50 \text{ km} \quad \text{Ans.}$$

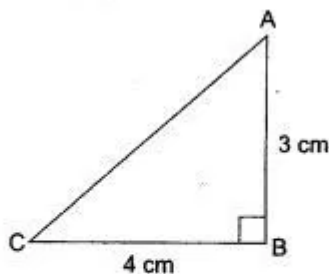
Question 9. On a map drawn to a scale of 1 : 2,50,000, a triangular plot of land has the following measurements, $AB = 3 \text{ cm}$, $BC = 4 \text{ cm}$, $\angle ABC = 90^\circ$. Calculate :

(i) The actual length of AB in kms.

(ii) The area of plot in sq. kms.

Solution : The scale of a map = 1 : 2,50,000 that is

$$\begin{aligned}
 1 \text{ cm} &= \frac{2,50,000}{1,000 \times 100} \\
 &= 2.5 \text{ kms}
 \end{aligned}$$



(i) Actual length of AB

$$= 3 \text{ cm} = 3 \times 2.5 \text{ kms}$$

$$= 7.5 \text{ kms} \quad \text{Ans.}$$

(ii) Area of triangular plot

$$= \frac{1}{2} AB \times BC$$

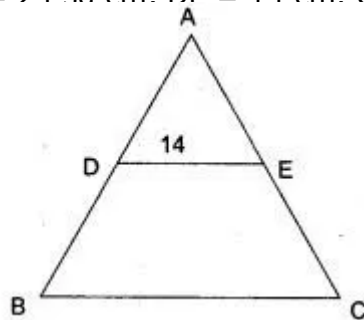
$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ cm}^2$$

$$= 6 \times 2.5 \times 2.5 \text{ km}^2$$

$$= 37.5 \text{ km}^2. \quad \text{Ans.}$$

Question 10. In the adjoining figure. BC is parallel to DE, area of $\triangle ABC = 25 \text{ sq cm}$, area of trapezium BCED = 24 sq cm. DE = 14 cm. Calculate the length of BC.



Solution. Given : Area of $\triangle ABC = 25 \text{ cm}^2$

Area of trapezium BCED = 24 cm^2

and $DE = 14 \text{ cm}$

$$\begin{aligned} \therefore \text{Area of } \triangle ADE &= \text{area of } \triangle ABC \\ &\quad - \text{area of trap. BCED} \\ &= 25 - 24 \\ &= 1 \text{ cm}^2 \end{aligned}$$

$$\therefore \triangle ABC \sim \triangle ADE$$

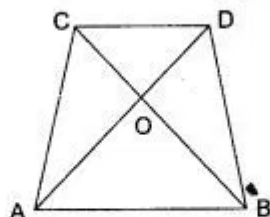
$$\therefore \frac{\text{Area of } (\triangle ABC)}{\text{Area of } (\triangle ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{1} = \frac{BC^2}{14^2}$$

$$\frac{BC}{14} = \frac{5}{1}$$

$$BC = 5 \times 14 = 70 \text{ cm} \quad \text{Ans.}$$

Question 11. In fig. ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2DC$. Determine the ratio between the areas of ΔAOB and ΔCOD .



Solution : In triangles AOB and COD, we have

$$\angle AOB = \angle COD,$$

[Vertically opposite angles]

and $\angle OAB = \angle OCD,$

[Corresponding angles]

So, by AA-criterion of similarity, we have

$$\Delta AOB \sim \Delta COD$$

$$\Rightarrow \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{AB^2}{DC^2}$$

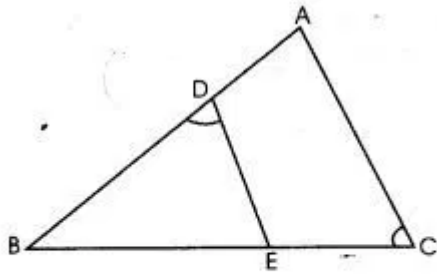
$$\begin{aligned} \Rightarrow \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} &= \frac{(2DC)^2}{(DC)^2} \\ &= \frac{4}{1} \end{aligned}$$

Hence, $\text{area}(\Delta AOB) : \text{area}(\Delta COD) = 4 : 1$.

Question 12. In the given figure ABC is a triangle with $\angle EDB = \angle ACB$.

(i) Prove that $\Delta ABC \sim \Delta EBD$.

(ii) If $BE = 6$ cm, $EC = 4$ cm, $BD = 5$ cm and area of $\Delta BED = 9$ cm². Calculate the length of AB and area of ΔABC .



Solution : (i) In ΔABC and ΔEBD ,

$$\angle ACB = \angle EDB \text{ (given)}$$

$$\angle ABC = \angle EBD \text{ (common)}$$

$$\therefore \Delta ABC \sim \Delta EBD$$

Hence Proved.

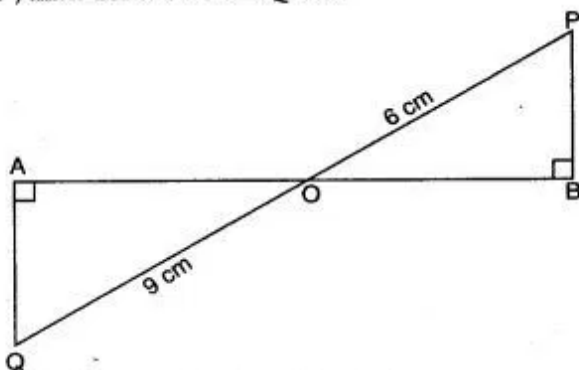
(ii) We have, $\frac{AB}{BE} = \frac{BC}{BD}$

$$\Rightarrow AB = \frac{6 \times 10}{5} = 12 \text{ cm. Ans.}$$

(iii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$$\begin{aligned} \Rightarrow \text{Area of } \Delta ABC &= \left(\frac{12}{6}\right)^2 \times 9 \text{ cm}^2 \\ &= 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2. \end{aligned}$$

Question 13. In the figure given below, PB and QA are perpendiculars to the line segment AB. If PO = 6 cm, QO = 9 cm and the area of $\Delta POB = 120 \text{ cm}^2$, find the area of ΔQOA .



Solution : PO = 6 cm, QO = 9 cm.

$$\text{Area of } \Delta POB = 120$$

In ΔPOB and ΔQOA ,

$$\angle B = \angle A \quad (\text{each } 90^\circ)$$

$$\angle POB = \angle QOA$$

(Opposite vertical angle)

$$\therefore \Delta POB \sim \Delta QOA$$

In similar Δ 's $\frac{\text{Area of } \Delta QOA}{\text{Area of } \Delta POB} = \frac{OQ^2}{OP^2}$

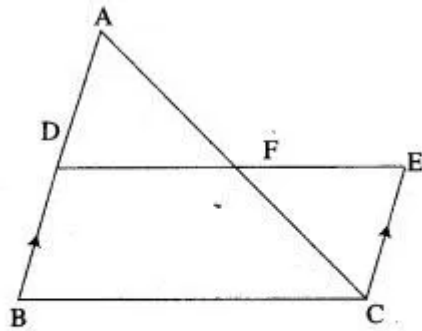
$$\frac{\text{Area of } \Delta QOA}{120} = \frac{9^2}{6^2}$$

$$\text{Area of } \Delta QOA = \frac{81 \times 120}{36}$$

$$= 27 \times 10$$

$$\text{Area of } \Delta QOA = 270 \text{ cm}^2 \text{ . Ans.}$$

Question 14. In the given figure ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.



(i) Prove that $\triangle ADF \sim \triangle CEF$

(ii) Find AD if $CE = 6$ cm.

(iii) If DF is parallel to BC find area of $\triangle ADF$:
area of $\triangle ABC$.

Solution : (i) In $\triangle ADF$ and $\triangle CFE$

$$\angle DAF = \angle FCE$$

(alternate angles)

$$\angle AFD = \angle CFE$$

(vertically opp. angles)

$$\angle ADF = \angle CEF$$

$$\therefore \triangle ADF \sim \triangle CEF \text{ (by A.A.)}$$

Hence Proved

(ii) $\triangle ADF \sim \triangle CEF$

$$\therefore \frac{AD}{CE} = \frac{AF}{FC}$$

$$FC = AC - AF$$

$$= 8 - 5 = 3$$

$$\therefore \frac{AD}{6} = \frac{5}{3}$$

$$\Rightarrow AD = 10 \text{ cm.}$$

(iii) $DF \parallel BC \therefore \triangle ADF \sim \triangle ABC$

$$\therefore \angle D = \angle B \text{ and } \angle F = \angle C.$$

$$\therefore \frac{\text{Ar. of } \triangle ADF}{\text{Ar. of } \triangle ABC} = \frac{AF^2}{AC^2}$$

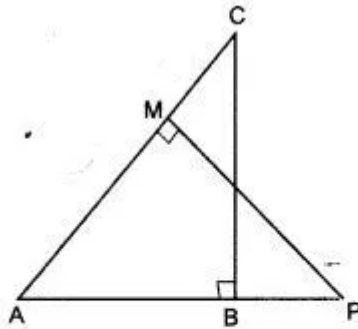
$$= \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

Question. 15. In the given figure ΔABC and ΔAMP are right angled at B and M respectively.

Given $AC = 10$ cm, $AP = 15$ cm and $PM = 12$ cm.

(i) Prove $\Delta ABC \sim \Delta AMP$.

(ii) Find AB and BC.



Solution. (i) In ΔABC and ΔAMP

$$\angle ABC = \angle AMP \quad (90^\circ \text{ each})$$

$$\angle A = \angle A \quad (\text{common})$$

$$\therefore \Delta ABC \sim \Delta AMP$$

(by AA similarity)

Hence proved.

(ii) $\Delta ABC \sim \Delta AMP$

$$\Rightarrow \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{BC}{12} = \frac{10}{15}$$

$$\Rightarrow BC = \frac{10}{15} \times 12$$

$$BC = 8$$

$$\text{Now, } AB^2 = AC^2 - BC^2$$

$$= 10^2 - 8^2$$

$$= 100 - 64 = 36$$

$$AB = 6 \text{ cm.}$$

Question 16. Triangles ABC and DEF are similar.

(i) If area (ΔABC) = 16 cm^2 , area (ΔDEF) = 25 cm^2 and $BC = 2.3 \text{ cm}$ find EF .

(ii) If area (ΔABC) = 9 cm^2 , area (ΔDEF) = 64 cm^2 and $DE = 5.1 \text{ cm}$, find AB .

(iii) If $AC = 19 \text{ cm}$ and $DF = 8 \text{ cm}$, find the ratio between the areas of two triangles.

(iv) If area (ΔABC) = 36 cm^2 , area (ΔDEF) = 64 cm^2 and $DE = 6.2 \text{ cm}$, find AB .

Solution : (i) We have

$$\text{area } (\Delta ABC) = 16 \text{ cm}^2$$

$$\text{area } (\Delta DEF) = 25 \text{ cm}^2$$

and $BC = 2.3 \text{ cm}$

$$\text{Since, } \frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$$

$$\Rightarrow \frac{2.3}{EF} = \frac{4}{5}$$

$$\Rightarrow 4 EF = 5 \times 2.3$$

$$\Rightarrow EF = \frac{11.5}{4}$$

$$\Rightarrow EF = 2.875 \text{ cm.}$$

(ii) We have

$$\text{area } (\Delta ABC) = 9 \text{ cm}^2$$

$$\text{area } (\Delta DEF) = 64 \text{ cm}^2$$

and $DE = 5.1 \text{ cm}$

$$\text{Since, } \frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{AB}{DE} = \frac{3}{8}$$

$$\Rightarrow \frac{AB}{5.1} = \frac{3}{8}$$

$$\Rightarrow AB = \frac{3}{8} \times 5.1 = \frac{15.3}{8}$$

$$\Rightarrow AB = 1.9125 \text{ cm.}$$

(iii) In ΔABC and ΔDEF , $AC = 19 \text{ cm}$,
 $DF = 8 \text{ cm}$.

$$\text{Since, } \frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{(8)^2} = \frac{361}{64}$$

Hence, the required ratio is 361 : 64.

$$(iv) \text{ Area } (\Delta ABC) = 36 \text{ cm}^2$$

$$\text{Area } (\Delta DEF) = 64 \text{ cm}^2.$$

$$DE = 6.2 \text{ cm}$$

$$AB = ?$$

We have

$$\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DEF)} = \frac{AB^2}{DE^2}$$

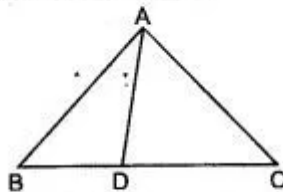
$$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2}$$

$$\Rightarrow \frac{AB}{6.2} = \frac{6}{8}$$

$$\Rightarrow AB = \frac{6 \times 6.2}{8}$$

$$\Rightarrow AB = 4.65 \text{ cm.}$$

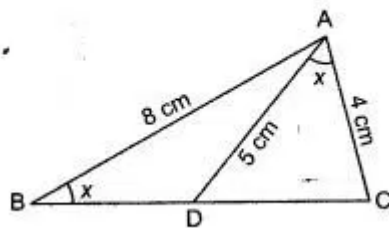
Question 17. In ΔABC , $\angle ABC = \angle DAC$. $AB = 8 \text{ cm}$, $AC = 4 \text{ cm}$, $AD = 5 \text{ cm}$.



- (i) Prove that ΔACD is similar to ΔBCA .
- (ii) Find BC and CD .
- (iii) Find area of ΔACD : area of ΔABC .

Solution :

$$\angle ABC = \angle DAC = x \text{ (say)}$$



$$AB = 8 \text{ cm,}$$

$$AC = 4 \text{ cm, } AD = 5 \text{ cm.}$$

- (i) In ΔACD and ΔBCA

$$\angle ABC = \angle DAC \text{ (Given)}$$

$$\angle ACD = \angle BCA$$

(Common)

$$\Rightarrow \Delta ACD \sim \Delta BCA \text{ (AA axiom).}$$

Hence ΔACD is similar to ΔBCA .

Hence proved.

(ii) As we have,

$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\Rightarrow \frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow \frac{4}{BC} = \frac{5}{8}$$

$$\Rightarrow BC = \frac{8 \times 4}{5} = \frac{32}{5}$$
$$= 6.4 \text{ cm.}$$

and $\frac{CD}{4} = \frac{5}{8}$

$$\Rightarrow CD = \frac{5 \times 4}{8}$$

$$\Rightarrow CD = 2.5 \text{ cm.} \quad \text{Ans.}$$

(iii) $\frac{\text{Area of } \Delta ACD}{\text{Area of } \Delta ABC} = \left(\frac{AC}{AB}\right)^2$

$$= \left(\frac{4}{8}\right)^2$$
$$= \frac{1}{4}$$

Thus area of ΔACD : area of $\Delta ABC = 1 : 4$.

Ans.