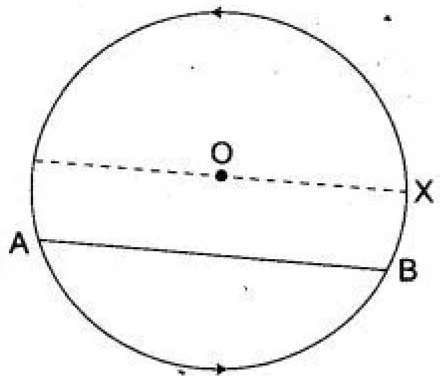
Chapter 15. Circles

Formulae

Theorems based on chord properties:

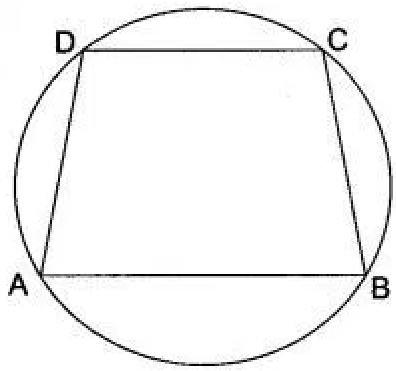
1. **Theorem:** A straight line drawn from the centre of the circle to bisect a chord, which is not a diameter, is at right angles to the chord.
   Conversely, the perpendicular to a chord, from the centre of the circle, bisects the chord.
2. **Theorem:** There is one circle, and only one, which passes through three given points not in a straight line.
3. **Theorem:** Equal chords of a circle are equidistant from the centre.
   Conversely, chords of a circle, equidistant from the centre of the circle, are equal.



Theorems based on Arc and Chord properties:

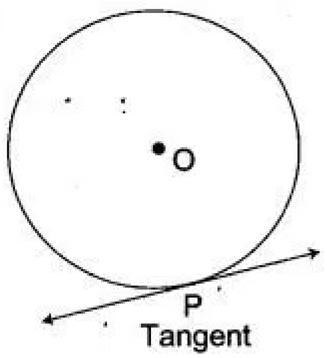
1. **Theorem:** The angle which an arc of a circle subtends at the centre is double, that which it subtends at any point on the remaining part of the circumference.
2. **Theorem:** Angles in the same segment of a circle are equal.
3. **Theorem:** The angle in a semicircle is a right angle.
4. **Theorem:** In equal circles (or, in the same circle), if two arcs subtends equal angles at the centre, they are equal.
   Conversely, in equal circles (or, in the same circle), if two arcs are equal, they subtend equal angles at the centre.
5. **Theorem:** In equal circles (or, in the same circle), if two chord are equal, they cut off equal arcs.
   Conversely, in equal circles (or, in the same circle, if two arcs are equal the chords of the arcs are also equal.

Theorems based on Cyclic properties: ABCD is a cyclic quadrilateral.



1. **Theorem:** The opposite angles of a cyclic quadrilateral (quadrilateral inscribed in a circle) are supplementary.
2. **Theorem:** The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Theorems based on Tangent Properties:

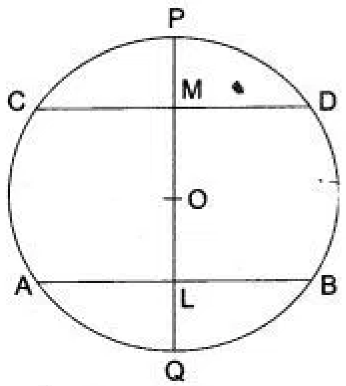


1. **Theorem:** The tangent at any point of a circle and the radius through this point are perpendicular to each other.
2. **Theorem:** If two circles touch each other, the point of contact lies on the straight line through the centres.
3. **Theorem:** From any point outside a circle two tangents can be drawn and they are equal in length.
4. **Theorem:** If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.
5. **Theorem:** If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Prove the Following

Question 1. If a diameter of a circle bisect each of the two chords of a circle, prove that the chords are parallel.

Solution : Let O be the centre of a circle and AB, CD be the two chords. Let PQ be the diameter bisecting chord AB and CD at L and M respectively.



L is the mid-point of AB

So $OL \perp AB \Rightarrow \angle ALO = 90°$

Similarly, $\angle CMO = 90°$

$\angle ALO = \angle CMO$

but these are alternate angles

So $AB \parallel CD$. Hence proved.

Question 2. If two chords of a circle are equally inclined to the diameter through their point of intersection, prove that the chords are equal.

Solution : Let AB and AC be two chords and AOD be a diameter such that

$\angle BAO = \angle CAO$

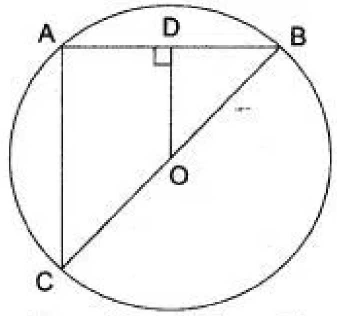
Draw $OL \perp AB$ and $OM \perp AC$

Now prove, $\Delta OLA = \Delta OMA$

Then $OL = OM \Rightarrow AB = CD$

(Chords which are equidistant from the centre are equal) Hence proved.

Question 3. In the given figure, OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2 OD.



Solution : Since, OD ⊥ AB and the perpendicular drawn from the centre to a chord bisects the chord.

∴ D is the mid-point of AB.

Also, O being the centre, is the mid-point of BC.

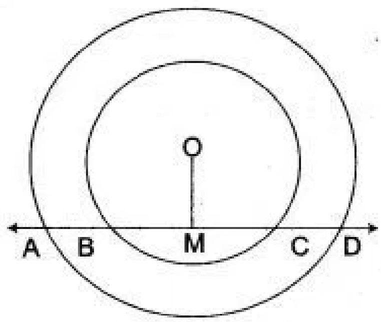
Thus, in ΔABC, D and O are mid-point of AB and BC respectively. Therefore, OD ∥ AC

and $OD = \frac{1}{2} CA$

[∵ Segment joining the mid-points of two sides of a triangle is half of the third side]

⇒ CA = 2OD. Hence proved.

Question 4. In Fig., l is a line intersecting the two concentric circles, whose common centre is O, at the points A, B, C and D. Show that AB = CD.



Solution : Let OM be perpendicular from O on line l. We know that the perpendicular from the centre of a circle to as chord; bisects the chord. Since BC is a chord of the smaller circle and OM ⊥ BC.

∴ BM = CM ...(i)

Again, AD is a chord of the larger circle and OM ⊥ AD.

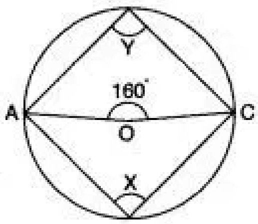
∴ AM = DM ...(ii)

Subtracting (i) from (ii), we get

AM – BM = DM – CM ⇒ AB = CD.

Hence proved.

Question 5. In the given below figure, O is the centre of the circle and $\angle AOC = 160°$. Prove that $3\angle y - 2\angle x = 140°$.



Solution : We know that angle by same arc at circle i.e., on circumference is half of the angle by same arc at centre.

$$\because \quad \angle x = \frac{1}{2} \times 160° = 80°$$

(Opposite triangles of a cyclic quadrilateral supplementary)

$$\therefore \quad \angle x + \angle y = 180°$$

$$\therefore \quad \angle y = 100°$$

$$\therefore \quad 3\angle y - 2\angle x = 3 \times 100° - 2 \times 80°$$

$$= 300° - 160°$$

$$= 140° \quad \text{Hence proved.}$$

Question 6. ABCD is a cyclic quadrilateral AB and DC are produced to meet in E. Prove that $\Delta EBC \sim \Delta EDA$.

Solution : In triangle EBC and EDA, we have

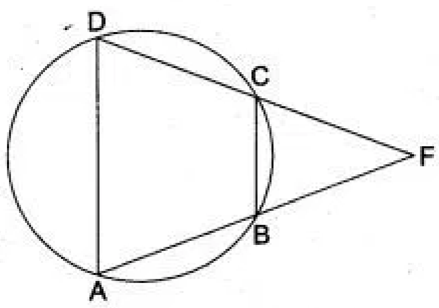
$$\angle EBC = \angle EDA$$

[$\because$ Exterior or angle in a cyclic quad. is equal to opposite interior angel]

$$\angle ECB = \angle EAD$$

[$\because$ Exterior angle in a cyclic quad. is equal to opposite interior angle]

and $\quad \angle E = \angle E$



So, by AAA enterior of similarly, we get

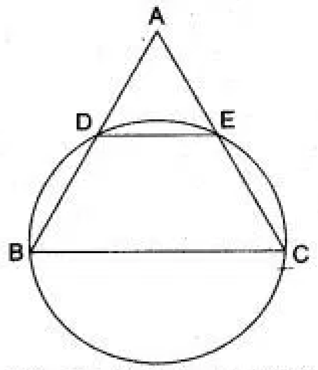
$\Delta EBC \sim \Delta EDA$. Hence Proved.

Question 7. In an isosceles triangle ABC with AB = AC, a circle passing through B and C intersects the sides. AB, and AC at D and E respectively. Prove that DE || BC.

Solution : In order to prove that DE ∥ BC, it is sufficient to show that $\angle B = \angle ADE$.

In Δ ABC, we have

$$AB = AC \Rightarrow \angle B = \angle C \quad ...(i)$$



In the cyclic quadrilateral CBDE, side BD is produced to A.

$$\therefore \quad \angle ADE = \angle C \quad ...(ii)$$

[∵ Exterior angle = Opposite interior angle]

From (i) and (ii), we get $\angle B = \angle ADE$.

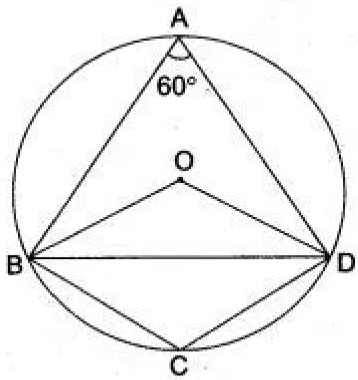
Hence, DE ∥ BC. Hence proved.

Question 8. ABCD is quadrilateral inscribed in circle, having ∠A = 60°, O is the centre of the circle, show that

$$\angle OBD + \angle ODB = \angle CBD + \angle CDB$$

Solution : Here, $\angle BOD = 2 \times \angle BAD$

$$= 2 \times 60^\circ$$
$$= 120^\circ$$



Now in Δ BOD,

$$\angle OBD + \angle ODB = 180^\circ - 120^\circ$$
$$= 60^\circ \quad ...(1)$$

Also $\angle DAB + \angle DCB = 180^\circ$

(ABCD is a cyclic quadrilateral)

$$\Rightarrow \quad \angle DCB = 180^\circ - 60^\circ = 120^\circ$$

∴ In Δ BCD,

$$\angle CBD + \angle CDB = 180^\circ - \angle DCB$$
$$= 180^\circ - 120^\circ$$
$$= 60^\circ \quad ... (2)$$

From (1) and (2) we get the required result.

Hence proved

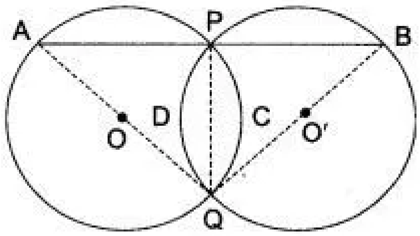
Question 9. Two equal circles intersect in P and Q. A straight line through P meets the circles in A and B. Prove that QA = QB

Solution : Let C(O, r) and C(O′, r) be two equal circles. Clearly C(O, r) ≅ C(O′, r).

Since, PQ is a common chord of two congruent circles. Therefore,

$$\text{arc } PCQ = \text{arc } PDQ$$

$$\Rightarrow \angle QAP = \angle QBP$$

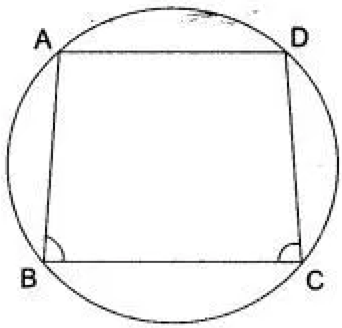


Thus, in ΔQAB , we have

$$\angle QAP = \angle QBP$$

$$\Rightarrow QA = QB.$$ Hence proved.

Question 10. If ABCD is a cyclic quadrilateral in which AD ∥ BC. Prove that ∠B = ∠C.



Solution : ABCD is a cyclic quadrilateral

So $\angle A + \angle C = 180°$ ...(i)

Since AD ∥ BC

So $\angle B + \angle A = 180°$ ...(ii)

From (i) & (ii)

$$\angle A + \angle C = \angle B + \angle A$$

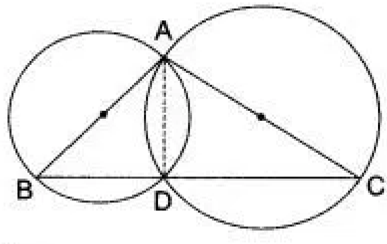
$$\Rightarrow \angle C = \angle B$$

or $\angle B = \angle C.$ Hence proved.

Question 11. Two circles are drawn with sides AB, AC of a triangle ABC as diameters. The circles intersect at a point D. Prove that D lies on BC.

Solution : Join AD.

Since, angle in a semi-circle is a right angle.



Therefore,

$$\angle ADB = 90° \text{ and } \angle ADC = 90°$$

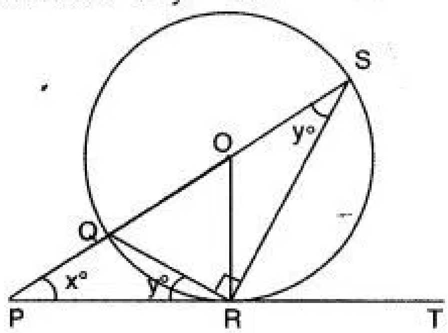
$$\Rightarrow \angle ADB + \angle ADC = 90° + 90°$$

$$\Rightarrow \angle ADB + \angle ADC = 180°$$

BDC is a straight line ⇒ D lies on BC.

Hence proved.

Question 12. In the given figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If ∠ SPR = x° and ∠QRP = y°. show that x° + 2y° = 90°.



Solution : PRT is tangent at R and QR is chord.

∴ ∠QRP = ∠QSR

(Angle in alternate segment)

= y°

and ∠QRS = 90°

(∵ QS is diameter and angle in semicircle is rt. angle)

Now in ΔPRS,

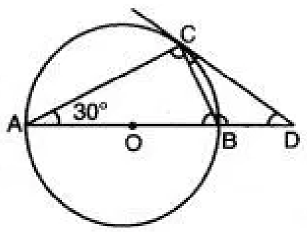
∠SPR + ∠PRS + ∠RSP = 180°

x° + y° + 90 + y° = 180°

x° + 2y° = 180° – 90°

x° + 2y° = 90° Hence proved.

Question 13. In the figure AB is a diameter and AC is a chord of a circle such that ∠ BAC = 30°. The tangent at C intersect AB produced at D. Prove that BC = BD.



Solution : Join OC.

∠ ACB = 90° (Angle of the semicircle).

∠ ABC = 60° (Angle sum property)

∠ CBD = 120° (adj to angle CBA 30°)

∠ OCD = 90° tangent

∠ COB = 60°

(Angle at the centre is equal to twice that of the circumference.

∠ OCB = 60° (Angle sum Property)

∠ BCD = ∠ OCD – ∠ OCB = 30°

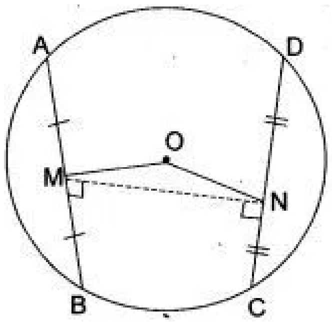
∴ ∠ BDC = ∠ BDC = 30°

BD = BC Hence proved.

Question 14. Prove that the line segment joining the midpoints of two equal chords of a circle substends equal angles with the chord.

Solution : Here, M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. We have to prove that

$$\angle BMN = \angle CNM$$
$$\angle AMN = \angle DNM$$



Join OM, ON and MN.

$\therefore$ $\left.\begin{array}{l}\angle OMA = \angle OMB = 90^\circ \\ \angle OND = \angle ONC = 90^\circ\end{array}\right\} \ldots (1)$

[Line joining the centre and mid-point of a chord is perpendicular to the chord]

Since, $AB = CD \Rightarrow OM = ON$

$\therefore$ In ΔOMN, $\angle OMN = \angle ONM$ ... (2)

(i) $\angle OMB = \angle ONC$

[Using (1) and (2)]

$\angle OMN = \angle ONM$

$\Rightarrow \angle OMB - \angle OMN = \angle ONC - \angle ONM$

$\Rightarrow \angle BMN = \angle CNM$

(ii) $\angle OMA = \angle OND$

$\angle OMN = \angle OND$

[Using (1) and (2)]

$\Rightarrow \angle OMA + \angle OMN = \angle OND + \angle ONM$

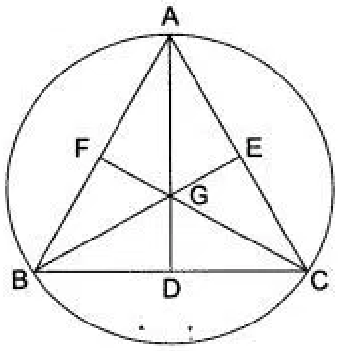
$\Rightarrow \angle AMN = \angle DNM$

Hence proved

Question 15. In an equilateral triangle, prove that the centroid and centre of the circum-circle (circumcentre) coincide.

Solution : Given : An equilateral triangle ABC in which D, E and F are the mid-points of sides BC, CA and AB respectively.

To prove : The centroid and circumcentre are coincident.



Construction : Draw medians AD, BE and CF.

Proof : Let G be the centroid of ΔABC *i.e.,* the point of intersection of AD, BE and CF. In triangles BEC and BFC, we have

$$\angle B = \angle C = 60^\circ$$

$$BC = BC$$

and $$BF = CE$$

$$\left[\because AB = AC \Rightarrow \frac{1}{2}AB = \frac{1}{2}AC \Rightarrow BF = CE\right]$$

$\therefore$ $\Delta BEC \cong \Delta BFC$

$\Rightarrow$ $BE = CF$ ...(i)

similarly, $\Delta CAF \cong \Delta CAD$

$CF = AD$ ...(ii)

From (i) & (ii) $AD = BE = CF$

$\Rightarrow$ $\frac{2}{3}AD = \frac{2}{3}, BE = \frac{2}{3}CF$

$$CG = \frac{2}{3}CF$$

$$GA = \frac{2}{3}AD,$$

$$GB = \frac{2}{3}BE$$

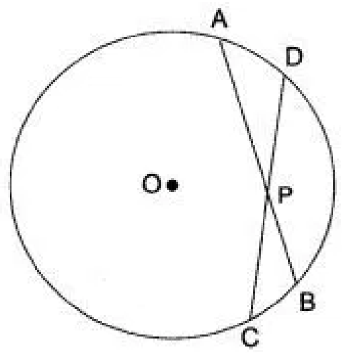
$\Rightarrow$ $GA = GB = GC$

$\Rightarrow$ G is equidistant from the vertices

$\Rightarrow$ G is the circumcentre of ΔABC

Hence, the centroid and circum centre are coincident.

Question 16. In Fig. AB and CD are two chords of a circle intersecting each other at P such that AP = CP. Show that AB = CD.



Solution : In order to prove the desired result, we shall first prove that $\Delta PAD \sim \Delta PCB$.

In triangles PAD and PCB, we have :

$$\angle PAD = \angle PCB$$

[Angles in the same segment of arc BD]

$$\angle APD = \angle CPB$$

[Vertically opposite angles]

So, by AAA criterion of similarity, we have

$$\Delta PAD \sim \Delta PCB$$

$$\Rightarrow \quad \frac{PA}{PC} = \frac{PD}{PB}$$

[$\because$ Corresponding sides of similar triangles are in the same ratio]

$$\Rightarrow \quad \frac{AP}{CP} = \frac{PD}{PB}$$

$$\Rightarrow \quad 1 = \frac{PD}{PB}$$

$$\left[\because AP = CP, \therefore \frac{AP}{CP} = 1\right]$$

$$\Rightarrow \quad PB = PD$$

$$\Rightarrow \quad AP + PB = AP + PD$$

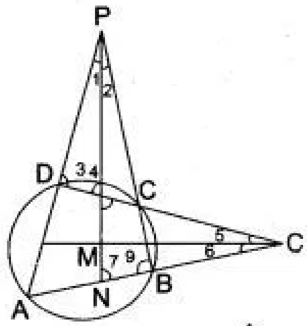
[Adding AP on both sides]

$$\Rightarrow \quad AP + PB = CP + PD \qquad [\because AP = CP]$$

$$\Rightarrow \quad AB = CD.$$ Hence proved.

Question 17. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (Provided they are not parallel) intersect at right angle.

Solution : Here, ABCD is a cyclic quadrilateral. PM is bisector of ∠ APB and QM is bisector of ∠ AQD.



In Δ PDL and Δ PBN,

∠ 1 = ∠ 2

(PM is the bisector of ∠ P)

∠ 3 = ∠ 9 (Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle)

∴ ∠ 4 = ∠ 7

But ∠ 4 = ∠ 8 (Vertically opposite angles)

∴ ∠ 7 = ∠ 8

Now in Δ QMN and Δ QML,

∠ 7 = ∠ 8 (Proved above)

∠ 5 = ∠ 6

(QM is bisector of Q)

Δ QMN ~ Δ QML

∴ ∠ QMN = ∠ QML

But ∠ QMN + ∠ QML = 180°

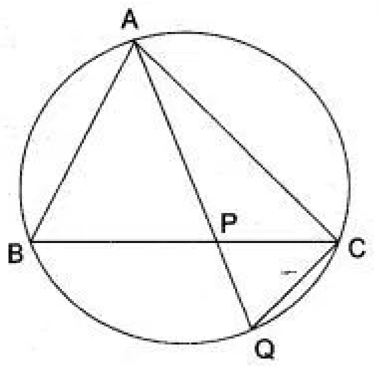
∴ ∠ QMN = ∠ QML = 90°

Hence, ∠ PMQ = 90°

(∵ ∠ PMQ = ∠ QML)

Hence proved.

Question 18. In Fig. P is any point on the chord BC of a circle such that AB = AP. Prove that CP = CQ.



Solution : We have to prove that CP = CQ i.e., ΔCPQ is an isosceles triangle. For this it is sufficient to prove that $\angle CPQ = \angle CQP$.

In ΔABP, we have

$$AB = AP$$

$$\Rightarrow \quad \angle APB = \angle ABP$$

$$\Rightarrow \quad \angle CPQ = \angle ABP \quad ...(i)$$

[$\because$ $\angle APB$ and $\angle CPQ$ are vertically opposite angles $\therefore$ $\angle APB = \angle CPQ$]

Now, consider arc AC. Clearly, it subtends $\angle ABC$ and $\angle AQC$ at points B and Q.

$$\therefore \quad \angle ABC = \angle AQC$$

[$\because$ Angles in the same segment]

$$\Rightarrow \quad \angle ABP = \angle PQC$$

[$\because$ $\angle ABC = \angle ABP$ and $\angle AQC = \angle PQC$]

$$\Rightarrow \quad \angle ABP = \angle CQP \quad ...(ii)$$

[$\because$ $\angle PQC = \angle CQP$]

From (i) and (ii), we get

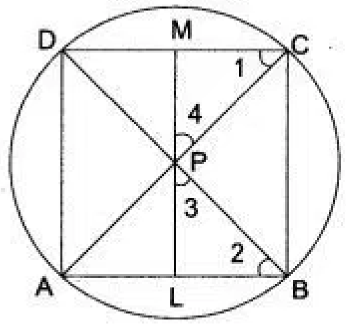
$$\angle CPQ = \angle CQP$$

$\Rightarrow$ CQ = CP. Hence proved.

Question 19. The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection on any side when produced backward bisects the opposite side.

Solution : Let ABCD be a cyclic quadrilateral such that its diagonals AC and BD intersect in P at right angles. Let $PL \perp AB$ such that LP produced to meet CD in M. We have to prove that M bisects CD *i.e.,*

$$CM = MD.$$



Consider arc AD. Clearly, it makes angles $\angle 1$ and $\angle 2$ in the same segment.

$$\angle 1 = \angle 2 \quad \ldots(i)$$

In right angled triangle PLB, we have

$$\angle 2 + \angle 3 + \angle PLB = 180°$$

$$\Rightarrow \quad \angle 2 + \angle 3 + 90° = 180°$$

$$\Rightarrow \quad \angle 2 + \angle 3 = 90° \quad \ldots(ii)$$

Since, LPM is a straight line.

$$\therefore \angle 3 + \angle BPD + \angle 4 = 180°$$

$$\Rightarrow \quad \angle 3 + 90° + \angle 4 = 180°$$

$$\Rightarrow \quad \angle 3 + \angle 4 = 90° \quad \ldots(iii)$$

From (ii) and (iii), we get

$$\angle 2 + \angle 3 = \angle 3 + \angle 4$$

$$\Rightarrow \quad \angle 2 = \angle 4 \quad \ldots(iv)$$

From (i) and (iv), we get

$$\angle 1 = \angle 4 \quad \ldots(v)$$

$$\Rightarrow \quad PM = CM$$

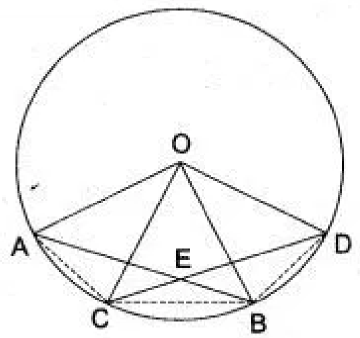
Similarly, $PM = DM$.

Hence, $CM = MD$. Hence proved.

Question 20. In a circle with centre O, chords AB and CD intersect inside the circumference at E. Prove that ∠AOC + ∠BOD = 2∠AEC.

Solution : Consider arc AC of the circle with centre at O.

Clearly, arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.



$$\therefore \quad \angle AOC = 2\angle ABC \quad \ldots(i)$$

Similarly, arc BD subtends $\angle BOD$ at the centre and $\angle BCD$ at the remaining part of the circle.

$$\therefore \quad \angle BOD = 2\angle BCD \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$\angle AOC + \angle BOD = 2(\angle ABC + \angle BCD)$$

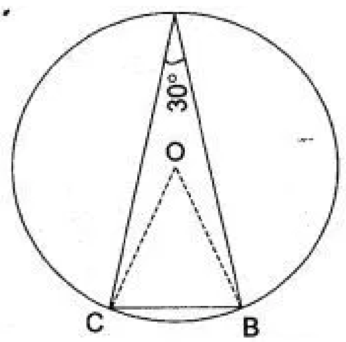
$$\Rightarrow \angle AOC + \angle BOD = 2\angle AEC$$

[$\because$ $\angle AEC$ is the exterior angle and $\angle ABC$ and $\angle BCD$ are other interior angles of ΔBEC

$\therefore \angle ABC + \angle BCD = \angle AEC$]

Hence proved.

Question 21. In Fig. ABC is a triangle in which ∠BAC = 30°. Show that BC is the radius of the circum circle of A ABC, whose centre is O.



Solution : Join OB and OC. Since, the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

$\therefore \quad \angle BOC = 2\angle BAC$

$\Rightarrow \quad \angle BOC = 2 \times 30° = 60°$

Now, in ΔBOC, we have

$OB = OC$

[Each equal to radius of the circle]

$\Rightarrow \quad \angle OBC = \angle OCB$

[∵ Angles opposite to equal sides of a triangle are equal]

But $\angle OBC + \angle OCB + \angle BOC = 180°$

$\therefore \quad 2\angle OBC + 60° = 180°$

$\Rightarrow \quad 2\angle OBC = 120° \Rightarrow \angle OBC = 60°$

Thus, $\angle OBC = \angle OCB$

$= \angle BOC = 60°$

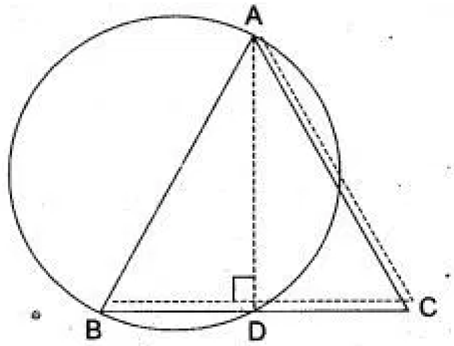
⇒ Triangle OBC is an equilateral

$\Rightarrow \quad OB = BC$ showed

⇒ BC is the radius of the circum circle of Δ ABC.

Hence proved.

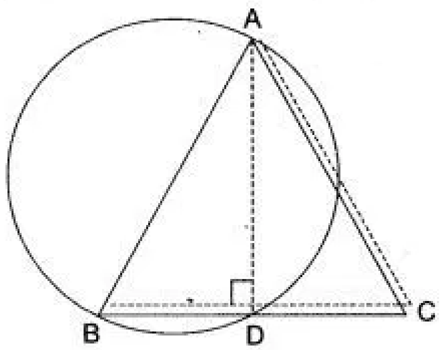
Question 22. Prove that the circle drawn on any one of the equal sides of an isosceles triangles as diameter bisects the base.



Solution :

Given : In Isosceles Δ ABC. A circle is drawn taken AB as diameter which intersect BC at D.

To prove : BD = DC.



Construction : Join AD.

Proof : $\angle ADB = 90°$ (Angle of semi-circle)

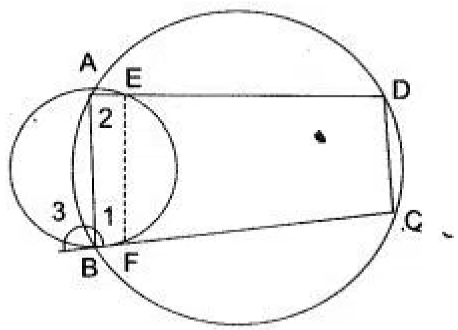
In Δ ABD & Δ ACD,

$AB = AC$		(Given)
$\angle ADB = \angle ADC$		(90°)
$AD = AD$		(Common)

$\therefore \quad \Delta ABD \cong \Delta ADC$

Hence $\quad BD = DC$.

Question 23. In Fig. ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in the points E and F respectively. Prove that EF || DC.



Solution : In order to prove that EF ∥ DC. It is sufficient to show that $\angle 2 = \angle 3$.

Since, ABCD is a cyclic quadrilateral.

$\therefore \quad \angle 1 + \angle 3 = 180° \quad ...(i)$

Similarly, in the cyclic quadrilateral ABFE, we have

$\angle 1 + \angle 2 = 180° \quad ...(ii)$

From (i) and (ii), we get

$\angle 1 + \angle 3 = \angle 1 + \angle 2 \Rightarrow \angle 2 = \angle 3$.

Hence, $\quad EF \parallel DC$.

Question 24. If O is the circumcentre of a Δ ABC and $OD \perp BC$, prove that $\angle BOD = \angle A$.

Solution : Join OB and OC

In ΔOBD and ΔOCD, we have

$$OB = OC$$

[Each equal to the radius of circumcircle]

$$\angle ODB = \angle ODC$$

[Each equal to 90°]

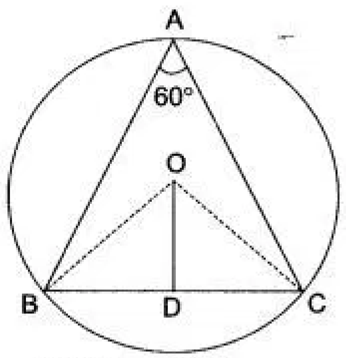
and $OD = OD$ [Common]

$\therefore \quad \Delta OBD \cong \Delta OCD$

$\Rightarrow \quad \angle BOD = \angle COD$

$\Rightarrow \quad \angle BOC = 2\angle BOD = 2\angle COD$

Now, arc BC subtends $\angle BOC$ at the centre and $\angle BAC = \angle A$ at a point in the remaining part of the circle.



$\therefore \quad \angle BOC = 2\angle A$

$\Rightarrow \quad 2\angle BOD = 2\angle A$

[$\because \angle BOC = 2\angle BOD$]

$\Rightarrow \quad \angle BOD = \angle A.$ Hence proved.

Question 25. If PA and PB are two tangent drawn from a point P to a circle with centre C touching it A and B, prove that CP is the perpendicular bisector of AB.

Solution : We shall prove that

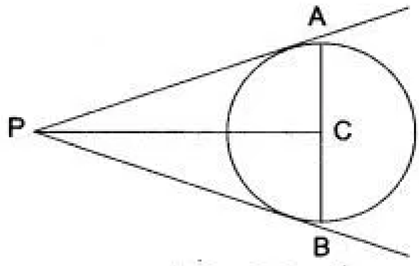
$$\angle ACP = \angle BCP = 90^\circ$$

and $AC = BC$

Now, $\angle APC = \angle BPC$

Since O lies on the bisector of $\angle APB$.

Δ s ACP and BCP are congruent traingles by SAS congruence criterion.



$\therefore$ $AC = BC$

and $\angle ACP = \angle BCP$

Since $\angle ACP + \angle BCP = 180^\circ$

$$2\angle ACP = 180^\circ$$

$$\angle ACP = 90^\circ$$

$\therefore$ $\angle ACP = \angle BCP = 90^\circ$

Hence proved.

Question 26. Two circle with radii r_1 and r_2 touch each other externally. Let r be the radius of a circle which touches these two circle as well as a common tangent to the two circles, Prove that:

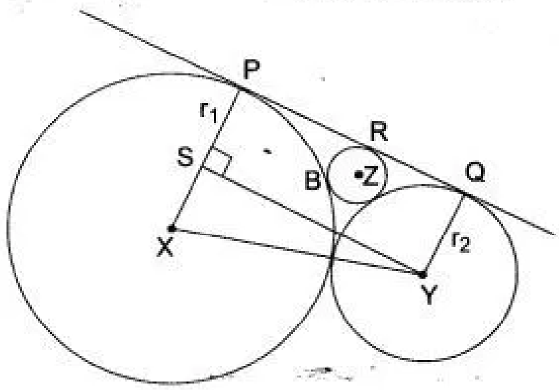
$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$$

Solution : From the adjoining figure,

$$PQ = SY = \sqrt{XY^2 - XS^2}$$

$$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$= \sqrt{4r_1r_2} = 2\sqrt{r_1r_2}$$



Similarly, $PR = 2\sqrt{r_1r}$ and $RQ = 2\sqrt{rr_2}$

Now $PQ = PR + RQ$

$\therefore$ $2\sqrt{r_1r_2} = 2\sqrt{rr_1} + 2\sqrt{rr_2}$

$\Rightarrow$ $\sqrt{r_1r_2} = \sqrt{rr_1} + \sqrt{rr_2}$

Dividing by $\sqrt{r\,r_1r_2}$ on both sides,

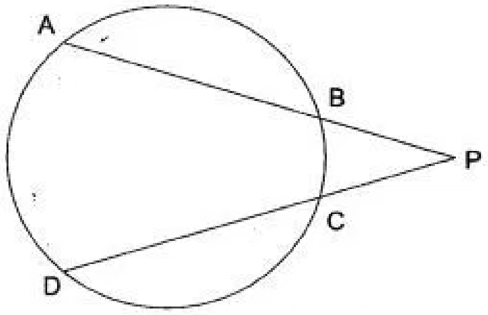
$\Rightarrow$ $\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$ Hence proved.

Question 27. If AB and CD are two chords which when produced meet at P and if AP = CP, show that AB = CD.

Solution : Here, chords AB and CD of the circle intersect at P.

$\therefore \quad PA \times PB = PC \times PD$

$\Rightarrow \quad PB = \dfrac{PC \times PD}{PA}$



$\Rightarrow \quad PB = \dfrac{AP \times PD}{AP}$

[$\because$ PC = AP (given)]

$\Rightarrow \quad PB = PD \qquad ...(i)$

Now, $\quad AB = AP - BP$

$\Rightarrow \quad AB = CP - PD$

[$\because$ AP = CP (given), BP = PD (From (i)]

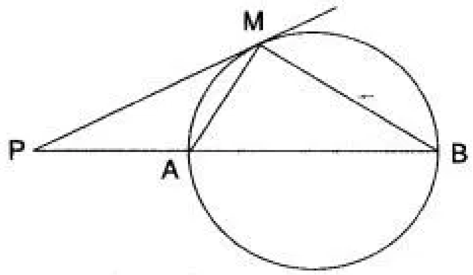
$\Rightarrow \quad AB = CD$

Hence, $\quad AB = CD.$ Hence proved.

Question 28. In the figure, PM is a tangent to the circle and PA = AM. Prove that:

(i) ΔPMB is isosceles.

(ii) $PA \times PB = MB^2$.



Solution : (i) In Δ PAM

$\angle APM = \angle AMP \qquad ...$ (i)

$PA = AM \qquad$ (Given)

by alternate segment property of tangent

$\angle ABM = \angle AMP$

$\therefore \quad \angle APM = \angle ABM$ from (i) and (ii)

$\therefore \quad PM = MB$

i.e. Δ PMB is an isosceles $\qquad$ Proved.

(ii) By rectangle property of tangent and chord

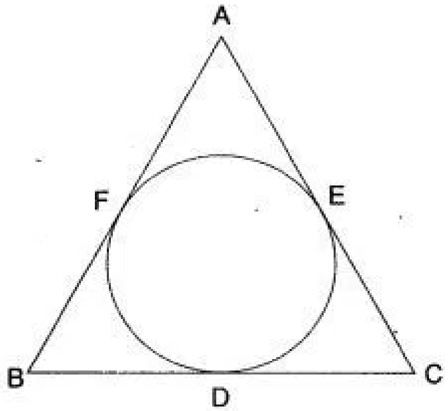
$PM^2 = PA \times PB$

$\therefore \quad MB^2 = PA \times PB \qquad$ [$\because$ PM = MB]

Hence proved.

Question 29. In Fig. the incircle of ΔABC, touches the sides BC, CA and AB at D, E respectively. Show that:

$$AF + BD + CE = AE + BF + CD$$
$$= \frac{1}{2}(\text{Perimeter of } \Delta ABC).$$



Solution : Since, lengths of the tangents drawn from an exterior point to a circle are equal.

$\therefore$ $AF = AE$ ...(i)

$BD = BF$ ...(ii)

and $CE = CD$ ...(iii)

Adding (i), (ii) and (iii), we get

$$AF + BD + CE = AE + BF + CD$$

Now, Perimeter of

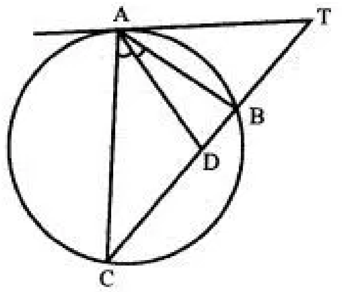
$$\begin{aligned} \Delta ABC &= AB + BC + AC \\ &= (AF + FB) + (BD + CD) + (AE + EC) \\ &= (AF + AE) + (BF + BD) + (CD + CE) \\ &= 2AF + 2BD + 2CE \\ &= 2(AF + BD + CE) \end{aligned}$$

[From (i), (ii) and (iii), we get $AE = AF$, $BD = BF$ and $CD = CE$]

$$\therefore AF + BD + CE = \frac{1}{2}(\text{Perimeter of } \Delta ABC)$$

Hence, $AF + BD + CE = AE + BF + CD = \frac{1}{2}$ (Perimeter of Δ ABC). Hence proved.

Question 30. In Fig. TA is a tangent to a circle from the point T and TBC is a secant to the circle. If AD is the bisector of ∠BAC, prove that ΔADT is isosceles.



Solution : In order to prove that Δ ADT is isosceles i.e., TA = TD, it is sufficient to show that $\angle$ TAD = $\angle$ TDA.

Since $\angle$ TAB and $\angle$ BCA are angles in the alternate segments of chord AB.

$\therefore \quad \angle TAB = \angle BCA \quad ...(i)$

It is given that AD is the bisector of $\angle$ BAC.

$\therefore \quad \angle BAD = \angle CAD \quad ...(ii)$

Now, $\quad \angle TAD = \angle TAB + \angle BAD$

$\Rightarrow \quad \angle TAD = \angle BCA + \angle CAD$

[Using (i) and (ii)]

$\Rightarrow \quad \angle TAD = \angle DCA + \angle CAD$

$[\because \angle BCA = \angle DCA]$

$\Rightarrow \quad \angle TAD = 180° - \angle CDA$

[In ΔCAD, $\angle CAD + \angle DCA + \angle CDA = 180°$

$\therefore \angle CAD + \angle BCA = 180° - \angle CDA$]

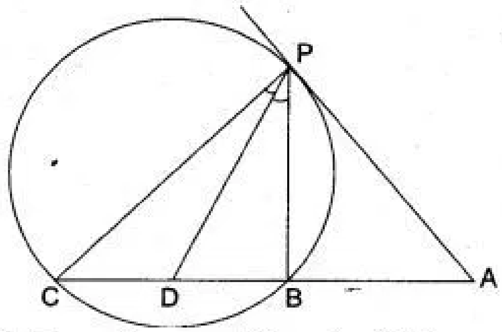
$\Rightarrow \quad \angle TAD = \angle TDA$

$[\because \angle CDA + \angle TDA = 180°]$

$\Rightarrow \quad TD = TA$

Hence, ΔADT is isosceles. Hence proved.

Question 31. In Fig. AP is a tangent to the circle at P, ABC is a secant and PD is the bisector of $\angle$BPC. Prove that $\angle BPD = \frac{1}{2}(\angle ABP - \angle APB)$.



Solution : Since, $\angle APB$ and $\angle BCP$ are angles in alternate segments of chord PB.

$\therefore \quad \angle APB = \angle BCP \quad \ldots(i)$

Since, PD is bisector of $\angle BPC$

$\therefore \quad \angle CPB = 2\angle BPD \quad \ldots(ii)$

In ΔPCB, side CB has been produced to A, forming exterior angle $\angle ABP$.

$\therefore \quad \angle ABP = \angle BCP + \angle CPB$

$\Rightarrow \quad \angle ABP = \angle APB + 2\angle BPD$

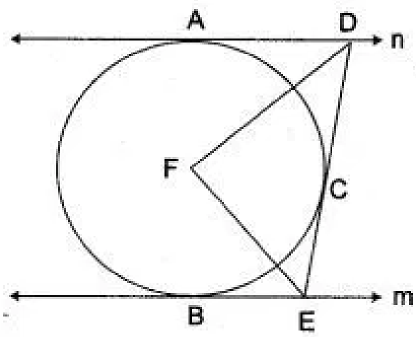
[Using (i) and (ii)]

$\Rightarrow \quad 2\angle BPD = \angle ABP - \angle APB$

$\Rightarrow \quad \angle BPD = \frac{1}{2}(\angle ABP - \angle APB).$

Hence proved.

Question 32. In Fig. l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between n and m. Prove that $\angle$ DFE = 90^0



Solution : In triangles ADF and DFC, we have

DA = DC

[Tangents drawn from an external point are equal in length]

DF = DF [Common]

AF = CF[Radii of the same circle]

So, by SSS-criterion of congruence, we get

$\Delta ADF \cong \Delta DFC$

$\Rightarrow \quad \angle ADF = \angle CDF$

$\Rightarrow \quad \angle ADC = 2\angle CDF \quad \ldots(i)$

Similarly, we can prove that

$\angle BEF = \angle CEF$

$\Rightarrow \quad \angle CEB = 2\angle CEF \quad \ldots(ii)$

Now, $\angle ADC + \angle CEB = 180°$

[Sum of the interior angles on the same side of transversal is 180°]

$\Rightarrow \quad 2(\angle CDF + \angle CDF) = 180°$

$\Rightarrow \quad \angle CDF + \angle CEF = \frac{180°}{2}$

$\Rightarrow \quad \angle CDF + \angle CEF = 90°$

Now, in $\angle DEF$;

$\angle DFE + \angle CDF + \angle CEF = 180°$

$\Rightarrow \quad \angle DFE + 90° = 180°$

$\Rightarrow \quad \angle DFE = 180° - 90°$

$\Rightarrow \quad \angle DFE = 90°$

Hence proved.

Question 33. A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

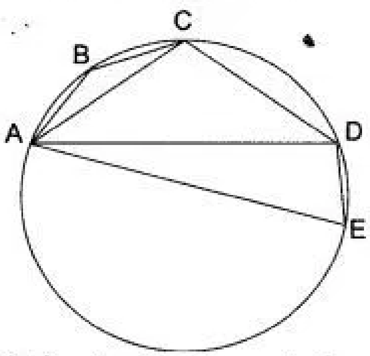
Solution : Given : A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral $ABCD$ at the points P, Q, R and S respectively.

To prove :

$\angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = 180°$

Construction : Join OP, OQ, OR and OS.



Proof : Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$ ...(i)

Now,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

[Sum of all the angles subtended at a point is 360°]

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

and $\quad 2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$

and $\quad (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

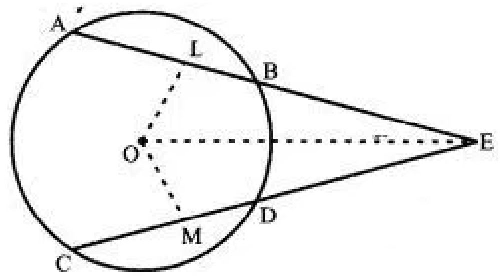
$\Rightarrow \quad \angle AOB + \angle COD = 180°$

[$\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD,$
$\angle 1 + \angle 8 = \angle AOD$ and $\angle 4 + \angle 5 = \angle BOC$]

and $\quad \angle AOD + \angle BOC = 180°$.

Hence proved.

Question 34. Two equal chords AB and CD of a circle with centre O, when produced meet at a point E, as shown in Fig. Prove that BE = DE and AE = CE.



Solution : Given : Two equal chords AB and CD intersecting at a point E.

To prove : BE = DE and AE = CE

Construction : Join OE. Draw $OL \perp AB$ and $OM \perp CD$.

Proof : We have

$$AB = CD$$

$$\Rightarrow \quad OL = OM$$

[$\because$ Equal chords are equidistant from the centre]

In triangles OLE and OME, we have

$$OL = OM$$

$$\angle OLE = \angle OME \quad \text{[Each equal to 90°]}$$

and $\quad OE = OE \quad$ [Common]

So, by SAS-criterion of congruences

$$\Delta OLE \cong \Delta OME$$

$$\Rightarrow \quad LE = ME \quad \text{...(i)}$$

Now, $\quad AB = CD$

$$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD \Rightarrow BL = DM \quad \text{...(ii)}$$

Subtracting (ii) from (i), we get

$$LE - BL = ME - DM$$

$$\Rightarrow \quad BE = DE$$

Again, $\quad AB = CD$ and $BE = DE$

$$\Rightarrow \quad AB + BE = CD + DE$$

$$\Rightarrow \quad AE = CE$$

Hence, $\quad BE = DE$ and $AE = CE$.

Hence proved.

Question 35. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral ABCD is also cyclic.

Solution : Given : In cyclic ABCD the bisectors formed a quadrilateral ABCD.

To prove : PQRS is a cyclic quadrilateral.

Proof : In cyclic quadrilateral ABCD, AR & BS be the bisectors of $\angle A$ and $\angle B$.

So $\angle 1 = \angle A/2$ and $\angle 2 = \angle B/2$

In Δ ASB, $\angle RSP$ is the exterior angle

so $\angle RSP = \angle 1 + \angle 2$

$$\angle RSP = \frac{\angle A}{2} + \frac{\angle B}{2} \quad \ldots(i)$$

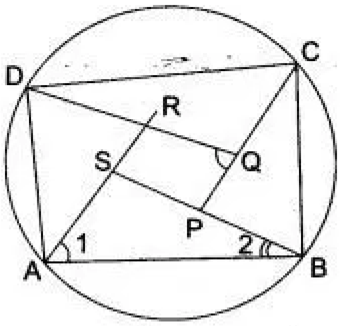
similarly $$\angle PQR = \frac{\angle C}{2} + \frac{\angle D}{2} \quad \ldots(ii)$$

Adding (i) & (ii)

$$\angle PQR + \angle RSP = \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$= \frac{1}{2} \times 360° = 180°$$

$\Rightarrow \angle PQR + \angle RSP = 180°$



But these are the opposite angles of quadrilateral PQRS

Hence PQRS is a cyclic quadrilateral.

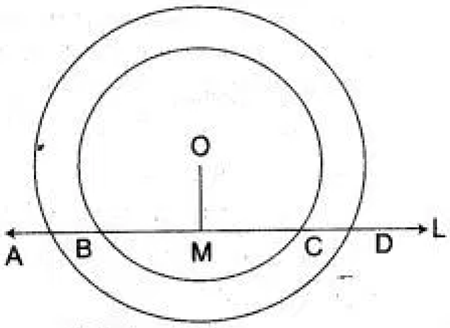
Hence proved.

Figure Based Questions

Question 1. Two concentric circles with centre 0 have A, B, C, D as the points of intersection with the lines L shown in figure. If AD = 12 cm and BC s = 8 cm, find the lengths of AB, CD, AC and BD.

Solution. Since, OM ⊥ BC

$$BM = CM = \frac{1}{2}BC = 4 \text{ cm}$$



also OM ⊥ AD

so $AM = DM = \frac{1}{2}AD = 6 \text{ cm}$

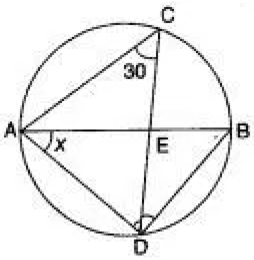
now $AB = AM - BM = (6 - 4) \text{ cm} = 2 \text{ cm}$

$CD = DM - CM = (6 - 4) \text{ cm} = 2 \text{ cm}$

∴ $AC = AB + BC = (2 + 8) \text{ cm} = 10 \text{ cm}$

and $BD = BC + CD = (8 + 2) \text{ cm} = 10 \text{ cm}$.

Question 2. In the given circle with diameter AB, find the value of x.



Solution. $\angle ABD = \angle ACD = 30°$

(∠ s of same segment)

$\angle ADB = 90°$

(∠ in the semi circle)

In Δ ADB,

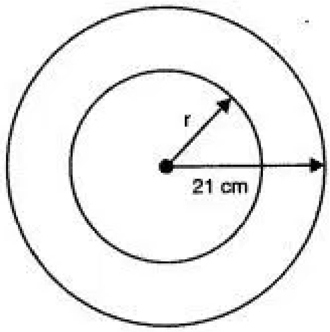
$x° + 90° + 30° = 180$

(Sum of all ∠s of triangle)

$x = 180 - 120$

$= 60°$ Ans.

Question 3. In the given figure, the area enclosed between the two concentric circles is 770 cm^2. If the radius of outer circle is 21 cm, calculate the radius of the inner circle.



Solution. Let the radius of inner circle be r.

Area enclosed between two concentric circles

$\Rightarrow \quad \pi[(21)^2 - (r)^2] = 770$

$\Rightarrow \quad (21)^2 - (r)^2 = \frac{770}{\pi} = \frac{770 \times 7}{22}$

$= 35 \times 7$

$\Rightarrow \quad (21)^2 - (r)^2 = 245$

$\Rightarrow \quad 441 - r^2 = 245$

$\Rightarrow \quad 441 - 245 = r^2$

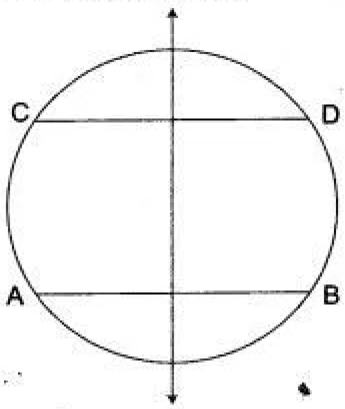
$\Rightarrow \quad 196 = r^2$

$\Rightarrow \quad 14 = r$

The radius of inner circle = 14 cm.

Question 4. Two chords AB and CD of a circle are parallel and a line L is the perpendicular bisector of AB. Show that L bisects CD.

Solution. We know that the perpendicular bisector of any chord of a circle always passes through the centre of the circle.



Since, L is the perpendicular bisector of AB. Therefore L passes through the centre of the circle.

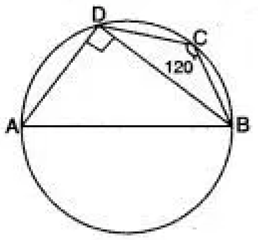
But $L \perp AB$ and $AB \parallel CD \Rightarrow L \perp CD$

Thus, $L \perp CD$ and passes through the centre of the circle.

So, L is perpendicular bisector of CD. Ans.

Question 5. In the adjoining figure, AB is the diameter of the circle with centre O. If ∠BCD = 120°, calculate:

(i) $\angle BAD$ (ii) $\angle DBA$



Solution : (i) Since AOB is a diameter

$\therefore \quad \angle ADB = 90^\circ$ (C is a semi circle)

Also, ABCD is a cyclic quadrilateral.

$$\therefore \quad \angle BCD + \angle BAD = 180^\circ$$

$$\angle BAD = 180^\circ - 120^\circ$$

$$\Rightarrow \quad \angle BAD = 60^\circ$$

(ii) Now, In Δ BAD.

$$\angle BAD + \angle BDA + \angle DBA = 180^\circ$$

$$60^\circ + 90^\circ + \angle DBA = 180^\circ$$

$$\angle DBA = 180^\circ - 150^\circ$$

$$\angle DBA = 30^\circ \quad \text{Ans.}$$

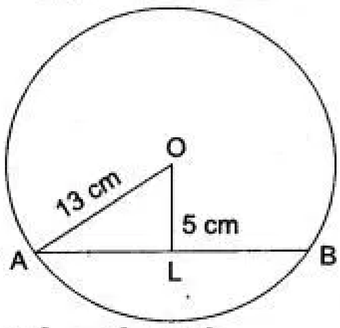
Question 6. Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 13 cm.

Solution : Let AB be a chord of a circle with centre O and radius 13 cm. Draw $OL \perp AB$.

Join OA. Clearly, OL = 5 cm and OA = 13 cm.

In the right triangle OLA, we have

$$OA^2 = OL^2 + AL^2$$



$$\Rightarrow \quad 13^2 = 5^2 + AL^2$$

$$\Rightarrow \quad AL^2 = 144 \text{ cm}^2$$

$$\Rightarrow \quad AL = 12 \text{ cm}$$

Since, the perpendicular from the centre to the chord bisects the chord. Therefore,

$$AB = 2AL = (2 \times 12) \text{ cm}$$

$$= 24 \text{ cm.} \quad \text{Ans.}$$

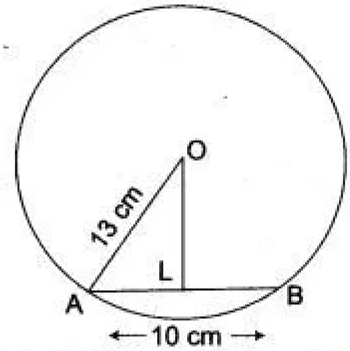
Question 7. The radius of a circle is 13 cm and the length of one of its chord is 10 cm. Find the distance of the chord from the centre.

Solution : Let AB be a chord of a circle with centre O and radius 13 cm such that AB = 10 cm.

From O, draw OL ⊥ AB. Join OA.

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AL = LB = \frac{1}{2}AB = 5 \text{ cm}$



Now, in right triangle OLA, we have

$$OA^2 = OL^2 + AL^2$$
$$\Rightarrow \quad 13^2 = OL^2 + 5^2$$
$$\Rightarrow \quad 13^2 - 5^2 = OL^2$$
$$\Rightarrow \quad OL^2 = 144$$
$$\Rightarrow \quad OL = 12 \text{ cm.}$$

Hence, the distance of the chord from the centre is 12 cm. Ans.

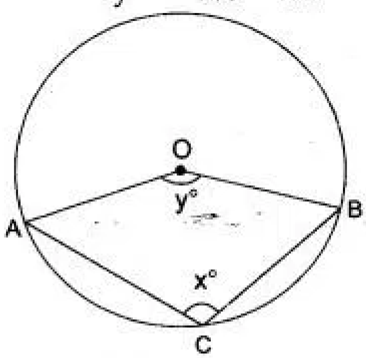
Question 8. C is a point on the minor arc AB of the circle, with centre O. Given ∠ ACB = x° and ∠ AOB = y° express y in terms of x. Calculate x, if ACBO is a parallelogram.

Solution. Clearly, major arc AB subtends x° at a point on the remaining part of the circle.

$\therefore \quad \text{reflex} \angle AOB = 2x^\circ$

$\Rightarrow \quad 360^\circ - y^\circ = 2x^\circ$

$y^\circ = 360^\circ - 2x^\circ$



Thus, $\quad y = 360^\circ - 2x^\circ$

If ACBO is a parallelogram, then

$x^\circ = y^\circ$ i.e., $x = y$

$\Rightarrow \quad x = 360^\circ - 2x \Rightarrow 3x = 360^\circ$

$\Rightarrow \quad x = 120^\circ.$ Ans.

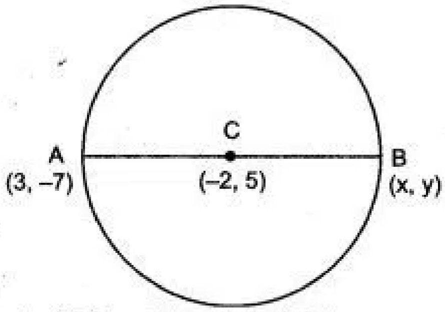
Question 9. AB is a diameter of a circle with centre C = (- 2, 5). If A = (3, – 7). Find
(i) the length of radius AC

(ii) the coordinates of B.

Solution. (i) $AC = \sqrt{(3+2)^2 + (-7-5)^2}$ ($\because$ Distance formula)

$= \sqrt{25 + 144}$

$\therefore$ Radius $= \sqrt{169} = 13$ units



(ii) As 'C' is mid point of AB

$-2 = \frac{3+x}{2}$ [$\because$ mid point formula]

or $-4 = 3 + x$

$x = -7$

and $5 = \frac{-7+y}{2}$

and $10 = -7 + y$

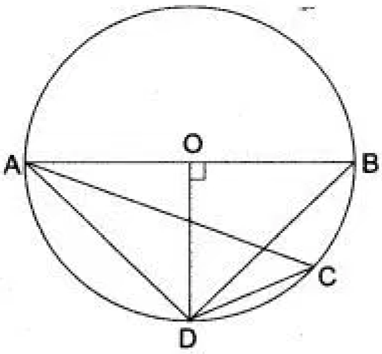
and $y = 17$

$\therefore$ B (–7, 17). Ans.

Question 10. AB is a diameter of a circle with centre O and radius OD is perpendicular to AB. If C is any point on arc DB, find ∠ BAD and ∠ ACD.

Solution. Since, chord BD makes $\angle BOD$ at the centre and $\angle BAD$ at A.

$\therefore \angle BAD = \frac{1}{2}\angle BOD = \frac{1}{2} \times (90°) = 45°$



Similarly, chord AD makes $\angle AOD$ at the centre and $\angle ACD$ at C.

$\therefore \angle ACD = \frac{1}{2}\angle AOD$

$= \frac{1}{2} \times (90°) = 45°$

Thus, $\angle BAD = \angle ACD = 45°$. Ans.

Question 11. In the given below figure,

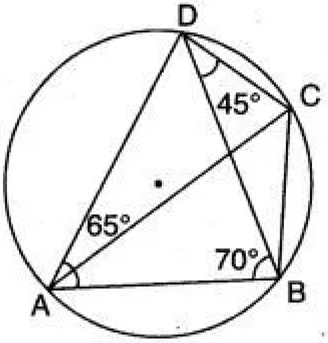
$\angle BAD = 65°$

$\angle ABD = 70°$

and $\angle BDC = 45°$. Find :

(i) $\angle BCD$, (ii) $\angle ADB$.

Hence show that AC is a diameter.



Solution. Given : $\angle BAD = 65°$,

$\angle ABD = 70°$

and $\angle BDC = 45°$.

(i) Quadrilateral ABCD is cyclic quadrilateral.

$\therefore \angle DAB + \angle BCD = 180°$

$65° + \angle BCD = 180°$

$\therefore \angle BCD = 180° - 65°$

$= 115°$ Ans.

(ii) In Δ ADB,

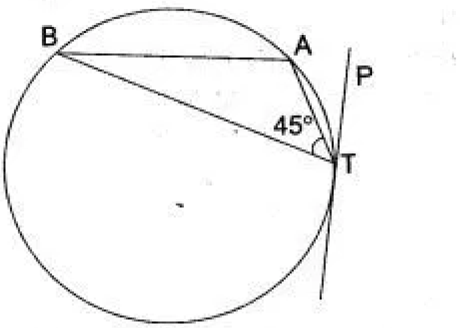
$\angle DAB + \angle ABD + \angle ADB = 180°$

$65° + 70° + \angle ADB = 180°$

$\angle ADB = 180° - 135°$

$\angle ADB = 45°$ **Ans.**

Question 12. If Fig. PT is a tangent to a circle. If m ($\angle$ BTA) = 45° and m ($\angle$ PTB) = 70°, find m $\angle$ (ABT).



Solution : $\angle ATP = \angle PTB - \angle BTA$

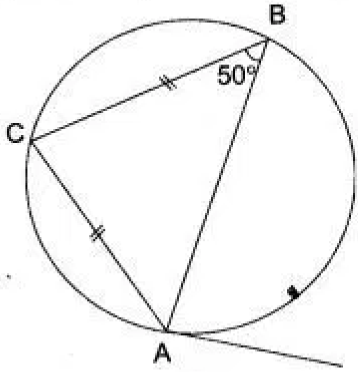
$= 70° - 45° = 25°$

$\therefore \angle ABT = \angle ATP$

(Angles are in alternate segments)

$\Rightarrow \angle ABT = 25°$. Ans.

Question 13. In Fig. AT is a tangent to the circle. If m∠ABC = 50°, AC = BC, find ∠BAT.



Solution : AC = BC

⇒ ∠CBA = ∠CAB

⇒ ∠CAB = 50°

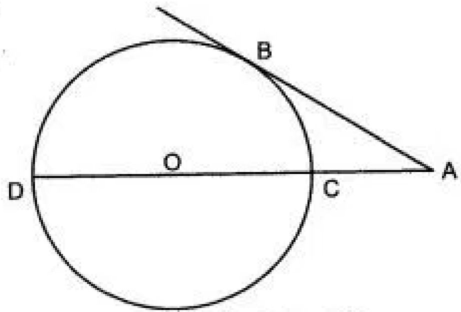
∴ ∠ACB = 180° – (50° + 50°) = 80°

Now, ∠BAT = ∠BCA

[Angles are in alternate segments]

⇒ ∠BAT = 80°. Ans.

Question 14. In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.



Solution : $AB^2 = AC \times AD$

$(PT^2 = PA \times PB)$

⇒ $15^2 = 7{\cdot}5 \times AD$

⇒ $AD = \frac{225}{7{\cdot}5} = 30$

⇒ $CD = AD - AC$

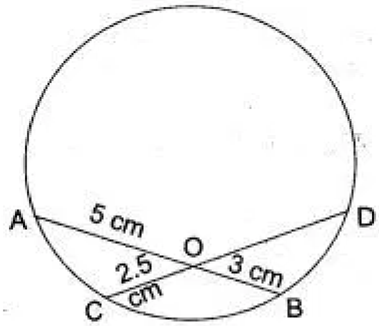
$= 30 - 7{\cdot}5 = 22{\cdot}5$

∴ Radius $= \frac{1}{2} \times CD$

Radius $= \frac{1}{2} \times 22{\cdot}5$

Radius $= 11{\cdot}25$ cm.

Question 15. In Fig., chords AB and CD of the circle intersect at O. AO = 5 cm, BO = 3 cm and CO = 2.5 cm. Determine the length of DO.



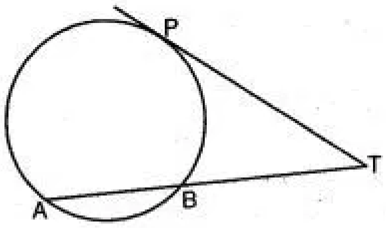
Solution : Clearly, chords AB and CD intersect at O.

$\therefore \quad OA \times OB = OC \times OD$

$\Rightarrow \quad 5 \times 3 = 2{\cdot}5 \times OD$

$\Rightarrow \quad OD = \left(\frac{5 \times 3}{2{\cdot}5}\right) = 6$ cm. Ans.

Question 16. In the figure given below, PT is a tangent to the circle. Find PT if AT = 16 cm and AB = 12 cm.



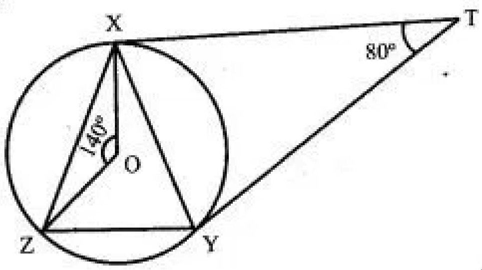
Solution : PT is tangent.

Hence by theorem,

$$\begin{aligned} PT^2 &= AT \times BT \\ &= 16 \times (AT - AB) \\ &= 16 \times (16 - 12) \\ &= 16 \times 4 = 64 \end{aligned}$$

$\therefore \quad PT = 8$ cm.

Question 17. In the alongside, figure, O is the centre of the circumcircle of triangle XYZ. Tangents at X and Y intersect at T. Given $\angle XTY = 80°$ and $\angle XOZ = 140°$. Calculate the value of $\angle ZXY$.



Solution :

$\angle TXY = \angle TYX = 50°$ (since XT = YT)

$\angle OXZ = \angle OZX = 20°$ (since OX = OZ)

[In ΔXOZ, $140° + \angle OXZ + \angle OZX = 180°$]

$\Rightarrow$ $\angle OXZ = 20°$

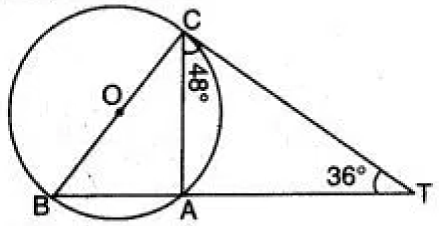
$\angle OXY = 40°$, (since $\angle OXT = 90°$)

$\angle ZXY = \angle OXZ + \angle OXY$

$= 20° + 40° = 60°$. Ans.

Question 18. A, B and C are three points on a circle. The tangent at C meets BN produced at T. Given that ∠ ATC = 36° and ∠ ACT = 48°, calculate the angle subtended by AB at the centre of the circle.

Solution : Join BC. Let O be the centre of the circle. Join OA and OB.



In Δ BCT, Δ ACT,

$\angle BTC = \angle ATC = 36°$

$\angle ACT = \angle ABC = 48°$

$\angle BAC = \angle ACT + \angle ATC$

$= 48° + 36° = 84°$

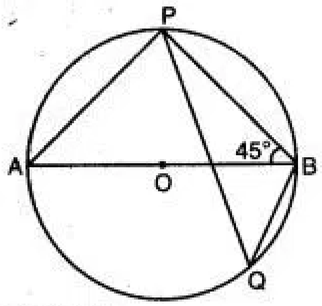
$\therefore$ $\angle BCA = 180° - (\angle ABC + \angle BAC)$

$= 180° - (48° + 84°) = 48°$

$\therefore$ $\angle BOA = 2 \angle BCA$

$= 2 \times 48° = 96°$ Ans.

Question 19. In the given figure, O is the centre of the circle and ∠PBA = 45°. Calculate the value of ∠PQB.



Solution : Given

$\angle PBA = 45°$

AOB is diameter of circle.

$\therefore$ $\angle APB = 90°$

(Angle in semi circle)

So in Δ APB, $\angle PAB = 180° - (90° + 45°)$

$= 45°$

$\angle PAB = \angle PQB$

(Angle in same segment)

$\therefore$ $\angle PQB = 45°$. Ans.

Question 20. Two chords AB, CD of lengths 16 cm and 30 cm, are parallel. If the distance between AB and CD is 23 cm. Find the radius of the circle.

Solution : Let AB and CD be the two parallel chords of a circle with centre O and radius r cm.

$$OM \perp AB \text{ and } ON \perp CD$$

$$AM = \frac{1}{2}AB = \frac{1}{2} \times 16 = 8 \text{ cm}$$

$$CN = \frac{1}{2}CD = \frac{1}{2} \times 30 \text{ cm} = 15 \text{ cm}$$

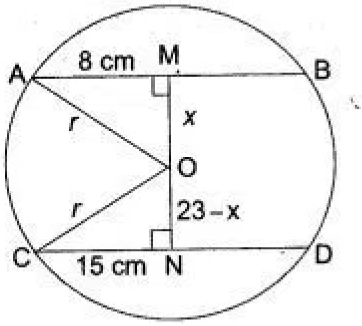
let $OM = x$ cm, $MN = 23$ cm

so $ON = (23 - x)$ cm

$OA = OC = r$ cm

InΔ OAM, $OA^2 = AM^2 + OM^2$

$\Rightarrow r^2 = (8)^2 + x^2$ ...(i)



In Δ OCN, $OC^2 = CN^2 + ON^2$

$\Rightarrow r^2 = (15)^2 + (23 - x)^2$ ...(ii)

From (i) & (ii),

$x^2 + 64 = 225 + (23 - x)^2$

$\Rightarrow x^2 + 64 = 225 + 529 - 46x + x^2$

$\Rightarrow 46x = 225 + 529 - 64$

$\Rightarrow 46x = 690 \Rightarrow x = 15$ cm

From (i), $r^2 = (8)^2 + (15)^2 = 64 + 225 = 289$

$\Rightarrow r = 17$ cm

Hence, the radius of a circle is 17 cm. Ans.

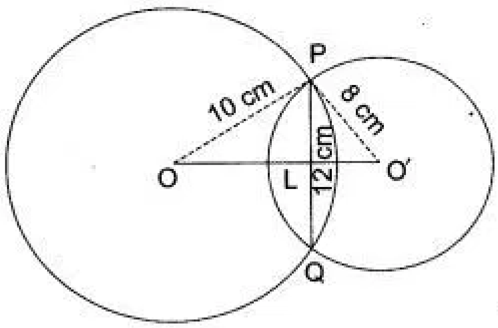
Question 21. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.

Solution : Let O and O′ be the centres of two circles with radii 10 cm and 8 cm respectively.

So $OP = 10$ cm, $O'P = 8$ cm

and $PQ = 12$ cm

then $PL = \frac{1}{2}PQ = 6$ cm



In ΔOLP, $OP^2 = OL^2 + LP^2$

$\Rightarrow \quad OL^2 = OP^2 - LP^2$

$OL = \sqrt{(10)^2 - 6^2} = \sqrt{64} = 8$ cm

In $\Delta O'LP$,

$O'L = \sqrt{O'P^2 - LP^2} = \sqrt{8^2 - 6^2}$

$= \sqrt{64 - 36}$

$O'L = \sqrt{28}$ cm

$= 5 \cdot 29$ cm

Distance between centres

$OO' = OL + LO'$

$= (8 + 5 \cdot 29)$ cm

$= 13 \cdot 29$ cm.

Question 22. AB and CD are two chords of a circle such that AB = 6 cm, CD = 12 cm and AB || CD. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution : Let AB and CD be two parallel chords

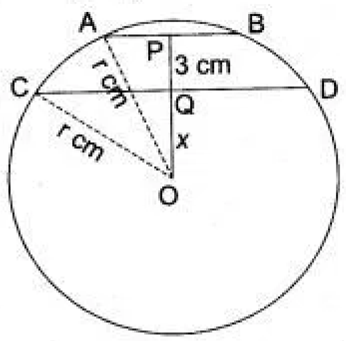
of a circle with centre O such that AB = 6 cm and CD = 12 cm. Let the radius of the circle be r cm. Draw $OP \perp AB$ and $OQ \perp CD$. Since, AB || CD and $OP \perp AB$, $OQ \perp CD$. Therefore, points O, Q and P are collinear. Clearly, PQ = 3 cm.

Let OQ = x cm. Then, OP = (x + 3) cm.

In right triangles OAP and OCQ, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$\Rightarrow \quad r^2 = (x+3)^2 + 3^2$ and $r^2 = x^2 + 6^2$



$\left[\because AP = \frac{1}{2} AB = 3 \text{ cm and } CQ = \frac{1}{2} CD = 6 \text{ cm}\right]$

$\Rightarrow \quad (x+3)^2 + 3^2 = x^2 + 6^2$

(on equating the value of r^2)

$\Rightarrow \quad x^2 + 6x + 9 + 9 = x^2 + 36$

$\Rightarrow \quad 6x = 18 \Rightarrow x = 3$ cm

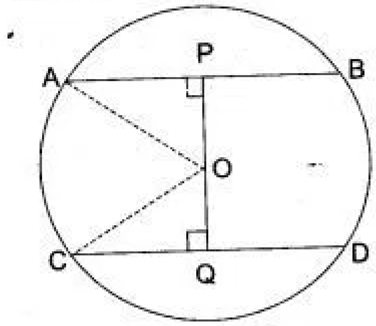
Putting the values of x in $r^2 = x^2 + 6^2$, we get

$r^2 = 3^2 + 6^2 = 45$

$\Rightarrow \quad r = \sqrt{45}$ cm $= 6 \cdot 7$ cm

Hence, the radius of the circle is $6 \cdot 7$ cm. Ans.

Question 23. In Fig. O is the centre of the circle with radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB || CD, AB = 8 cm and CD = 6 cm. Determine PQ.



Solution : Join OA and OC.

Since, the perpendicular from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently, $AP = PB = \frac{1}{2}AB = 3$ cm

and $CQ = QD = \frac{1}{2}CD = 4$ cm

In right triangles OAP and OCQ, we have

$$OA^2 = OP^2 + AP^2 \text{ and } OC^2 = OQ^2 + CQ^2$$

$$\Rightarrow 5^2 = OP^2 + 3^2 \text{ and } 5^2 = OQ^2 + 4^2$$

$$\Rightarrow OP^2 = 5^2 - 3^2 \text{ and } OQ^2 = 5^2 - 4^2$$

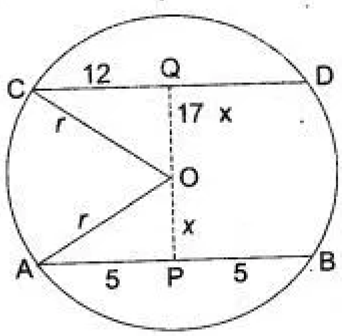
$$\Rightarrow OP^2 = 16 \text{ and } OQ^2 = 9$$

$$\Rightarrow OP = 4 \text{ and } OQ = 3$$

$$\therefore PQ = OP + OQ = (4 + 3) \text{ cm} = 7 \text{ cm.}$$

Question 24. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, find the radius of the circle.

Solution : Let O be the centre of the given circle and let it's radius be r cm. Draw OP ⊥ AB and OQ ⊥ CD. Since, AB || CD. Therefore, points P, O and Q are collinear. So, PQ = 17 cm.



Let OP = x cm. Then, OQ = (17 – x) cm.

Join OA and OC. Then, OA = OC = r.

Since, the perpendicular from the centre to a chord of the circle bisects the chord.

$\therefore AP = PB = 5$ cm and $CQ = QD = 12$ cm

In right triangles OAP and OCQ, we have

$$OA^2 = OP^2 + AP^2 \text{ and } OC^2 = OQ^2 + CQ^2$$

$$\Rightarrow \quad r^2 = x^2 + 5^2 \quad \ldots(i)$$

and $$r^2 = (17 - x)^2 + 12^2 \quad \ldots(ii)$$

$$\Rightarrow \quad x^2 + 5^2 = (17 - x)^2 + 12^2$$

[On equating the values of r^2]

$$\Rightarrow \quad x^2 + 25 = 289 - 34x + x^2 + 144$$

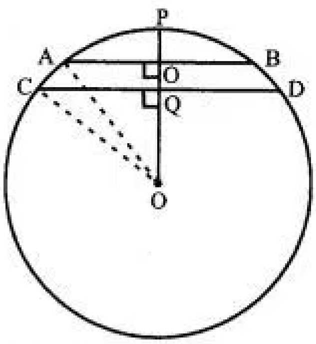
$$\Rightarrow \quad 34x = 408 \Rightarrow x = 12 \text{ cm.}$$

Putting $x = 12$ cm in (i), we get

$$r^2 = 12^2 + 5^2 = 169 \Rightarrow r = 13 \text{ cm}$$

Hence, the radius of the circle is 13 cm. Ans.

Question 25. In Fig. O is the centre of the circle of radius 5 cm. $OP \perp AB$, $OQ \perp CD$, $AB \parallel CD$, AB = 6 cm and CD = 8 cm. Determine PQ.



Solution : Join OA and OC.

Since, the perpendicular from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently,

$$AP = PB = \frac{1}{2}AB = 3 \text{ cm}$$

and $$CQ = QD = \frac{1}{2}CD = 4 \text{ cm}$$

In the right angled triangle OAP, we have

$$OA^2 = OP^2 + AP^2$$

$$\Rightarrow \quad 5^2 = OP^2 + 3^2$$

$$\Rightarrow \quad OP^2 = 5^2 - 3^2 = 16 \text{ cm}^2 \Rightarrow OP = 4 \text{ cm}^2$$

In the right angled triangle OCQ we have

$$OC^2 = OQ^2 + CQ^2$$

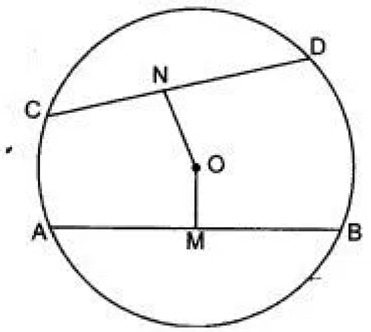
$$\Rightarrow \quad 5^2 = OQ^2 + 4^2$$

$$\Rightarrow \quad OQ^2 = 5^2 - 4^2 = 9 \text{ cm}^2 \Rightarrow OQ = 3 \text{ cm}$$

$$\therefore \quad PQ = PO - QO$$

$$= OP - OQ = (4 - 3) \text{ cm} = 1 \text{ cm.}$$

Question 26. In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD.



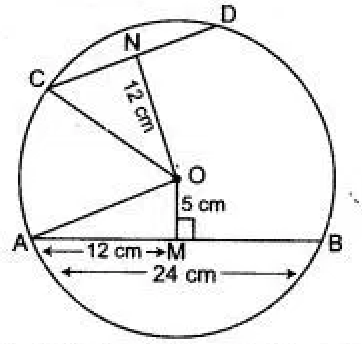
AB = 24 cm. OM = 5 cm, ON = 12 cm. Find the

(i) radius of the circle.

(ii) length of chord CD.

Solution :

AB = 24 cm, ON = 12 cm, OM = 5 cm.



(i) In Δ AOM, $OA^2 = OM^2 + AM^2$

$= (5)^2 + (12)^2$

$= 25 + 144 = 169$

$OA = 13$ cm

Thus, radius of the circle is 13 cm.

(ii) In Δ CON, $OC^2 = ON^2 + CN^2$

$(13)^2 = (12)^2 + CN^2$

$[\because OC = OA = 13 \text{ (Radius)}]$

$169 - 144 = CN^2$

$CN^2 = 25$

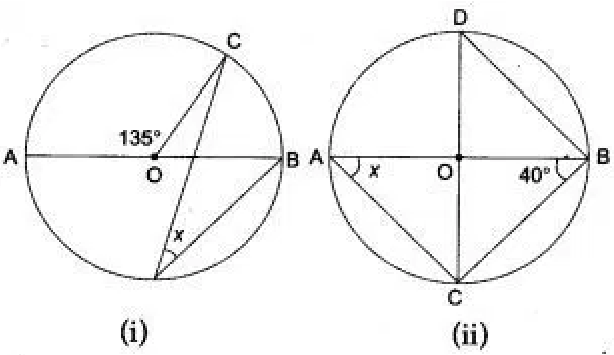
$CN = 5$

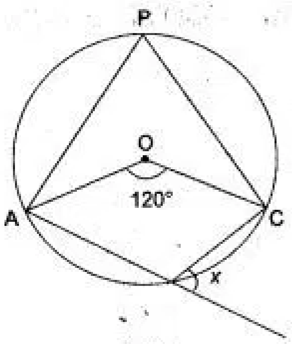
Thus length of chord CD $= 2\,CN$

$= 2 \times 5$

$= 10$ cm. Ans.

Question 27. If O is the centre of the circle, find the value of x in each of the following figures





(iii)

Solution : (i) We have $AOC = 135°$

then $COB = 180° - 135° = 45°$

but $x = \frac{COB}{2} = \frac{45}{2}° = 22\frac{1}{2}°$ Ans.

(ii) In BOC,

$OB = OC$ (radii)

So $OCB = OBC = 40°$

In BOC,

$BOC = 180° - (40 + 40)°$

$= 180° - 80°$

So $x = \frac{BOC}{2}$

$= \frac{100°}{2} = 50°$. Ans.

(i $= \frac{1}{2}\ 120° = 60°$

Since, $ABCD$ is a cyclic quadrilateral

$CBD = x$ is the exterior angle

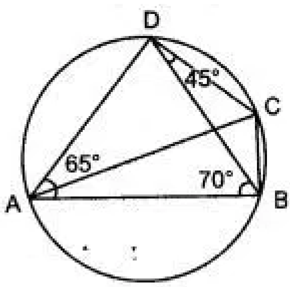
so $x = APC = 60°$.

Question 28. In the given figure, BAD = 65°

ABD = 70°, BDC = 45°

(i) Prove that AC is a diameter of the circle.

(ii) Find ACB



Solution : Given

BAD = 65°

ABD = 70°

BDC = 45°

(i) In ABD,

BAD + ABD + ADB = 180°

65° + 70° + ADB = 180°

{Sum of three angles of a }

ADB = 180° – (65° + 70°)

= 45°

ADC = ADB + BDC

45° + 45° = 90°

AC is the diameter of the circle.

[Angle in a semi circle is 90°] Proved

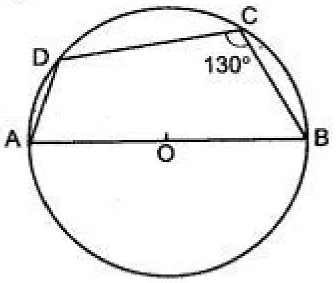
(ii) ACB = ADB = 45°

{Angles in the same segment of a circle}

Question 29. In the given figure, AB is the diameter of a circle with centre O. BCD = 130°. Find :

(i) DAB

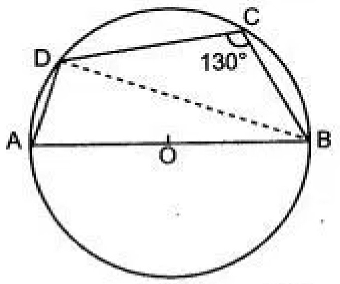
(ii) DBA



Solution : (i) DAB + BCD = 180°

[opp. angles of a cyclic quadrilateral]

DAB + 130° = 180°



DAB = 180° – 130°

DAB = 50° Ans.

(ii) ADB = 90° (angle in semi-circle)

In ADB

DAB + ADB + DBA =180°

(angle sum property)

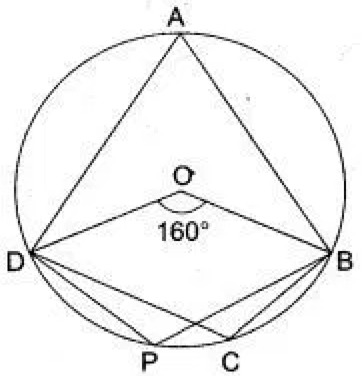
50° + 90° + DBA = 180°

DBA = 180° – 140°

DBA = 40° Ans.

Question 30. In ABCD is a cyclic quadrilateral; O is the centre of the circle. If BOD = 160°, find the measure of BPD.

Solution : Consider the arc BCD of the circle. This arc makes angle $\angle$ BOD =160 ° at the centre of the circle and $\angle$ BAD at a point A on the circumference.



$\therefore \quad \angle BAD = \frac{1}{2} \angle BOD = 80°$

Now, ABPD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BPD = 180°$

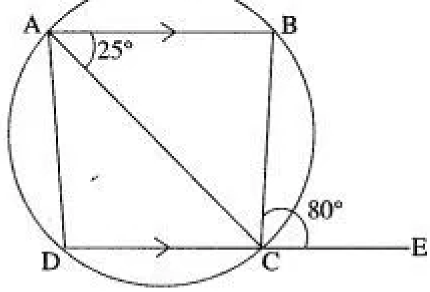
$\Rightarrow \quad 80° + \angle BPD = 180°$

$\Rightarrow \quad \angle BPD = 100°$

$\Rightarrow \quad \angle BCD = 100°$

[$\because$ $\angle$ BPD and $\angle$ BCD are angles in the same segment $\therefore$ $\angle BCD = \angle BPD$] = 100° Ans.

Question 31. In the given below figure, AB is parallel to DC, $\angle BCE = 80°$ and $\angle BAC = 25°$, find (i) $\angle$ CAD, (ii) $\angle$ CBD, (iii) $\angle$ ADC



Solution : (i) $\angle CAD = \angle BCE - \angle CAB$

$= 80° - 25°$

$= 55°$

$\because$ Ext. of cyclic is equal to opp. int. Ans.

(ii) $\angle CBD = \angle CAD$

[Angles in the same segment]

$= 55°$ Ans.

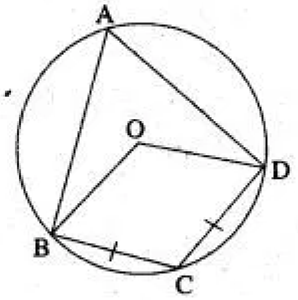
(iii) $\angle ADC = 180° - \angle DAB$

$= 180° - 80°$

$= 100°$ Ans.

Question 32. In the given figure O is the centre of the circle, ∠ BAD = 75° and chord BC = chord CD. Find:

(i) ∠ BOC (ii) ∠ OBD (iii) ∠ BCD.



Solution :

(i) $\angle BOD = 2 \cdot \angle BAD$

$= 2 \times 75° = 150°$

$\angle BOC = \angle COD$

$\because$ BC = CD

$\therefore \angle BOD = 2 \angle BOC$

$\therefore \angle BOC = \frac{1}{2} \angle BOD = 75°$ Ans.

(ii) $\angle OBD = \frac{1}{2}(180° - \angle BOD)$

$= \frac{1}{2}(180° - 150°) = 15°$ Ans.

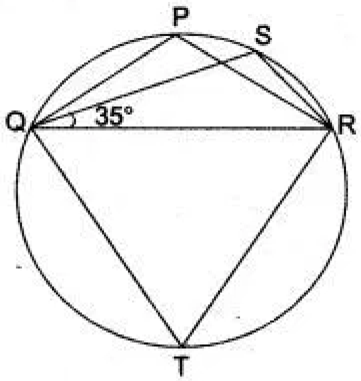
(iii) $\angle BCD = 180° - \angle BAD$

(opp. ∠s of a cyclic quardilateral is supplementary)

$= 180° - 75°$

$= 105°$ Ans.

Question 33. In Fig. ΔPQR is an isosceles triangle with PQ = PR and m∠PQR = 35°. Find m∠ QSR and m∠QTR.



Solution : In ΔPQR, we have

PQ = PR

$\Rightarrow \angle PQR = \angle PRQ$

$\Rightarrow \angle PRQ = 35°$

$\therefore \angle QPR = 180° - (\angle PQR + \angle PRQ)$

$= 180° - (35° + 35°) = 110°.$

Since, PQTR is a cyclic quadrilateral.

$\therefore \angle P + \angle T = 180°$

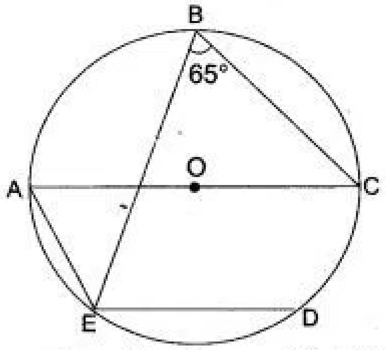
$\Rightarrow \angle T = 180° - 110° = 70°$ Ans.

In cyclic quadrilateral QSRT, we have

$\angle S + \angle T = 180°$

$\Rightarrow \angle S = 180° - 70° = 110°.$ Ans.

Question 34. In Fig., chord ED is parallel to the diameter AC of the circle. Given ∠ CBE = 65°, calculate ∠ DEC.



Solution : Consider the arc CDE. We find that ∠CBE and ∠CAE are the angles in the same segment of arc CDE.

$\therefore$ ∠ CAE = ∠ CBE

$\Rightarrow$ ∠ CAE = 65° [∵ ∠ CBE = 65°]

Since, AC is the diameter of the circle and the angle in a semi-circle is a right angle. Therefore, ∠ AEC = 90°.

Now, in Δ ACE, we have

∠ ACE + ∠ AEC + ∠ CAE = 180°

$\Rightarrow$ ∠ ACE + 90° + 65° = 180°

$\Rightarrow$ ∠ ACE = 25°

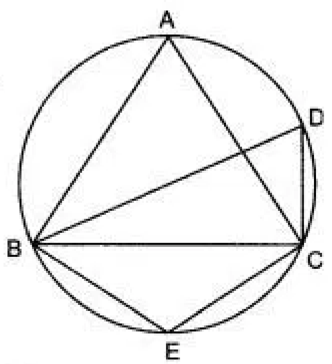
But ∠DEC and ∠ ACE are alternate angles, because AC | | DE.

$\therefore$ ∠ DEC = ∠ ACE = 25°. Ans.

Question 35. In the figure, ∠ DBC = 58°, BD is diameter of the circle. Calculate :

(i) ∠ BDC (ii) ∠ BEC

(iii) ∠ BAC



Solution :

∠ DBC = 58° (given)

Now, BD is the diameter

∠ BCD = 90°

(angle in a semicircle)

In Δ BDC

∠BDC + 90° + 58° = 180°

(Sum of the angles of a triangle)

$\therefore$ ∠ BDC = 180 – (90 + 58) = 32°

(ii) BECD is a cyclic quadrilateral

$\because \angle BEC + \angle BDC = 180°$

(Opp. angles of a cyclic quadrilateral)

$\therefore \angle BEC = 180 - \angle BDC$

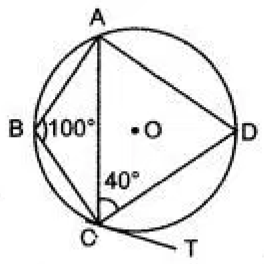
$= 180° - 32° = 148°$ Ans.

(iii) $\angle BAC = \angle BDC = 32°$

(angles in the same segment of a circle)

Ans.

Question 36. In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.



Solution : Given

$\angle ABC = 100°$
$\angle ACD = 40°$ } Given

$\angle ABC + \angle ADC = 180°$

{opposite angles of a cyclic quadrilateral}

$100 + \angle ADC = 180°$

$\therefore \angle ADC = 180 - 100 = 80°$

Also

$\angle ACD + \angle ADC + \angle CAD = 180°$

{sum of angles of a Δ}

$40° + 80° + \angle CAD = 180°$

$\angle CAD = 180 - 120 = 60°$

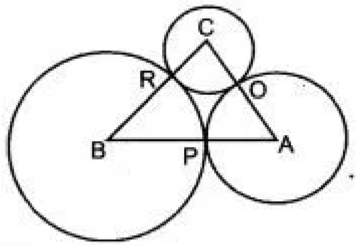
Now, $\angle DCT = \angle CAD$

$= 60°$

{Alternate segment theorem}

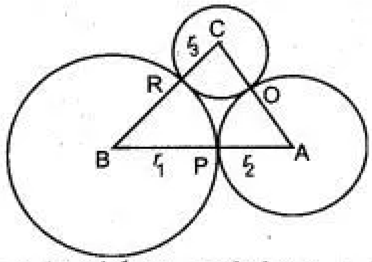
Ans.

Question 37. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.



Solution :

AB = 10 cm
BC = 8 cm
AC = 6 cm



Let the radii of three circle be r_1, r_2 & r_3 (shown in fig.)

$$r_1 + r_2 = 10 = AB \quad ...(1)$$
$$r_2 + r_3 = 6 = AC \quad ...(2)$$
$$r_3 + r_1 = 8 = BC \quad ...(3)$$

Adding (1), (2) and (3), we get

$$2(r_1 + r_2 + r_3) = 10 + 6 + 8 = 24$$
$$r_1 + r_2 + r_3 = 12 \quad ...(4)$$

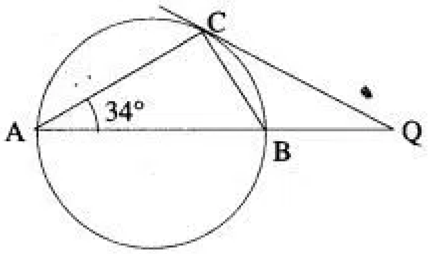
Subtract (4) and (1) $\Rightarrow r_3 = 12 - 10 = 2$ cm

Subtract (4) and (2) $\Rightarrow r_1 = 12 - 6 = 6$ cm

Subtract (4) and (3) $\Rightarrow r_2 = 12 - 8 = 4$ cm

Question 38. In the given figure, AB is a diameter. The tangent at C meets AB produced at Q. If $\angle CAB = 34°$, find :

(i) $\angle CBA$, (ii) $\angle CQA$.



(i) AB is diameter.

$\therefore \quad \angle ACB = 90°$

Angle in semicircle is rt. angle

$\therefore$ In Δ ACB,

$$\angle A + \angle C + \angle B = 180$$
$$34 + 90 + \angle B = 180$$
$$\angle B = 180 - (90 + 34)$$
$$= 180 - 124$$

$\therefore \quad \angle CBA = 56°$

(ii) Now CQ is tangent

$\therefore \quad \angle QCB = \angle CAB$

(Alternate segment angle)

$= 34°$

and $\angle CBQ = 180 - \angle CBA$

$= 180 - 56 = 124°$

$\therefore \quad \angle CQA = 180 - (\angle QCB + \angle CBQ)$

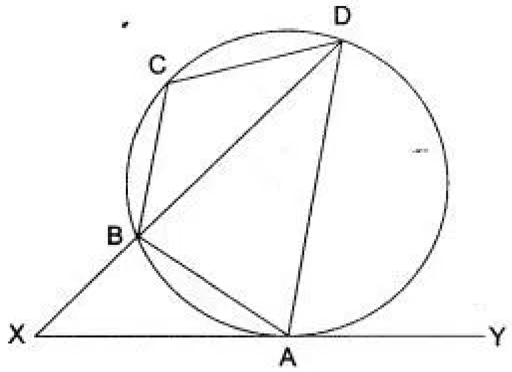
$= 180 - (34 + 124)$

$= 180 - 158 = 22°$ Ans.

Question 39. In the joining figure shown XAY is a tangent. If ∠ BDA = 44°, ∠ BXA = 36°, Calculate :

(i) ∠ BAX (ii) ∠ DAY (iii) ∠ DAB

(iv) ∠ BCD.



Solution : (i) ∠ BAX = ∠ BDA = 44°

[Angles in the alternate segment] Ans.

(ii) ∠ ABD = ∠ BXA + ∠ BAX

= 36° + 44° = 80°

[Ext. angle of a Δ = sum of int. opp. angles]

∴ ∠ DAY = ∠ ABD = 80°

[Angles in alternate segment] Ans.

(iii) ∠ DAB = 180 – [∠BAX + ∠ DAY]

= 180 – (44° + 80°) = 56°

Ans.

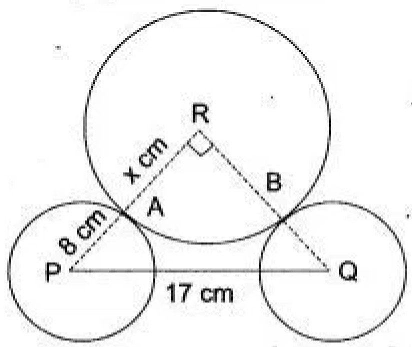
(iv) ∠ BCD = 180° – ∠ DAB

= 180° – 56° = 124° Ans.

[Opp. ∠s of a cyclic quad. are supplementary]

Question 40. P and Q are the centre of circles of radius 9 cm and 2 cm respectively; PQ = 17 cm. R is the centre of circle of radius x cm, which touches the above circles externally, given that ∠ PRQ = 90°. Write an equation in x and solve it.

Solution : Let the circle with centre R touch the given two circles at A and B. Then, P, A, R are collinear and Q, B, R are collinear.



Since, ∠ PRQ = 90°, by Pythagoras theorem,

$$PQ^2 = PR^2 + QR^2$$

$$\Rightarrow \quad 17^2 = (9 + x)^2 + (2 + x)^2$$

$$\Rightarrow \quad x^2 + 11x - 102 = 0$$

$$\Rightarrow \quad (x + 17)(x - 6) = 0$$

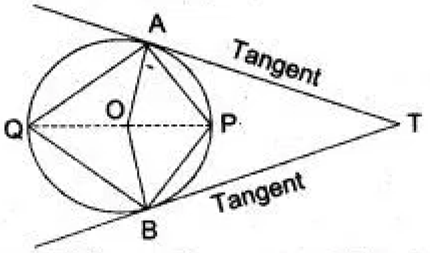
$$\Rightarrow \quad x = 6 \text{ cm}$$

($x = -17$ is not possible).

Ans.

Question 41. In the adjoining diagram TA and TB are tangents, O is the centre. If ∠PAT = 35° and ∠PBT = 40°. Calculate :

(i) ∠ AQP (ii) ∠ BQP
(iii) ∠ AQB (iv) ∠ APB
(v) ∠ AOB (vi) ∠ ATB.



Solution : (i) ∠ AQP = ∠ PAT = 35°

[Angles are in alternate segment] Ans.

(ii) ∠ BQP = ∠ PBT = 40°

[Angles are in alternate segment] Ans.

(iii) ∠ AQB = ∠ AQP + ∠ BQP
= 35° + 40° = 75°. Ans.

(iv) ∠ APB + ∠ AQB = 180°

[Opp. ∠s of a cyclic quadrilateral are supplementary]

∴ ∠ APB + 75° = 180°

∴ ∠ APB = 105°. Ans.

(v) ∠ AOB = 2∠ AQB
= 2(75°) = 150°. Ans.

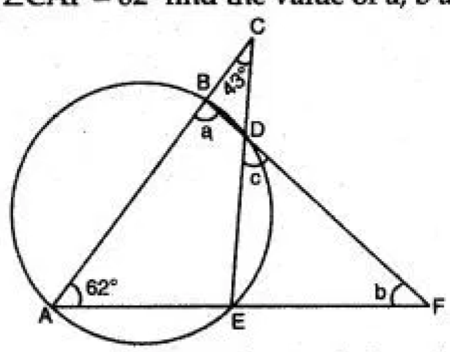
[Angle at the centre = 2 Angle at the circumference]

(vi) In quadrilateral AOBT :

∠ ATB = 360° – (∠ OAT + ∠ OBT + ∠ AOB)
= 360° – (90°+ 150° + 90°) = 30°

[∠ OAT = ∠ OBT = 90° rad, ⊥ tangent] Ans.

Question 42. In the given figure, if ∠ACE = 43° and ∠CAF = 62° find the value of a, b and c.



Solution : ABDE is cyclic quadrilateral.

∴ ∠ ABD + ∠ AED = 180°

and ∠ EAB + ∠ BDE = 180°

Now in Δ ACE

∠ A + ∠ C + ∠ E = 180°

62° + 43° + ∠ E = 180°

∠ E = 180° – 105° = 75°

so $\angle ABD + \angle AED = 180°$

$\therefore \quad a + 75° = 180°$

$\therefore \quad a = 105°$

$\angle EDF = \angle BAE$

(exterior angle of cyclic quadrilateral)

$62° = c$

$\therefore \quad c = 62°$

In Δ ABF,

$\angle ABF + \angle BAF + \angle BFA = 180°$

$105° + 62° + b = 180°$

$167° + b = 180°$

$b = 180° - 167°$

$b = 13°$

$\therefore \quad a = 105°, b = 13°$

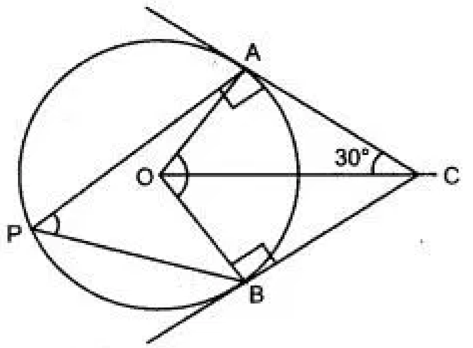
and $\quad c = 62°$. Ans.

Question 43. In the given figure O is the centre of the circle. Tangents at A and B meet at C.

If $\angle ACO = 30°$, find

(i) $\angle BCO$ (ii) $\angle AOB$

(iii) $\angle APB$



Solution :

$\angle BCO = \angle ACO = 30°$ Ans.

($\therefore$ C is the intensecting point of tangents AC and BC)

(ii) $\angle OAC = \angle OBC = 90°$

$\angle ACO = 30°$ Given

$\therefore \quad \angle AOC = \angle BOC$

$= 180° - (90° + 30°)$

($\therefore$ sum of the 3 angles of a Δ is 180°)

$\angle AOC = 180° - 120°$

$\angle AOC = 60°$

$\therefore \quad \angle AOB = \angle AOC + \angle BOC$

$= 60° + 60°$

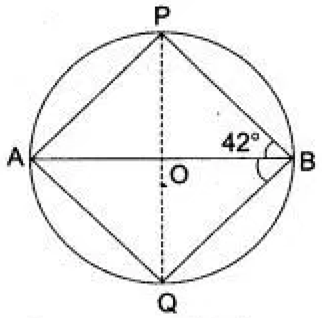
$\angle AOB = 120°$ Ans.

(iii) $\angle APB = \frac{1}{2} \angle AOB = \frac{120°}{2} = 60°$

Ans.

($\therefore$ Angle substended at the remaining part of the circle is half the $\angle$ substended at the centre}

Question 44. In the following figure, O is the centre of the circle, $\angle PBA = 42°$. Calculate :
(i) $\angle APB$ (ii) $\angle PQB$ (iii) $\angle AQB$.



Solution : (i) In circle C(O, *r*)
AB is the diameter
So $\angle APB = 90°$ (Angle in semi-circle)
(ii) Now in ΔAPB

$$\angle PAB = 180 - (\angle APB + \angle ABP)$$
$$= 180 - (90° + 42°)$$
$$= 180° - 132° = 48°$$
$$\angle PQB = \angle PAB = 48°$$

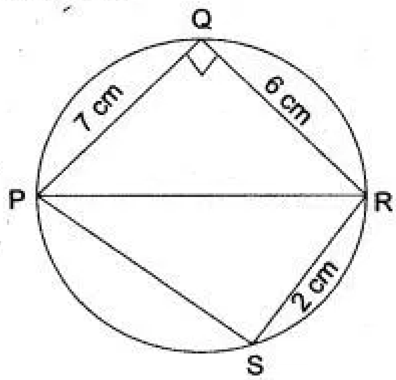
(Angles of the same segment)

Hence
$\angle PQB = 48°$. Ans.

(iii) AQBP is a cyclic quadrilateral.
Therefore

$$\angle APB + \angle AQB = 180°$$
$$\Rightarrow 90° + \angle AQB = 180°$$
$$\Rightarrow \angle AQB = 180° - 90° = 90°. \text{ Ans.}$$

Question 45. In the figure alongside PR is a diameter of the circle, PQ = 7 cm; QR = 6 cm and RS = 2 cm. Calculate the perimeter of the cyclic quadrilateral PQRS.



Solution : PR is the diameter of the circle
then $\angle PQR = 90°$,
$\because$ (Angle in semi-circle)

In ΔPQR, $PR = \sqrt{(7)^2 + (6)^2}$
$= \sqrt{49 + 36} = \sqrt{85}$

Similarly
$\angle PSR = 90°$

In ΔPSR, $\quad PS = \sqrt{PR^2 - SR^2}$

$= \sqrt{85 - 4} = \sqrt{81}$

$\therefore \quad PS = 9$ cm

Perimeter of cyclic quadrilateral

$PQRS = PQ + QR + RS + PS$

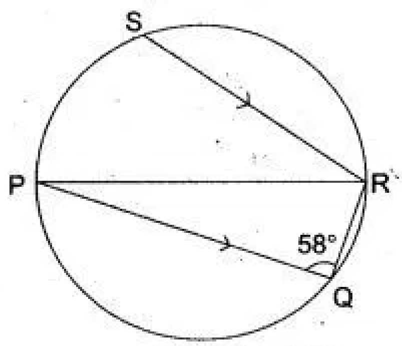
$= 7 \text{ cm} + 6 \text{ cm} + 2 \text{ cm} + 9 \text{ cm}$

$= 24$ cm. Ans.

Question 46. In the adjoining figure, PQ is the diameter, chord SR is parallel to PQ. Given $\angle PQR = 58°$. Calculate :

(i) $\angle RPQ$

(ii) $\angle STP$ (T is a point on the minor arc).



Solution : $\quad \angle PRQ = 90°$

($\angle$ in a semi-circle)

In ΔPQR,

$\angle RPQ + \angle PQR + \angle PRQ = 180°$

($\because$ The sum of the three $\angle$s of a Δ is 180°)

$\Rightarrow \quad \angle RPQ + 58° + 90° = 180°$

$\Rightarrow \quad \angle RPQ + 148° = 180°$

$\Rightarrow \quad \angle RPQ = 180° - 148°$

$\Rightarrow \quad \angle RPQ = 32°$ Ans.

(ii) $\because \quad PQ \parallel SR$

and RP intersects them

$\angle PRS = \angle RPQ$

(Alternate angles)

$\therefore \quad \angle PRS = 32°$

$\because$ PTSR is a cyclic quadrilateral.

$\therefore \ \angle PTS + \angle PRS = 180°$

($\because$ Opposite $\angle$s of a cyclic quadrilateral are supplementary)

$\Rightarrow \quad \angle PTS + 32° = 180°$

$\Rightarrow \quad \angle PTS = 180° - 32° = 148°$

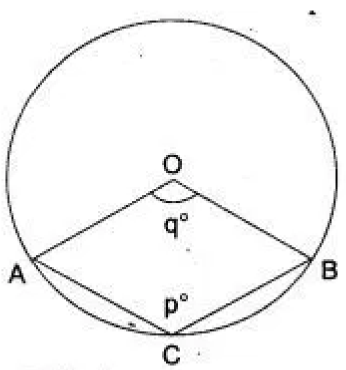
$\Rightarrow \quad \angle STP = 148°$. Ans.

Question 47. C is a point on the minor arc AB of the circle, with centre O. Given $\angle ACB = p°$, $\angle AOB = q°$.

(i) Express q in terms of p.

(ii) Calculate p if ACBO is a parallelogram.

(iii) If ACBO is a parallelogram, then find the value of q + p.



Solution : (i) Reflex

$$AOB = 360° - q°$$

$$ACB = \frac{1}{2} \text{reflex} \quad AOB$$

(angle at the centre property)

$$p° = \frac{1}{2}(360° - q°)$$

$$2p° = 360° - q°$$

$$q° = 360° - 2p°$$

$$q = 360° - 2p. \qquad \text{Ans.}$$

(ii) If ACBO is a parallelogram, then

$$p = q$$

$$q = 360° - 2p$$

$$p = 360° - 2p$$

$$p + 2p = 360°$$

$$3p = 360°$$

$$p = \frac{360°}{3} = 120°. \qquad \text{Ans.}$$

(iii) If ACBO is a parallelogram, then

$$p = q$$

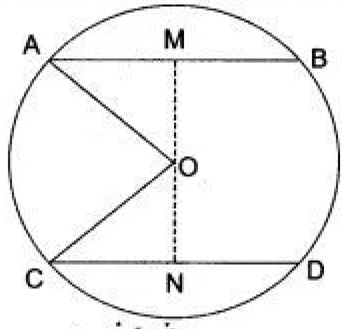
Also $\quad p = 120° \quad$ [From (ii)]

$$p + q = p + p = 2p$$

$$= 2 \quad 120° = 240°. \qquad \text{Ans.}$$

Question 48. AB, CD are parallel chords of a circle 7 cm apart. If AB = 6 cm, CD = 8 cm, find the radius of the circle.

Solution : Let O be the centre of the circle OM and ON are perpendiculars on AB and CD.



MON is one straight line.

Here $AM = \frac{1}{2} AB = 3$ cm, $CN = \frac{1}{2} CD = 4$ cm

Let $\quad ON = x$ cm and radius $OA = OC = r$ cm

From right angled triangle OCN,

$ON^2 = OC^2 - CN^2$

[By Pythagoras Theorem]

$x^2 = r^2 - 16$ ...(1)

From right angled triangle *OAM*,

$OM^2 = OA^2 - AM^2$

[By Pythagoras Theorem]

$(7-x)^2 = r^2 - 9$ ...(2)

From (1) and (2),

$$(7-x)^2 - x^2 = 7$$

$$49 + x^2 - 14x - x^2 = 7$$

$$14x = 42$$

$$x = 3$$

From (1), $r^2 = x^2 + 16$

$$= 9 + 16 = 25$$

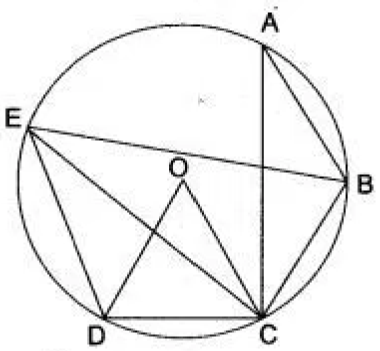
$$r = 5 \text{ cm}$$

Hence, the radius of the circle is 5 cm. Ans.

Question 49. In the adjoining diagram, chords AB, BC and CD are equal. O is the centre of the circle. If ABC = 120°, calculate :

(i) BAC (ii) BEC

(iii) BED (iv) COD.



Solution : (i) In ABC

ABC + BAC + BCA = 180°

(∵ The sum of three s of a is 180°)

120° + BAC + BCA = 180°

[∵ ABC = 120° (given)]

BAC + BCA = 60°

But, BA = BC

BAC + BAC = 60°

2 BAC = 60°

BAC = 30°. Ans

(ii) BEC = BAC = 30°. Ans.

(iii) AB = BC = CD

Arc AB = Arc BC = Arc CD

Now, $\quad COB = 2 \quad CAB$

$= 2 \quad 30° = 60°$

$DOC = \quad COB = 60°$

$DEC = \frac{1}{2} \quad DOC = \frac{1}{2} \quad 60°$

$= 30°$

$\therefore \quad \angle BED = \angle BEC + \angle DEC$

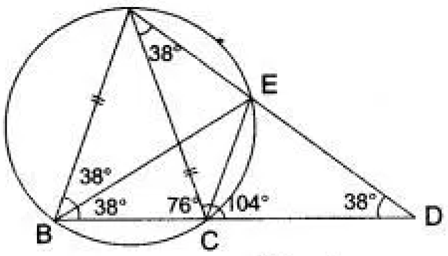
$= \angle BAC + \angle DEC$

$= 30° + 30° = 60°.$ Ans.

(iv) $\angle COD = 60°.$ Ans.

Question 50. In the figure, $AB = AC = CD$, angle, $ADC = 38°$. Calculate :

(i) Angle ABC (ii) Angle BEC.



Solution : $\because \quad AC = CD$

$\therefore \quad \angle CAD = \angle ADC = 38°$

Now in Δ ACD,

$\angle ACD + \angle CAD + \angle ADC = 180°$

$\Rightarrow \quad \angle ACD + 38° + 38° = 180°$

$\Rightarrow \quad \angle ACD = 104°$

Now $\quad \angle ACB + \angle ACD = 180°$

$\Rightarrow \quad \angle ACB + 104° = 180°$

$\Rightarrow \quad \angle ACB = 76°$

Again, $\because \quad AB = AC$

$\therefore \quad \angle ABC = \angle ACB = 76°.$ Ans.

(ii) In ΔABC

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow \quad \angle BAC + 76° + 76° = 180°$

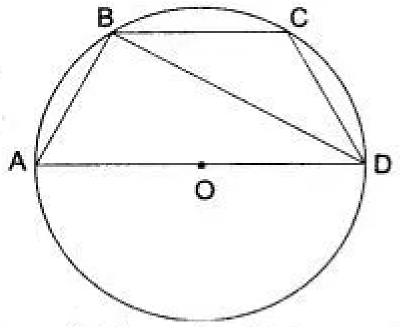
$\Rightarrow \quad \angle BAC = 28°$

Now $\quad \angle BEC = \angle BAC = 28°.$

[Angles subtended by the same chord] Ans.

Question 51. In the figure given alongside, AD is the diameter of the circle. If $\angle BCD = 130°$, calculate :

(i) $\angle DAB$ (ii) $\angle ADB$.



Solution : (i) Since ABCD is a cyclic quadrilateral.

$\therefore$ Its opposite angles are supplementary.

$$\therefore \angle DAB + \angle BCD = 180°$$

$$\Rightarrow \angle DAB = 180° - \angle BCD$$

$$= 180° - 130°$$

$$= 50°. \quad \text{Ans.}$$

(ii) Since, angle in the semi-circle is a right angle.

$\therefore$ In Δ ABD, $\angle ABD = 90°$

Since, the sum of the angle of a Δ is 180°

$\therefore$ From Δ ABD,

$$\angle ABD + \angle ADB + \angle DAB = 180°$$

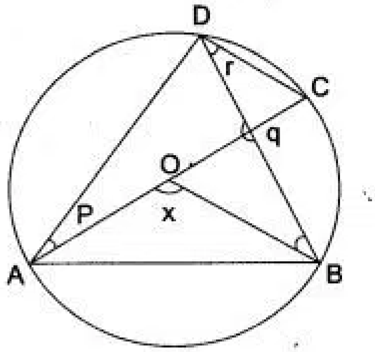
$$90° + \angle ADB + 50° = 180°$$

$$\angle ADB = 180° - (90° - 50°)$$

$$\angle ADB = 180° - 140°$$

$$\angle ADB = 40° \quad \text{Ans.}$$

Question 52. In the figure AC is the diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles p, q, r in term of x.



Solution. $\angle ADB = \frac{1}{2}\angle AOB = \frac{x}{2}$

$$\angle ADB = 90° - r$$

$$\angle ADB = \angle ACB = q$$

Combining these, we get

$$\frac{x}{2} = 90° - r = q$$

$\Rightarrow$ $2r = 180° - x$

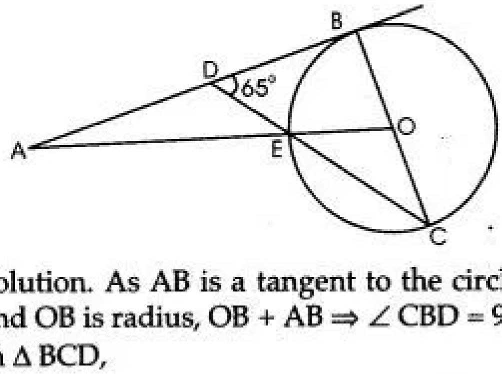
and $x = 2q$

$$\angle DAC = \angle CAB$$

$$= \angle BDC$$

$\Rightarrow$ $p = r = \frac{1}{2}(180° - x)$ Ans.

Question 53. In the following figure O is the centre of the circle and AB is a tangent to it at point B. $\angle$ BDC = 65°. Find $\angle$ BAO.



Solution. As AB is a tangent to the circle at B and OB is radius, OB + AB $\Rightarrow \angle$ CBD = 90°

In Δ BCD,

$$\angle BCD + \angle CBD + \angle BDC = 180°$$

$$\angle BCD + 90° + 65° = 180°$$
$$\angle BCD + 155° = 180°$$
$$\angle BCD = 180° - 155°$$
$$\angle BCD = 25°$$
$$\angle BOE = 2\angle BCE$$
[angle at centre = double the angle at the remaining part of circle]

$$\Rightarrow \angle BOE = 2 \times 25° = 50°$$
$$\angle BOA = 50°$$

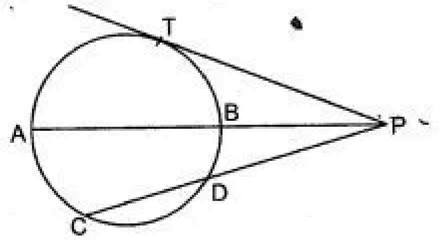
In ΔBOA,

$$\angle BAO + \angle ABO + \angle BOA = 180°$$
$$\Rightarrow \angle BAO + 90° + 50° = 180°$$
$$\Rightarrow \angle BAO + 140° = 180°$$
$$\Rightarrow \angle BAO = 180° - 140°$$
$$\Rightarrow \angle BAO = 40° \quad \text{Ans.}$$

Question 54. In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7·8 cm, PD = 5 cm, PB = 4 cm. Find :

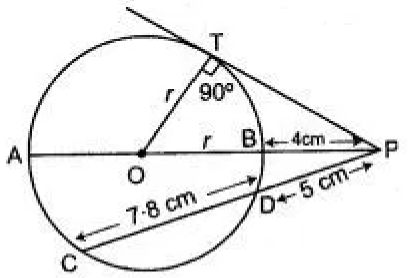
(i) AB.

(ii) The length of tangent PT.



Solution : Given that

$$CD = 7·8 \text{ cm}, PD = 5 \text{ cm},$$
$$PB = 4 \text{ cm}$$



As we know,

$$PT^2 = PD \times PC$$
$$PT^2 = PD \times (PD + CD)$$
$$PT^2 = 5 \times 12·8$$
$$PT^2 = 64$$
$$\Rightarrow PT = 8 \text{ cm}$$

Now in ΔPOT

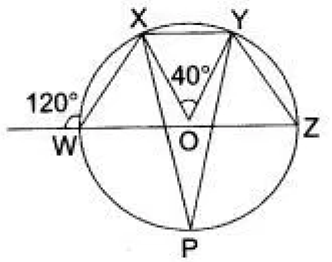
$$PO^2 = OT^2 + PT^2$$
$$(r+4)^2 = r^2 + 64$$

$$r^2 + 16 + 8r = r^2 + 64$$
$$8r = 48$$
$$r = 6$$

(i) Thus $AB = 2r = 12$ cm

(ii) Length of tangent $PT = 8$ cm.

Question 55. In the figure alongside O is the centre of circle $\angle XOY = 40°$, $\angle TWX = 120°$ and XY is parallel to TZ.



Find :

(i) $\angle XZY$, (ii) $\angle YXZ$, (iii) $\angle TZY$.

Solution : Construction : Take a point P on the circumference of the circle. Join XP and YP.

Determination of Angles : (i) $\angle XOY = 2\angle XPY$

(Angle subtended by an arc of a circle at the centre is twice the angle subtended by that arc at any point on the circumference of the circle)

$\Rightarrow \quad 40° = 2\angle XPY$

[$\because \angle XOY = 40°$ (given)]

$\Rightarrow \quad \angle XPY = \dfrac{40°}{2} = 20°$

$\angle XZY = 20°$ [$\because \angle XPY = \angle XZY$]

Angles in a same segment of a circle are equal.

(ii) $\angle XWT + \angle XWZ = 180°$

(Linear Pair Axiom)

$120° + \angle XWZ = 180°$

$\Rightarrow \quad \angle XWZ = 180° - 120° = 60°$

$\Rightarrow \quad \angle XWZ + \angle XYZ = 180°$

(Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow \quad 60° + \angle XYZ = 180°$

$\Rightarrow \quad \angle XYZ = 180° - 60° = 120°$

$\therefore \Delta XYZ$, $\angle YXZ + \angle XYZ + \angle XZY = 180°$

(The sum of the three angles of triangle is 180°)

$\Rightarrow \angle YXZ + 120° + 20° = 180°$

$\Rightarrow \quad \angle YXZ + 140° = 180°$

$\Rightarrow \quad \angle YXZ = 180° - 140° = 40°$. Ans.

(iii) $\because$ XY || TZ and transversal YZ intersects then

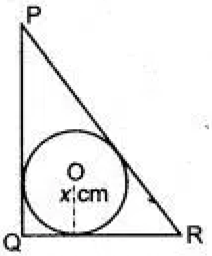
$\angle XYZ + \angle TZY = 180°$

(Sum of the consecutive interior angles is 180°)

$\Rightarrow \quad 120° + \angle TZY = 180°$

$\Rightarrow \quad \angle TZY = 180° - 120° = 60°$.

Question 56. In Δ PQR, PQ = 24 cm, QR = 7 cm and $\angle$ PQR = 90°. Find the radius of the inscribed circle.



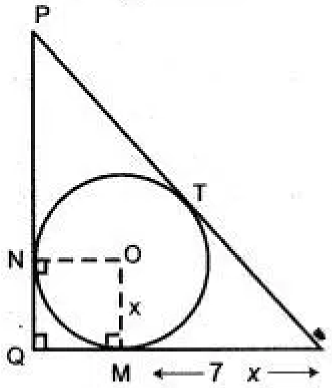
Solution :

$$OM \perp QR$$
$$ON \perp PQ$$

[Tangents and radius perpendicular to each other]

$$OM = ON\ (r)$$
$$QM = QN$$

(Tangents from an external point)



$\Rightarrow$ QMON is a square.

$\Rightarrow$ $QM = OM = ON = QN = x$ cm.

So, $mR = (7 - x)$ cm

$PN = (24 - x)$ cm.

$PT = PN = 24 - x$

and, $mR = RT = 7 - x$

[Tangents from an external point]

$\Rightarrow$ $PR = PT + RT$

$= 24 - x + 7 - x = 31 - 2x$

Now, In Δ PQR

$$PR^2 = PQ^2 + QR^2$$
$$= 24^2 + 7^2$$
$$= 576 + 49 = 625$$
$$PR = 25 \text{ cm}$$

$\Rightarrow$ $31 - 2x = 25$

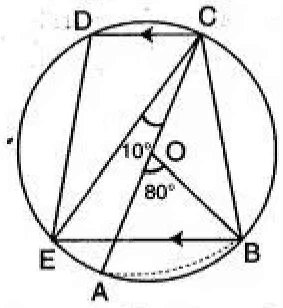
$\Rightarrow$ $2x = 31 - 25$

$2x = 6$

$\Rightarrow$ $x = 3$ cm. Ans.

Question 57. In the diagram given alongside, AC is the diameter of the circle, with centre O. CD and BE are parallel. Angle AOB = 80° and angle ACE = 10°. Calculate :

(i) ∠ BEC (ii) ∠ BCD (iii) ∠ CED.



Solution : From the figure, we have

$$\angle AOB = 80°$$
$$\angle ACE = 10°$$

(i) $\angle BOC = 180° - \angle AOB$
$= 180° - 80°$
$= 100°$

$\angle BEC = \frac{1}{2} \angle BOC$

[∵ ∠ subtended at the centre and ∠ subtend by E by arc BC]

$= \frac{1}{2} \times 100°$

∴ $\angle BEC = 50°$. Ans.

(ii) $\angle ACB = \frac{1}{2} \angle AOB$

[∵ ∠s subtended by arc AB at the centre and at C]

$= \frac{1}{2} \times 80°$
$= 40°$

$\angle ECD = \angle BEC$

[∵ Alt. ∠s as CD || BE]

$= 50°$

$\angle BCD = \angle ACB + \angle ECA + \angle ECD$
$= 40° + 10° + 50°$
$= 100°$. Ans.

(iii) BCDE is a cyclic quadrilateral,

[∵ Its opposite ∠s are supplementary]

⇒ $\angle BED = \angle 180° - \angle BCD$
$= 180° - 100°$ [From (ii)]
$= 80°$

$\angle BEC + \angle CED = 80°$

⇒ $\angle CED = 80° - \angle BED$
$= 80° - 50°$ [From (i)]

∴ $\angle CED = 30°$. Ans.