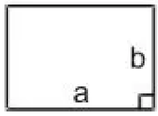
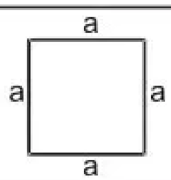
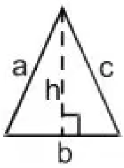
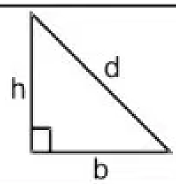
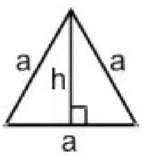
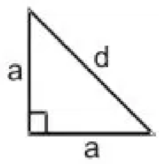
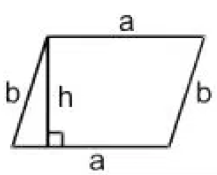
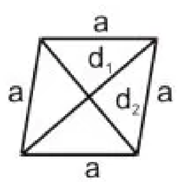
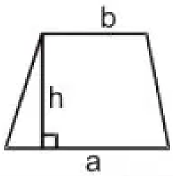
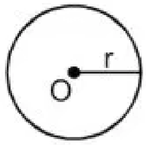
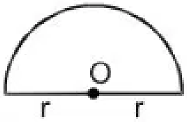
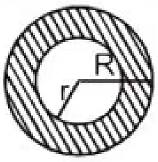
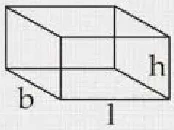
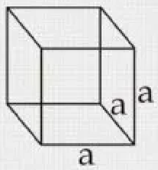
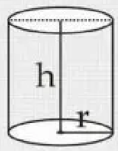
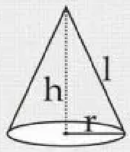
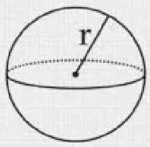
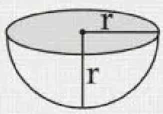
Chapter 17. Mensuration

Name	Figure	Perimeter	Area
Rectangle		2 (a + b)	ab
Square		4a	a^2
Triangle		a + b + c = 2s	$1 = \frac{1}{2} \times b \times h$ $2 = \sqrt{s(s-a)(s-b)(s-c)}$
Right triangle		b + h + d	$\frac{1}{2}$ bh
Equilateral triangle		3a	1. $\frac{1}{2}$ ah 2. $\frac{\sqrt{3}}{4} a^2$
Isosceles right triangle		2a + d	$\frac{1}{2} a^2$
Parallelogram		2 (a + b)	ah
Rhombus		4a	$\frac{1}{2} d_1 d_2$

Trapezium		Sum of its four sides	$\frac{1}{2}h(a+b)$
Circle		$2\pi r$	πr^2
Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$
Ring (shaded region)		----	$\pi(R^2 - r^2)$
Sector of a circle		$l + 2r$ where $l = (\theta/360) \times 2\pi r$	$\theta/360° \times \pi r^2$

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

Formulae

Perimeter:

1. Perimeter of a plane figure = sum of lengths of its sides.
2. Circumference of a circle = $2\pi r$,
   where r is the radius of the circle.

Area (of Plane Figures):

1. Area of a triangle = ½ × base × height.
2. Area of a triangle (Heron's formula)
   $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are lengths of sides and $s = \frac{1}{2}(a + b + c)$.
   (Heron's formula).
3. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$
   where a is its side.
4. Area of an isosceles triangle
   $= \frac{b}{4}\sqrt{4a^2 - b^2}$, where b is the base and a is an equal side.
5. Area of a quadrilateral (when diagonals intersect at right angles)
   = ½ × product of diagonals.
6. Area of a rectangle = length × breadth.
7. Area of a square = $(\text{side})^2$.
8. Area of a parallelogram = base × height.
9. Area of a rhombus = ½ × product of diagonals.
10. Area of a trapezium = ½ × (sum of parallel sides) × height.
11. Area of a circle = πr^2
    where r is the radius of the circle.
12. Area of a circular ring = $\pi (R^2 - r^2)$
    where R and r are the radii of the outer and the inner circles.

Surface Area and Volume of Solids:

1. **Cube Cuboid:**
   (i) Surface area of a cube = $6a^2$
   where a is its edge (side).
   (ii) Surface area of a cuboid = $2(\ell b + bh + \ell h)$
   where ℓ, b and h are its edges.
   (iii) Surface area of four walls (lateral surface area) of a cuboid = $2h(\ell + b)$
   where ℓ, b and h are its edges.
   (iv) Volume of a cube = $(\text{side})^3$.
   (v) Volume of a cuboid = length × breadth × height.
2. **Solid Cylinder:**
   Let r and h be the radius and height of a solid cylinder, then
   (i) Curved (lateral) surface area = $2\pi rh$.

(ii) Total surface area = $2\pi r (h + r)$.
(iii) Volume = $\pi r^2 h$.

3. **Hollow Cylinder:**
   Let R and r be the external and internal radii, and h be the height of a hollow cylinder, then
   (i) External curved surface area = $2\pi Rh$.
   (ii) Internal curved surface area = $2\pi rh$.
   (iii) Total surface area = $2\pi (Rh + rh + R^2 - r^2)$.
   (iv) Volume of material = $\pi (R^2 - r^2)h$.
4. **Cone:**
   Let r and h be the radius and height, and l be the slant height of a cone, then
   (i) Slant height = $l = \sqrt{r^2 + h^2}$.
   (ii) Curved (lateral) surface area = πrl.
   (iii) Total surface area = $\pi r (l + r)$.
   (iv) Volume = $\frac{1}{3} \pi r^2 h$.
5. **Solid Sphere:**
   Let r be the radius of a solid sphere, then
   (i) Surface area = $4\pi r^2$.
   (ii) Volume = $\frac{4}{3} \pi r^3$.
6. **Spherical Shell:**
   Let R and r be the radii of the outer and inner spheres, then
   (i) Thickness of the shell = $R - r$.
   (ii) Volume of material = $\frac{4}{3} \pi (R^3 - r^3)$.
7. **Solid Hemisphere:**
   Let r be the radius of a hemisphere, then
   (i) Curved (lateral) surface area = $2\pi r^2$.
   (ii) Total surface area = $3\pi r^2$.
   (iii) Volume = $\frac{2}{3} \pi r^3$.

8. **Hemispherical Shell:**

Let R and r be the radii of the outer and inner hemispheres, then

(i) Thickness of the shell = $R - r$.

(ii) Area of base = $\pi (R^2 - r^2)$.

(iii) External curved surface area = $2\pi R^2$.

(iv) Internal curved surface area = $2\pi r^2$.

(v) Total surface area = $\pi (3R^2 + r^2)$.

(vi) Volume of material = $\frac{2}{3}\pi (R^3 - r^3)$.

Formulae Based Questions

Question 1. Find the area of a circle whose circumference is 22 cm.

Solution : Let r be the radius of the circle, then

circumference = $2\pi r$

$$\Rightarrow \quad 2\pi r = 22 \text{ cm}$$

$$\Rightarrow \quad r = \frac{22}{2\pi} = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

$$\therefore \text{ Area of circle } = \pi r^2$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 = \frac{77}{2} \text{ cm}^2$$

$$= 38{\cdot}5 \text{ cm}^2.$$

Ans.

Question 2. If the perimeter of a semi circular protractor is 36 cm. Find its diameter.

Solution : Let r cm be the radius of the protractor, then

$$\text{Perimeter} = \pi r + 2r$$

According to the question,

$$36 = \pi r + 2r$$

$$\Rightarrow \quad 36 = r\left(\frac{22}{7} + 2\right)$$

$$\Rightarrow \quad r = 7 \text{ cm.}$$

$\therefore$ The diameter of the protractor is (2×7) cm = 14 cm.

Ans.

Question 3. A well 28.8 m deep and of diameter 2 m is dug up. The soil dug out is spread all around the well to make a platform 1 m high considering the fact losse soil settled to a height in the ratio 6 : 5 find the width of the platform.

Solution : Volume of soil dug out

$$= \pi (1)^2 (28.8) \text{ cu m.}$$

∴ Volume of soil that would be settled on the platform is

$$\frac{5 \times \pi \times 28.8}{6} \text{ cu m.}$$

Let the width of the platform be r cm. Then,

$$\pi (1 + r^2) - (1)^2 (1) = \frac{5 \times \pi \times 28.8}{6}$$

$$\Rightarrow \quad (1 + r)^2 - 1 = \frac{5 \times 28.8}{6} = 24$$

$$\Rightarrow \quad (1 + r)^2 = 25$$

$$\Rightarrow \quad 1 + r = 5$$

$$\Rightarrow \quad r = 4 \text{ cm.}$$ Ans.

Question 4. Two cylinder have bases of same size. The diameter of each is 14 cm. One of the cone is 10 cm high and the other is 20 cm high. Find the ratio between their volumes.

Solution :	Cone I	Cone II
Base diameter	14 cm	14 cm
Base radius	$r_1 = 7$ cm	$r_2 = 7$ cm
Height	$h_1 = 10$ cm	$h_2 = 20$ cm

Volume $V_1 = \pi r_1^2 h_1$ $V_2 = \pi r_2^2 h_2$

$$= \pi \times (7)^2 \times 10 \text{ cm}^3 : \pi \times (7)^2 \times 20 \text{ cm}^3$$

$$= 490\pi \text{ cm}^3 : 980\pi \text{ cm}^3$$

$$\therefore \frac{V_1}{V_2} = \frac{490\pi}{980\pi} = \frac{1}{2} \Rightarrow V_1 : V_2 = 1 : 2.$$ Ans.

Question 5. A glass cylinder with diameter 20 cm water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. (Take π = 3.142)

Solution : Suppose the water rises by h cm. Clearly water in the cylinder forms a cylinder of height h cm and radius 10 cm.

∴ Volume of the water displaced = Volume of the cube of edge 8 cm

$$\Rightarrow \quad \pi r^2 h = 8^3$$

$$\Rightarrow \quad 3.142 \times 10^2 \times h = 8 \times 8 \times 8$$

$$\Rightarrow \quad \frac{8 \times 8 \times 8}{3.142 \times 10 \times 10} \text{ cm} = h$$

$$h = 1.6 \text{ cm.}$$ Ans.

Question 6. Water is being pumped out through a circular pipe whose external diameter is 7 cm. If the flow of water is 72 cm per second how many litres of water are being pump out in one hour.

Solution : Volume of water that will be pumped out in 1 seconds

$$= \pi\left(\frac{7}{2}\right)^2 (72) \text{ cu cm.}$$

Volume of water that will be pumped out in one hour

$$= \pi\left(\frac{7}{2}\right)^2 (72)(3600) \text{ cu cm}$$

$$= 99,79,200 \text{ cu cm}$$

$$= 9979.2 \text{ litres.} \qquad \text{Ans.}$$

Question 7. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions? (Take π = 22/7)

Solution : Clearly,

Area covered = Curved surface area × No. of revolutions

Here, $r = \frac{1 \cdot 4}{2} \text{ m} = 0 \cdot 7 \text{ m and } h = 2 \text{ m}$

∴ Curved surface area

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0 \cdot 7 \times 2 \text{ m}^2$$

$$= 8 \cdot 8 \text{ m}^2$$

Hence, area covered

$$= 8 \cdot 8 \text{ m}^2 \times 5$$

$$= 44 \text{ m}^2. \qquad \text{Ans.}$$

Question 8. The radius and height of a cylinder are in the ratio of 5 : 7 and its volume is 550 cm. Find its radius. (Take π = 22/7)

Solution : Let the radius of the base and height of the cylinder be 5x cm and 7x cm respectively. Then,

$$\text{Volume} = 550 \text{ cm}^3$$

$$\Rightarrow \frac{22}{7} \times (5x)^2 \times 7x = 550$$

[Use, $r = 5x$, $h = 7x$ and volume $= \pi r^2 h$]

$$\Rightarrow \frac{22}{7} \times 25x^2 \times 7x = 550$$

$$\Rightarrow 22 \times 25x^3 = 550$$

$$\Rightarrow 550x^3 = 550$$

$$\Rightarrow x^3 = 1$$

$$\Rightarrow x = 1 \text{ cm}$$

Hence, radius of the cylinder

$$= 5x \text{ cm} = (5 \times 1) \text{ cm}$$

$$= 5 \text{ cm.} \qquad \text{Ans.}$$

Question 9. The ratio of the base area and curved surface of a conical tent is 40 : 41. If the height is 18 m, Find the air capacity of tent in term of π.

Solution : Given : $\frac{\text{base area}}{\text{curved surface}} = \frac{40}{41}$

$$\Rightarrow \quad \frac{\pi r^2}{\pi r \sqrt{h^2 + r^2}} = \frac{40}{41}$$

{Where h is the height and r is the radius of conical tent}

$$\Rightarrow \quad \frac{r}{\sqrt{18^2 + r^2}} = \frac{40}{41} \qquad (\because h = 18 \text{ m})$$

$$\Rightarrow \quad r = 80 \text{ m}$$

$$\text{Air capacity} = \frac{1}{3}\pi (80)^2 \times 18$$

$$= 38{,}400\, \pi \text{ cu m.}$$ Ans.

Question 10. The diameter of two cones are equal. If their slant heights be in the ratio of 5 : 4. Find the ratio of their curved surface areas?

Solution : Let their slant height be 5x and 4x.

Given that diameter of two cones are equal

$$\therefore \quad R_1 = R_2 = R$$

$$S_1 = \pi R l = \pi R (5x)$$

$$S_2 = \pi R l = \pi R (4x)$$

$$\frac{S_1}{S_2} = \frac{\pi R (5x)}{\pi R (4x)} = \frac{5}{4}$$

$$\frac{S_1}{S_2} = \frac{5}{4}.$$

$$S_1 : S_2 = 5 : 4$$ Ans.

Question 11. The radius and height of cone are in the ratio 3 : 4. If its volume is 301.44 cm^3. What is its radius? What is its slant height? (Take π = 3.14)

Solution : Let the radius of cone be 3x cm and the height 4x cm, then

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad \frac{1}{3}\pi (3x)^2 (4x) = 301{\cdot}44$$

$$\Rightarrow \quad x^3 = 8 \Rightarrow x = 2$$

Thus, radius of cone is 6 cm and height 8 cm.

Now, slant height of cone

$$= \sqrt{(6)^2 + (8)^2}$$

$$= 10 \text{ cm.}$$ Ans.

Question 12. Find the volume and surface area of a sphere of diameter 21 cm.

Solution :

$$\text{Diameter of sphere} = 21 \text{ cm}$$

$$\therefore \quad \text{Radius of sphere} = \frac{21}{2} \text{ cm}$$

$$\therefore \quad \text{Surface area of sphere} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 1386 \text{ cm}^2$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 4851 \text{ cm}^3. \qquad \text{Ans.}$$

Question 13. The volume of a sphere is 905 1/7 cm^3, find its diameter.

Solution : Volume of a sphere $= 905\frac{1}{7}$

$$\frac{4}{3}\pi r^3 = \frac{6336}{7}$$

$$r^3 = \frac{6336 \times 7 \times 3}{4 \times 22 \times 7}$$

$$r^3 = 216$$

$$r = \sqrt[3]{216}$$

$$r = 6 \text{ cm} \qquad \text{Ans.}$$

$$\therefore \text{Diameter of a sphere} = 2r = 2 \times 6 = 12 \text{ cm.}$$

Question 14. There is surface area.and volume of sphere equal, find the radius of sphere.

Solution : Let the radius of sphere = r

∵ Volume of sphere = Surface area of sphere

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\Rightarrow \quad r^3 = \frac{4\pi r^2 \times 3}{4\pi}$$

$$\Rightarrow \quad r^3 = 3r^2$$

$$\Rightarrow \quad r = 3 \text{ cm} \qquad \text{Ans.}$$

Hence, the radius of sphere = 3 cm.

Question 15. There is a ratio 1 : 4 between surface area of two spheres, find the ratio between their radius.

Solution : Let radius of spheres are r_1 and r_2.

So, surface area of spheres is $4\pi r_1^2$ and $4\pi r_2^2$

$$\text{Ratio of surface area} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4}$$

$$= \frac{r_1^2}{r_2^2} = \frac{1}{4}$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{1}{4}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{1}{4}}$$

$$\frac{r_1}{r_2} = \frac{1}{2}$$

Hence, the ratio between their radius = 1 : 2.

Question 16. Marbles of diameter 1.4 cm are dropped into a beaker containing some water are fully submerged. The diameter of beaker is 7 cm. Find how many marbles have been drapped in it if the water rises by 5.6 cm.

Solution : Volume of water that rises in putting marbles in the beaker.

$$= \pi\left(\frac{7}{2}\right)^2 \times (5.6) \text{ cu cm.}$$

Volume of one marble

$$= \frac{4}{3}\pi (0.7)^3 \text{ cu cm}$$

$\therefore$ Number of marbles

$$= \frac{\pi\left(\frac{7}{2}\right)^2 \times 5.6}{\frac{4}{3}\pi (0.7)^3} = 150 \qquad \text{Ans.}$$

Question 17. A spherical cannon ball, 28 cm in diameter is melted and recast into a right circular conical mould, the base of which is 35 cm in diameter. Find the height of the cone, correct to one place of decimal.

Solution : Let h be the height of cone. Then

$$\frac{1}{3}\pi\left(\frac{35}{2}\right)^2 h = \frac{4}{3}\pi (14)^3$$

(Because the volume of conical mould is the same as that of the spherical cannon ball.)

$$\Rightarrow \quad h = \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35}$$

$$= \frac{896}{25} = 35.84 \text{ cm.} \qquad \text{Ans.}$$

Question 18. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Solution : Let, height of cone $= h$

and height of hemisphere $= H$

$\therefore$ Volume of cone = Volume of hemisphere

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 H \qquad (\because H = r)$$

$$\frac{h}{H} = \frac{2\pi r^2 \times 3}{3\pi r^2}$$

$$\frac{h}{H} = \frac{2}{1}$$

$$h : H = 2 : 1. \qquad \text{Ans.}$$

Question 19. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

Solution : Number of cones

$$= \frac{\text{Volume of solid sphere}}{\text{Volume of 1 cone}}$$

$$= \frac{\frac{4}{3}\pi (15)^3}{\frac{1}{3}\pi (2\cdot5)^2 \times 8}$$

$$= \frac{4 \times 15 \times 15 \times 15}{2\cdot5 \times 2\cdot5 \times 8}$$

$$= 180$$

Question 20. The radius of two spheres are in the ratio of 1 : 3. Find the ratio between their volume.

Solution : Let the radius of two sphere number is r_1 and r_2

$$\because \quad \frac{r_1}{r_2} = \frac{1}{3}$$

Volume of spheres,

$$V_1 = \frac{4}{3}\pi r_1^3$$

and

$$V_2 = \frac{4}{3}\pi r_2^3$$

Now,

$$\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\therefore \quad V_1 : V_2 = 1 : 27$$

Hence, the volume of two spheres are in the ratio of 1: 27. Ans.

Question 21. A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones.

Solution : Volume of metal in hollow sphere

$$= \frac{4}{3}\pi(8^3 - 6^3)$$

$$= \frac{1184}{3}\pi \text{ cm}^3$$

Volume of metal in one cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 2^2 \times 8$$

$$= \frac{32}{3}\pi \text{ cm}^3$$

Number of cones

$$= \frac{\text{Volume of metal in sphere}}{\text{Volume of metal in one cone}}$$

$$= \frac{\frac{1184}{3}\pi}{\frac{32}{3}\pi}$$

$$= \frac{1184}{32}$$

$$= 37 \qquad \text{Ans.}$$

Question 22. A sphere cut out from a side of 7 cm cubes. Find the volume of this sphere?

Solution : $\because$ Diameter of sphere equal to sides of cube

$\therefore$ Radius of sphere $= \frac{7}{2}$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{539}{3} = 179{\cdot}66 \text{ cm}^3. \quad \text{Ans.}$$

Question 23. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter?

Solution : Let the total number of bullets be x

Radius of a spherical bullet $= \frac{4}{2} = 2$ cm

Now, volume of a spherical bullet

$$= \frac{4}{3}\pi \times (2)^3 = \frac{4}{3} \times \frac{22}{7} \times 8$$

∴ Volume of x spherical bullets

$$= \frac{4}{3} \times \frac{22}{7} \times 8 \times x \text{ cm}^3$$

Volume of the solid cube $= (44)^3$ cm^3

clearly, volume of x spherical bullets = volume of cube

$$\frac{4}{3} \times \frac{22}{7} \times 8 \times x = 44 \times 44 \times 44$$

$$x = \frac{44 \times 44 \times 44 \times 7 \times 3}{4 \times 22 \times 8}$$

$$= 2541$$

Hence, total number of spherical bullets are 2541.

Question 24. A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone.

Solution : Volume of hemispherical bowl

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi (3\cdot6)^3$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \times (4\cdot8)^2 \times h$$

Volume of bowl = Volume of cone

$$\Rightarrow \quad \frac{2}{3}\pi \times (3\cdot6)^3 = \frac{1}{3}\pi \times (4\cdot8)^2 \times h$$

$$\Rightarrow \quad h = \frac{2 \times 3\cdot6 \times 3\cdot6 \times 3\cdot6}{4\cdot8 \times 4\cdot8}$$

$$= 4\cdot05 \text{ cm}$$

Question 25. The total surface area of a hollow metal cylinder, open at both ends of external radius 8 cm and height 10 cm is 338π cm^2. Taking r to be inner radius, write down an equation in r and use it to state the thickness of the metal in the cylinder.

Solution : Surface area of the hollow cylinder

$$= 338\pi, cm^2$$

$$2\pi(8)10 + 2\pi(r)10 + 2[\pi(8)^2 - \pi(r)^2] = 338\pi$$

$$\Rightarrow 160 + 20r + 2(64 - r^2) = 338$$

$$\Rightarrow -2r^2 + 20r - 50 = 0$$

$$\Rightarrow r^2 - 10r + 25 = 0$$

$$\Rightarrow (r-5)^2 = 0$$

$$\Rightarrow r = 5$$

∴ Thickness of the metal in the cylinder = 8 − r

$$= 8 - 5 = 3 \text{ cm.}$$ Ans.

Question 26. Find the weight of a lead pipe 35 cm long. The external diameter of the pipe is 2.4 cm and thickness of the pipe is 2mm, given 1 cm^3 of lead weighs 10 gm.

Solution : External radius 'R' = 1·2 cm

Internal radius 'r' = 1·0 cm

[Since internal radius = external radius – thickness]

Height 'h' = 35 cm

∴ Volume of the pipe

$$= \pi h[R^2 - r^2]$$

$$= \frac{22}{7} \times 35 \times [(1{\cdot}2)^2 - (1{\cdot}0)^2]$$

$$= 48{\cdot}4 \text{ cm}^3$$

∴ Weight of lead pie

$$= 10 \times 48{\cdot}4$$

$$= 484 \text{ gm.}$$

Question 27. A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which the water will up in the cylinder. Answer correct of the nearest mm. (Take π = 3.142)

Solution : Let the height by which the water ups be

$$= h \text{ cm}$$

Volume of the increase in water

$$= 3{\cdot}142 \times 10 \times 10 \times h \text{ cm}^3$$

Volume of the cube = $8 \times 8 \times 8 \text{ cm}^3$

Both the above volumes are equal

$$\therefore 3{\cdot}142 \times 10 \times 10 \times h = 8 \times 8 \times 8$$

$$h = \frac{8 \times 8 \times 8}{3{\cdot}142 \times 10 \times 10}$$

$$= 1{\cdot}6 \text{ cm}$$

The height h = 16 mm. Ans.

Question 28. A road roller is cylindrical in shape, its circular end has a diameter of 1.4 m and its width is 4 m. It is used to level a play ground measuring 70 m × 40 m. Find the minimum number of complete revolutions that the roller must take in order to cover the entire ground once.

Solution : Curved surface area of the road roller

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 0{\cdot}7 \times 4 = 17{\cdot}6 \text{ m}^2$$

Area of the play ground $= 70 \times 40 = 2800 \text{ m}^2$

$\therefore$ Number of revolutions to cover the entire ground

$$= \frac{2800}{17{\cdot}6} = 159\frac{1}{11}$$

$\therefore$ Number of complete revolutions = 160 Ans.

Question 29. A vessel is in he form of an inverted cone. Its height is 11 cm., and the radius of its top which is open, is 2.5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.25 cm., are dropped 2 into the vessel, 2/5 th of the water flows out. Find the number of lead shots dropped into the vessel.

Solution : Vol. of n lead shots = volume of water displaced

$$n \times \frac{4}{3}\pi r^3 = \frac{2}{5} \times \frac{1}{3}\pi R^2 H$$

$$\therefore \quad n = \frac{\frac{2}{5} \times \frac{1}{3}\pi R^2 H}{\frac{4}{3}\pi r^3}$$

$$= \frac{2R^2 H}{5 \times 4r^3}$$

$$= \frac{2 \times 2{\cdot}5^2 \times 11}{5 \times 4 \times (0{\cdot}25)^3}$$

$$= 440 \text{ shots}$$

Prove the Following

Question 1. The circumference of the base of a 10 m high conical tent is 44 metres. Calculate the length of canvas used in making the tent if width of canvas is 2m. (Take π = 22/7)

Solution : Let r m be the radius of the base, h m be the height and l m be the slant height of the cone. Then,

$$\text{Circumference} = 44 \text{ metres}$$

$$\Rightarrow \quad 2\pi r = 44$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7 \text{ metres}$$

It is given that $h = 10$ metres

$$\therefore \quad l^2 = r^2 + h^2 \Rightarrow l = \sqrt{r^2 + h^2}$$

$$l = \sqrt{49 + 100} = \sqrt{149} = 12{\cdot}2 \text{ m}$$

Now, surface area of the tent

$$= \pi r l \text{ m}^2$$

$$= \frac{22}{7} \times 7 \times 12{\cdot}2 \text{ m}^2$$

$$= 268{\cdot}4 \text{ m}^2$$

$\therefore$ Area of the canvas used $= 268{\cdot}4 \text{ m}^2$

It is given that the width of the canvas is 2 m

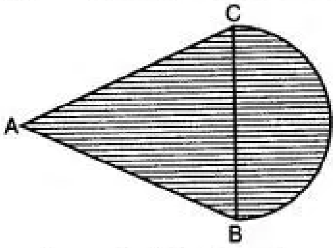
$$\therefore \text{Length of the canvas used} = \frac{\text{Area}}{\text{Width}}$$

$$= \frac{268{\cdot}4}{2}$$

$$= 134{\cdot}2 \text{ metres}$$

Figure Based Questions

Question 1. In an equilateral Δ ABC of side 14 cm, side BC is the diameter of a semi-circle as shown in the figure below. Find the area of the shaded region. (Take $\pi = 22/7$ and $\sqrt{3} = 1.732$)



Solution : Area of shaded part

= Area of equilateral ΔABC + Area of semi circle

$$= \frac{\sqrt{3}}{4} a^2 + \frac{1}{2} \pi r^2$$

Given $a = 14$ cm and $r = \frac{14}{2} = 7$ cm

$\therefore$ Area of shaded part

$$= \frac{\sqrt{3}}{4} \times 14^2 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

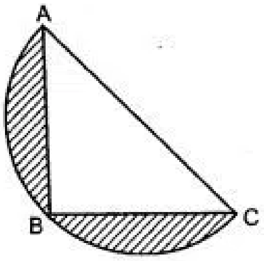
$$= \frac{1{\cdot}732 \times 14 \times 14}{4} + 11 \times 7$$

$$= 1{\cdot}732 \times 7 \times 7 + 77 = 84{\cdot}868 + 77$$

$$= 161{\cdot}868 \text{ sq. cm.}$$

Ans.

Question 2. ABC is an isosceles right angled triangle with $\angle ABC = 90°$. A semi-circle is drawn with AC as the diameter. If AB = BC = 7 cm, find the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7}\right)$



Solution : In Δ ABC,

$$AC^2 = AB^2 + BC^2$$
$$= 7^2 + 7^2$$
$$= 49 + 49$$
$$= 98$$
$$\Rightarrow \quad AC = \sqrt{98}$$
$$= 7\sqrt{2} \text{ cm}$$

Radius of semi circle

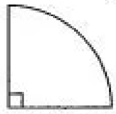
$$= \frac{7\sqrt{2}}{2} \text{ cm}$$

$\therefore$ Area of shaded region

$$= (\text{Area of semi circle}) - \text{Area of } \Delta ABC$$
$$= \frac{1}{2}(\pi r^2) - \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2}\pi\left(\frac{7\sqrt{2}}{2}\right)^2 - \frac{1}{2} \times 7 \times 7$$
$$= \frac{1}{2} \times \frac{22}{7} \times \frac{49 \times 2}{4} - \frac{1}{2} \times 49$$
$$= \frac{77}{2} - \frac{49}{2} = \frac{28}{2} = 14 \text{ cm}^2 \quad \text{Ans.}$$

Question 3. In the adjoining figure, the radius is 3.5 cm. Find:
(i) The area of the quarter of the circle correct to one decimal place.

(ii) The perimeter of the quarter of the circle correct to one decimal place. $\left(\text{Take } \pi = \frac{22}{7}\right)$.



Solution : (i) $r = 3.5 \text{ cm} = \frac{7}{2} \text{ cm}$ (given)

Area of the quarter of the circle $= \frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \left[\frac{7}{2} \times \frac{7}{2}\right]$$

$$= \frac{77}{8} \text{ cm}^2 = 9.625$$

$$= 9.6 \text{ cm}^2$$

(Correct to one decimal place)

Ans.

(ii) Perimeter of the quarter of the circle

$$= \frac{2\pi r}{4} + r + r$$

$$= \frac{\pi r}{2} + 2r$$

$$= \frac{1}{2} \times \frac{22}{7} \times (3.5) \text{ cm} + 2 \times (3.5) \text{ cm}$$

$$= [5.5 + 7] \text{ cm}$$

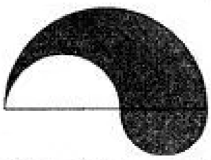
$$= 12.5 \text{ cm.}$$ Ans.

Question 4. The boundary of the shaded region in the given diagram consists of three semicircular areas, the smaller ones being equal and it's diameter 5 cm, if the diameter of the larger one is 10 cm, calculate:

(i) The length of the boundary,

(ii) The area of the shaded region.

(Take $\pi = 3.14$)



Solution : (i) Diameter = 10 cm

$\therefore$ Radius $(r) = \frac{10}{2} \text{ cm} = 5 \text{ cm}$

Length of the boundary $= \pi(5) + \pi(2.5) + \pi(2.5)$

$$= 10\pi$$

$$= 10 \times 3.14 \text{ cm}$$

$$= 31.4 \text{ cm.}$$ Ans.

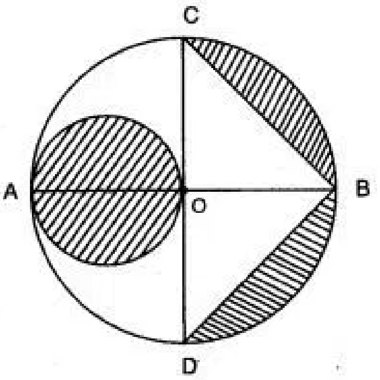
(ii) Area of the shaded region

$$= \frac{1}{2}\pi(5)^2 - \frac{1}{2}\pi(2.5)^2 + \frac{1}{2}\pi(2.5)^2$$

$$= \frac{25}{2}\pi = \frac{25}{2} \times 3.14$$

$$= 25 \times 1.57 = 39.25 \text{ cm}^2.$$ Ans.

Question 5. In the given figure, AB is the diameter of a circle with centre O and OA = 7 cm. Find the area of the shaded region.



Solution : Given $OA = 7$ cm.

Area of small circle $= \pi r^2$

where $r = 7/2$

$\therefore$ Area of circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2} = 38{\cdot}5$ sq. cm.

Area of two shaded segment

$=$ Area of big semi-circle – Area of $\Delta\, CBD$

$= \frac{1}{2}\pi \times 7^2 - \frac{1}{2} \times 7 \times 14$

$= \frac{1}{2} \times \frac{22}{7} \times 49 - 7 \times 7$

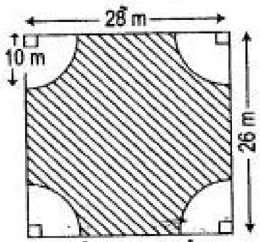
$= 11 \times 7 - 7 \times 7$

$= 77 - 49 = 28$ sq. cm.

$\therefore$ Total shaded Area $= 38{\cdot}5 + 28$

$= 66{\cdot}5$ sq. cm.

Question 6. Find the perimeter and area of the shaded portion of the following diagram; give your answer correct to 3 significant figures. (Take π = 22/7).



Solution : Area of rectangle

$= 28 \text{ m} \times 26 \text{ m}$

$= 728 \text{ m}^2$

(i) Perimeter of the shaded position

$= 2 \times 28 + 2 \times 26 - 4 \times 20 + 2 \times \frac{22}{7} \times 10$

$= 56 + 52 - 80 + \frac{440}{7} = 108 - 80 + 62{\cdot}85$

$= 170{\cdot}85 - 80 = 90{\cdot}85 \text{ m}$ Ans.

(ii) Area of one corner (unshaded)

$= \frac{1}{4} \pi \times (10)^2$

$= \frac{1}{4} \times (3{\cdot}14)(100 \text{ m}^2)$

$= \frac{314}{4}$

$= 78{\cdot}5 \text{ m}^2$

$\therefore$ Area of 4 corners (unshaded)

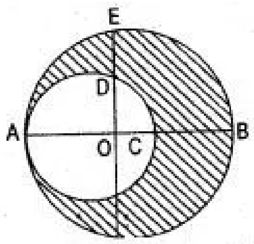
$= 78{\cdot}5 \times 4 = 314 \text{ m}^2$

$\therefore$ Area of the shaded portion

$= [728 - 314] \text{ m}^2$

$= 414 \text{ m}^2.$ Ans.

Question 7. In the adjoining figure, crescent is formed by two circles which touch at the point A. O is the centre of bigger circle. If CB = 9 cm and DE = 5 cm, find the area of the shaded portion.

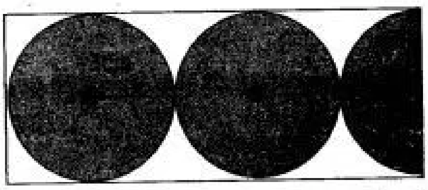


Solution : Let R and r be the radii of two circles. Then, $2(R - r) = 9$. Join AD and CD, $\Delta AOD \sim \Delta DOC$

$\therefore \quad \frac{OD}{OC} = \frac{OA}{OD}$

$$\Rightarrow \quad OD^2 = OA.OC$$
$$\Rightarrow \quad (R-5)^2 = R(R-9)$$
$$\Rightarrow \quad R^2 + 25 - 10R = R^2 - 9R$$
$$\therefore \quad R = 25$$
So, $2(25 - r) = 9$
$$\therefore \quad r = 20{\cdot}5 \text{ cm}$$
Area of the shaded portion
$$= \pi R^2 - \pi r^2$$
$$= \pi [(25)^2 - (20{\cdot}5)^2] \text{ cm}^2$$
$$= \frac{22}{7}[625 - 420{\cdot}25] \text{ cm}^2$$
$$= \frac{22}{7} \times 204{\cdot}75 \text{ cm}^2$$
$$= 643{\cdot}5 \text{ cm}^2.$$

Question 8. In the given figure, find the area of the unshaded portion within the rectangle. (Take π = 22/7).

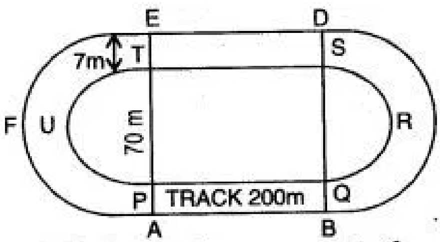


Solution : In the given figure

Radius of circle = 3 cm
Length of rectangle = 15 cm
Breadth of rectangle = 6 cm

Area of rectangle $- \left[\text{Ar. of I + II} + \frac{1}{2}\text{ III}\right]$ circles
$$= 15 \times 6 - \left[\text{Ar. of } 2\frac{1}{2} \text{ circles}\right]$$
$$= 90 - \frac{5}{2} \times \pi r^2 = 90 - \frac{5}{2} \times 3{\cdot}14 \times 3^2$$
$$= 90 - 70{\cdot}65$$
$$= 19{\cdot}35 \text{ cm}^2 \qquad \text{Ans.}$$

Question 9. The figure shows a running track surrounding a grassed enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semicircular region at each end, PQ = 200 m; PT = 70 meter



(i) Calculate the area of the grassed enclosure in m²;

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF of the track. (Take $\pi = 22/7$)

Solution : (i) Area of the grassed enclosure

$$= PQ \times PT + 2 \times \frac{1}{2} \times \pi \times (35)^2$$

$$\left(\because r = \frac{70}{2} = 35 \text{ m}\right)$$

$$= 200 \times 70 + \frac{22}{7} \times 35 \text{ m} \times 35 \text{ m}$$

$$= 14000 + 22 \times 5 \times 35 \text{ m}^2$$

$$= [14000 + 110 \times 35] \text{ m}^2$$

$$= 14000 + 3850 = 17850 \text{ m}^2. \quad \text{Ans.}$$

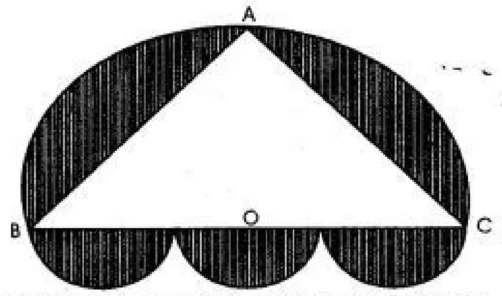
(ii) $$R = \left[\frac{70 + 14}{2}\right] \text{m} = \frac{84}{2} = 42 \text{ m}$$

Outer perimeter $= 2l + 2 \times \frac{1}{2} \times 2\pi R$

$$= 2 \times 200 \text{ m} + 2 \times \frac{22}{7} \times 42 \text{ m}$$

$$= 400 \text{ m} + 264 \text{ m} = 664 \text{ m}. \quad \text{Ans.}$$

Question 10. A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi-circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC. If BO = OC, find the area of the shaded region. (Take π = 22/7).



Solution : As angle in a semi circle is 90°,

$$\angle A = 90°$$

From Δ ABC, by pythagoras theorem, we get

$$AB^2 + AC^2 = BC^2$$

$$\Rightarrow x^2 + x^2 = 84^2$$

$$\Rightarrow 2x^2 = 84 \times 84$$

$$\Rightarrow x^2 = 84 \times 42$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 84 \text{ cm} \times 42 \text{ cm}$$

$$= 1764 \text{ cm}^2$$

Radius of semicircle with BC as diameter

$$= \frac{1}{2} \times 84 \text{ cm}$$

$$= 42 \text{ cm}$$

Diameter of each of three equal semicircles

$$= \frac{1}{3} \times 84 = 28 \text{ cm}$$

⇒ Radius of each of 3 equal semicircles

$$= 14 \text{ cm}$$

Area of the shaded region

= Area of semicircle with 42 cm as Radius

+ Area of three equal semi-circles of radius

14 cm – area of Δ ABC

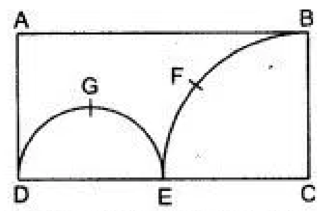
$$= \frac{1}{2} \pi \times 42^2 \text{ cm}^2 + 3 \times \frac{1}{2} \pi \times 14^2 \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= \frac{1}{2} \pi (42^2 + 3 \times 14^2) \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 14^2 (9 + 3) \text{ cm}^2 - 1764 \text{ cm}^2$$

$$= 3696 \text{ cm}^2 - 1764 \text{ cm}^2 = 1932 \text{ cm}^2$$ Ans.

Question 11. In the figure given below, ABCD is a rectangle. AB 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle. (Take π = 22/7).



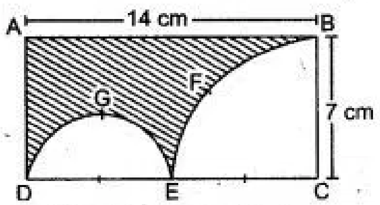
Solution : Length of rectangle (l)

$$= 14 \text{ cm},$$

breadth = 7 cm,

Radius of quarter circle r_1 = 7 cm,

Radius of semicircle $r_2 = \frac{7}{2}$ cm.



Area of remaining portion

= Area of rectangle

– (area of quadrant + area of semi-circle)

$$= l \times b - \left(\frac{\pi r_1^2}{4} + \frac{\pi r_2^2}{2}\right)$$

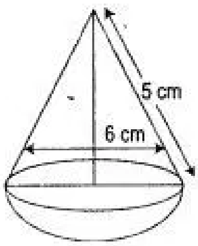
$$= 14 \times 7 - \left(\frac{22}{7} \times \frac{1}{4} \times 7 \times 7 + \frac{22}{7} \times \frac{1}{2} \times \frac{7}{2} \times \frac{7}{2}\right)$$

$$= 98 - \frac{77}{2} - \frac{77}{4}$$

$$= \frac{392 - 154 - 77}{4} = \frac{161}{4}$$

$$= 40\frac{1}{4} \text{ sq. units.}$$

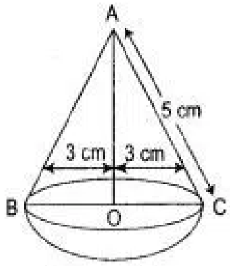
Question 12. The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate:
(i) the height of the cone.
(ii) the vol. of the solid.



Solution : (i) In ΔADC

$$AC^2 = AO^2 + OC^2$$

$$25 = AO^2 + 9$$



$$AO^2 = 16$$

$$AO = 4 \text{ cm}$$

$\therefore$ Height of cone = 4 cm

(ii) Vol. of solid = Vol. of cone + Vol. of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{\frac{4}{3}\pi r^3}{2}$$

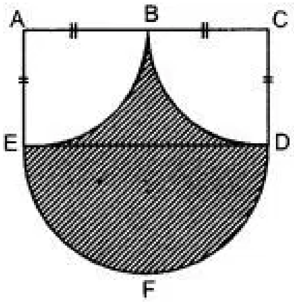
$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times (3)^3$$

$$= 37{\cdot}68 + 56{\cdot}52$$

$$= 94{\cdot}20 \text{ cm}^3 \qquad \text{Ans.}$$

Question 13. Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm. (Take π = 22/7).



Solution : AC = DE = 14 cm (given)
$\Rightarrow$ AB = BC = 7 cm (given)

Area of the shaded region

= Area of rectangle ACDE + Area of semicircle DEF – 2 × Area of quadrant

$\therefore$ Area of rectangle = Length × Breadth

Area of semicircle $= \frac{1}{2}\pi(r^2)$

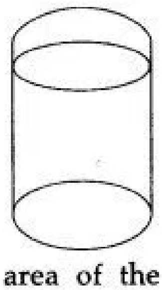
2 × Area of quadrant $= \frac{\pi}{4}(r^2)$

$\therefore$ Area of shaded region $= 14 \times 7 + \frac{\pi}{2}(7)^2 - 2 \times \frac{\pi}{4}(7)^2$

$= 98 + \frac{49\pi}{2} - \frac{49\pi}{2}$

$= 98 \text{ cm}^2$ Ans.

Question 14. With reference to the figure given a alongside, a metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3.5 m. Calculate:



(i) The total area of the internal surface, excluding the base;

(ii) The internal volume of the container in m^3.

(Take $\pi = 22/7$)

Solution. (i) Total area of the internal surface, encluding base

$= 2\pi rh + 2\pi r^2$, where $r = 3{\cdot}5$ m, $h = 7$ m

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 3{\cdot}5 \times (7 + 3{\cdot}5) \text{ m}^2$

$= \left(2 \times \frac{22}{7} \times 3{\cdot}5 \times 10{\cdot}5\right) \text{m}^2$

$= 231 \text{ m}^2$. Ans.

(ii) Internal volume of the container

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \times (21 + 7)\ m^3$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \times 28\right) m^3$$

$$= 359{\cdot}33\ m^3. \qquad \text{Ans.}$$

Concept Based Questions

Question 1. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/sec, calculate the number of complete revolutions the wheel makes in raising the bucket. (Take п =22/7)

Solution :

Distance moved by the bucket

$$= \text{Speed} \times \text{Time}$$

$$= [1{\cdot}1 \times 88]\ \text{metre}$$

$$[\because 1\ \text{min. } 28\ \text{sec} = 60 + 28 = 88\ \text{sec.}]$$

$$= 96{\cdot}8\ \text{metres}$$

Circumference of wheel $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{77}{2}\ \text{cm}$$

$$\left(\because r = \frac{d}{2} = \frac{77}{2}\ \text{cm}\right)$$

$$= 242\ \text{cm} = 2{\cdot}42\ \text{metres}$$

$\therefore$ Number of complete revolutions the wheel makes in raising the bucket

$$= \frac{\text{Distance}}{\text{Circumference}}$$

$$= \left(\frac{96{\cdot}8}{2{\cdot}42}\right) = 40. \qquad \text{Ans.}$$

Qustion 2. The radius of two right circular cylinder are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 4 calculate the ratio of their curved surface areas and also the ratio of their volumes.

Solution : Let the radii of two cylinders be $2r$ and $3r$ respectively and their heights be $5h$ and $4h$ respectively. Let S_1 and S_2 be Curved Surface Area of the two cylinders and V_1 and V_2 be their volumes.

Then, S_1 = Curved surface area of the cylinders of height $5h$ and radius $2r$

$\Rightarrow \quad 2\pi \times 2r \times 5h = 20\pi rh$ sq., units

S_2 = Curved surface area of cylinder of height $4h$ and radius $3r$

$$= 2\pi \times 3r \times 4h = 24\pi rh$$

$$\frac{S_1}{S_2} = \frac{20\pi rh}{24\pi rh} = \frac{5}{6} \Rightarrow S_1 : S_2 = 5 : 6$$

V_1 = Volume of cylinder of height $5h$ and radius $2r$

$$= \pi \times (2r)^2 \times 5h = 20\pi r^2 h \text{ cubic units}$$

V_2 = Volume of cylinder of height $4h$ and radius $3r$

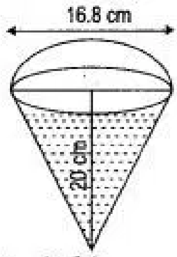
$$= \pi \times (3r)^2 \times 4h = 36\pi r^2 h \text{ cubic units}$$

$$\therefore \frac{V_1}{V_2} = \frac{20\pi r^2 h}{36\pi r^2 h} = \frac{5}{9} \Rightarrow V_1 : V_2 = 5 : 9. \quad \text{Ans.}$$

Question 3. A vessel in the form of an inverted cone is filled with water to the brim: Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in its so that they are fully submerged. As a result, one third of the water in the original cone overflows. What is the volume of each of the solid cones submerged?

Solution : height = 20 cm, diameter = 16·8 cm

or radius $= \frac{16 \cdot 8}{2} = 8 \cdot 4$ cm.



Volume of water in bigger cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 8 \cdot 4 \times 8 \cdot 4 \times 20$$

$$= 1478 \cdot 4 \text{ cm}^3$$

Volume of water overflows when two equal cone is submerged

$$= \frac{1}{3} \times 1478 \cdot 4$$

$$= 492 \cdot 8 \text{ cm}^3$$

$\therefore$ Volume of two equal cones

$$= 492 \cdot 8$$

So volume of each cone

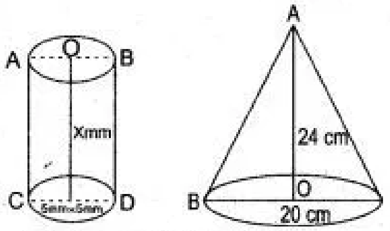
$$= \frac{1}{2} \times 492 \cdot 8$$

$$= 246 \cdot 4 \text{ cm}^3 \quad \text{Ans.}$$

Question 4. Water flows at the rate of 10 m per minute through a cylindrical pipe 5 mm of diameter. How much time would it take to fill a conical vessel whose diameter at he surface is 40 cm and depth is 24 cm?

Solution : Volume that flows in 1 min.

$$= [\pi \times (0.25)^2 \times 1000] \text{ cm}^3$$



Volume of the conical vessel

$$= \frac{1}{3}\pi \times (20)^2 \times 24 \text{ cm}^3$$

$$\text{Required time} = \frac{\frac{1}{3}\pi \times (20)^2 \times 24}{\pi \times (2.5)^2 \times 10000}$$

$$= 51 \text{ min } 12 \text{ sec.} \qquad \text{Ans.}$$

Question 5. A conical tent is accommodate to 11 persons each person must have 4 sq. metre of the space on the ground and 20 cubic metre of air to breath. Find the height of the cone.

Solution : Area of the base

$$= 11 \times 4 = 44 \text{ m}^2$$

And volume of the cone

$$= 11 \times 20 = 220 \text{ m}^3$$

$$\frac{1}{3} \times \pi R^2 h = 220 \text{ m}^3 \qquad \ldots\text{(i)}$$

$$\text{Area of the base} = \pi R^2$$

$$\pi R^2 = 44$$

$$R^2 = \frac{44}{22} \times 7;$$

$$R^2 = 14$$

$$R = \sqrt{14} \qquad \ldots\text{(ii)}$$

By equation (i) and (ii), we get

$$\frac{1}{3} \times \frac{22}{7} \times \sqrt{14} \times \sqrt{14} \times h = 220$$

$$h = \frac{220 \times 3}{22 \times 2}$$

$$h = \frac{30}{2} = 15 \text{ cm.} \qquad \text{Ans.}$$

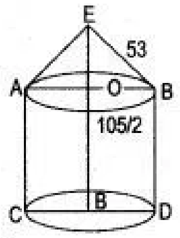
Question 6. A circus tent is cylindrical to a height of 3 meters and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m calculate the length of the canvas which is 5m wide to make the required tent.

Solution : Cylindrical area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{105}{2} \times 3 \text{ m}^2$$

and conical area = πrl

$$= \frac{22}{7} \times \frac{105}{2} \times 53 \text{ m}^2$$



Area of canvas

$$= 2 \times \frac{22}{7} \times \frac{105}{2} \times 3 + \frac{22}{7} \times \frac{105}{2} \times 53$$

$$= 15 \times 11(2 \times 3 + 53)$$

$$= 15 \times 11 \times 59 = 165 \times 59 \text{ m}^3$$

Length of canvas $= \frac{165 \times 59}{5}$ m

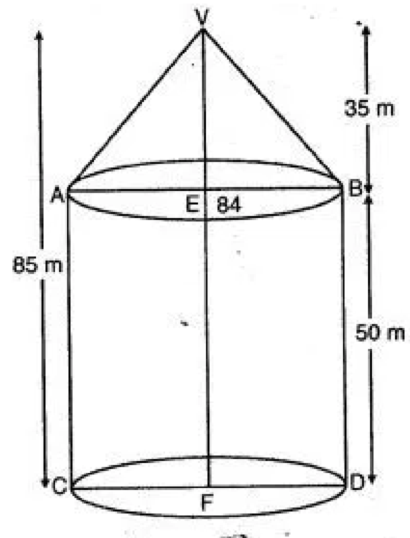
$$= 33 \times 59 \text{ m}$$

$\Rightarrow$ = 1947 m. Ans.

Question 7. An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds' and for stitching. Give your answer to the nearest m2.

Solution : Radius of box

$$= \frac{168}{2} = 84 \text{ m}$$



Height of the cone = 35 m

Height of cylinder = 50 m

Curved surface of the tent

$$= 2\pi rh + \pi rl$$

$$= 2 \times \frac{22}{7} \times 84 \times 50 + \frac{22}{7} \times 84 \times 91$$

$$[l^2 = (35)^2 + (84)^2] \text{ i.e., } l = 91$$

$$= 44 \times 12 \times 50 + 22 \times 12 \times 91$$

$$= 26,400 + 24,024$$

$$= 50,424 \text{ sq. m.}$$

Area of the canvas required with (20% extra).

$$= \frac{120}{100} \times 50,424 = 60,508 \cdot 8 \text{ sq. m}$$

$$= 60,509 \text{ m}^2 \quad \text{Ans.}$$

Question 8. The radius of a sphere is 10 cm. If we increase the radius 5% then how many % will increase in volume?

Solution : Volume of sphere $= \frac{4}{3}\pi r^3$

$\because$ Radius $= r = 10$ cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi \times 10 \times 10 \times 10$

$$= \frac{4000\pi}{3} \text{ cm}^3$$

Now, increase the radius 5%

Radius of new sphere $= \frac{10 \times 105}{100}$

$$= \frac{21}{2} \text{ cm}$$

Volume of new sphere $= \frac{4}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

$$= \frac{9261\pi}{6} \text{ cm}^3$$

Increase volume = Volume of new sphere − Volume of sphere

$$= \frac{9261\pi}{6} - \frac{4000\pi}{3}$$

$$= \frac{9261\pi - 8000\pi}{6}$$

$$= \frac{1261\pi}{6} \text{ cm}$$

Percentage of increasing volume

$$= \frac{\frac{1261\pi}{6} \times 100}{\frac{4000\pi}{3}}$$

$$= \frac{1261\,\pi \times 100 \times 3}{4000\pi \times 6} = \frac{1261}{80}\%$$

$$= 15\frac{61}{80}\%. \quad \text{Ans.}$$

Question 9. The cylinder of radius 12 cm have filled the 20 cm with water. One piece of iron drop in the stands of water goes up 6.75 cm. Find the radius of sphere piece.

Solution : Radius of cylinder

$$= 12 \text{ cm}$$

Height of cylinder $= 6{\cdot}75$ cm

Volume of water $= \pi r^2 h$

$$= \pi \times 12 \times 12 \times 6{\cdot}75 \text{ cm}^3$$

Let the radius of iron sphere piece = R cm

$\because$ Volume of sphere = Volume of water

$$\frac{4}{3}\pi R^3 = \pi \times 12 \times 12 \times 6{\cdot}75$$

$$R^3 = \frac{\pi \times 12 \times 12 \times 6{\cdot}75 \times 3}{4\pi}$$

$$R^3 = 729$$

$$R = \sqrt[3]{729} = 9 \text{ cm}$$

Hence, the radius of sphere piece = 9 cm. Ans.

Question 10. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively! If it is melted and recast into a solid cylinder of height 8/3 cm, find the diameter of the cylinder.

Solution : Internal radius of hollow spherical shell

$$(r_1) = 3 \text{ cm}$$

External radius of hollow spherical

$$(r_2) = 5 \text{ cm}$$

Volume of hollow spherical

$$= \frac{4}{3}\pi(r_2{}^3 - r_1{}^3)$$

$$= \frac{4}{3}\pi[(5)^3 - (3)^3]$$

$$= \frac{4}{3}\pi(125 - 27)$$

$$= \frac{4\pi \times 98}{3} = \frac{392\pi}{3} \text{ cm}^3$$

Volume of cylinder

$$= \pi r^2 h$$

$$= \pi r^2 \times \frac{8}{3}$$

$$= \frac{8\pi r^2}{3} \text{ cm}^3$$

Volume of cylinder

= Volume of hollow sphere

$$\frac{8\pi r^2}{3} = \frac{392\pi}{3}$$

$$r^2 = \frac{392\pi \times 3}{8\pi \times 3}$$

$$r^2 = 49$$

$$r = \sqrt{49}$$

$$r = 7 \text{ cm}$$

The diameter of cylinder $= 2r = 2 \times 7 = 14$ cm.

Question 11. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained?

Solution : The surface area of sphere $= 4\pi r^2$

$$4\pi r^2 = 616$$

$$r^2 = \frac{616 \times 7}{4 \times 22} = 49$$

$$r = \sqrt{49}$$

$$r = 7 \text{ cm}$$

$$\text{Vol. of big sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3$$

$$\text{Vol. of small sphere} = \frac{4}{3}\pi\left(\frac{3\cdot5}{2}\right)^3$$

$$\therefore \text{No of smaller sphere} = \frac{\text{Vol. of big sphere}}{\text{Vol. of small sphere}}$$

$$= \frac{\frac{4}{3}\pi(7)^3}{\frac{4}{3}\pi\left(\frac{3\cdot5}{2}\right)^3}$$

$$= 64 \qquad \text{Ans.}$$

Question 12. The diameter of the cross section of a water pipe is 5 cm. Water flows through it at 10km/hr into a cistern in the form of a cylinder. If the radius of the base of the cistern is 2.5 m, find the height to which the water will rise in the cistern in 24 minutes.

Solution : Area of the cross section of the water pipe

$$= \pi r^2$$

$$= 3\cdot142 \times \frac{5}{2} \times \frac{5}{2}$$

Speed of the water

$$= 10 \text{ km/hr}$$

$$= \frac{10 \times 1000 \times 100}{60} \text{ cm/minutes}$$

$\therefore$ Quantity of water supplied in 24 minutes

$$= 3\cdot142 \times \frac{5}{2} \times \frac{5}{2} \times \frac{10,00,00}{6} \times 24$$

$$= 78,55,000 \text{ cm}^3.$$

Let the height of water in the cistern be h cm.

The quantity of water collected in the cistern

$$= 3\cdot142 \times 250 \times 250 \times h \text{ cm}^3$$

Both the above quantities must be equal

$$\therefore 3\cdot142 \times 250 \times 250 \times h = 78,55,000$$

$$h = 78,55,000 \times \frac{1}{3\cdot142 \times 250 \times 250}$$

$$= 40 \text{ cm.} \qquad \text{Ans.}$$

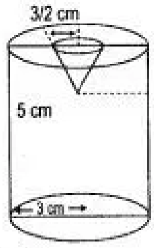
Question 13. A metallic cylinder has radius 3 cm and height 5 cm. It is made of metal A. To reduce its weight, a conical hole is drilled in the cylinder, as shown and it is completely filled with a lighter metal B. The conical hole has a radius of 3/2 cm and its depth is 8/9 cm. Calculate the ratio of the volume of the metal A to the volume of the metal B in the solid.

Solution : Volume of metal A = Volume of the cylinder – volume of the cone

$$= \pi (3)^2 \times 5 - \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \frac{8}{9}$$

$$= \pi\left(45 - \frac{2}{3}\right)$$

$$= \frac{133}{3}\pi \text{ cm}^3$$



Volume of metal B

= Volume of the conical cavity

$$= \frac{1}{3}\times\left(\frac{3}{2}\right)^2 \frac{8}{9} = \frac{2}{3}\pi$$

Hence, ratio of the volume of the metal A to the volume of the metal B

$$= \frac{\frac{133}{3}\pi}{\frac{2\pi}{3}} = \frac{133}{2}$$

$= 66{\cdot}5 : 1.$ Ans.

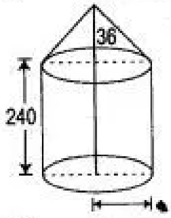
Question 14. An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cubic cm of iron weight is 7.8 grams.

Solution : Let r_1 cm and r_2 cm denote the radii of the base of the cylinder and cone respectively. Then,

$$r_1 = r_2 = 8 \text{ cm}$$

Let h_1 and h_2 cm be the height of the cylinder and the cone respectively. Then,

$$h_1 = 240 \text{ cm and } h_2 = 36 \text{ cm}$$



Now, volume of the cylinder

$$= \pi r_1^2 h_1 \text{ cm}^3$$
$$= (\pi \times 8 \times 8 \times 240) \text{ cm}^3$$
$$= (\pi \times 64 \times 240) \text{ cm}^3$$

Volume of the cone $= \frac{1}{3}\pi r_2^2 h_2 \text{ cm}^3$

$$= \left(\frac{1}{3}\pi \times 8 \times 8 \times 36\right) \text{cm}^3$$
$$= \left(\frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$$

$\therefore$ Total volume of iron

= Volume of the cylinder + Volume of the cone

$$= \left(\pi \times 64 \times 240 + \frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$$
$$= \pi \times 64 \times (240 + 12) \text{ cm}^3$$
$$= \frac{22}{7} \times 64 \times 252 \text{ cm}^3$$
$$= 22 \times 64 \times 36 \text{ cm}^3$$

Hence, total weight of the pillar

= Volume × Weight per cm^3

$= (22 \times 64 \times 36) \times 7{\cdot}8$ gms

= 395366·4 gms

= 395·3664 kg. Ans.

Question 15. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Solution : The radius of spherical ball = 3 cm

Volume of spherical ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3 \times 3 \times 3$

$= 36\pi \text{ cm}^3$

$\because$ Volume of spherical ball = Total volume of three small spherical ball

$\because$ The radii of the ball are 1·5 cm and 2 cm

$\therefore$ Let the radius of third ball = r

$\therefore$ Volume of spherical ball

= Total volume of three small spherical balls

$$36\pi = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi r^3$$

$$36\pi = \frac{4}{3}\pi \times \frac{27}{8} + \frac{4}{3}\pi \times 8 + \frac{4}{3}\pi r^3$$

$$36\pi = \frac{4}{3}\pi\left(\frac{27}{8} + 8 + r^3\right)$$

$$\frac{36\pi \times 3}{4\pi} = \frac{27}{8} + 8 + r^3$$

$$27 = \frac{27 + 64}{8} + r^3$$

$$27 = \frac{91}{8} + r^3$$

$$27 - \frac{91}{8} = r^3$$

$$\frac{216 - 91}{8} = r^3$$

$$\frac{125}{8} = r^3$$

$$r = \sqrt[3]{\frac{125}{8}}$$

$$r = \frac{5}{2} \text{ cm}$$

The diameter of the third ball $= 2r = 2 \times \frac{5}{2}$

= 5 cm.

Question 16. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of hemisphere. The radius of the base of the cone is 3.5 metres and its volume is two thirds of the hemisphere. Calculate the height of the cone and the surface area of buoy correct to two places of decimal.

Solution : According to question.

$\frac{2}{3}$ (Volume of hemisphere) = Volume of Cone

$$\frac{2}{3}\left(\frac{2}{3}\pi r^3\right) = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{9}(3\cdot5)^3 = \frac{1}{3}(3\cdot5)^2 . h$$

$$h = \frac{4 \times 3\cdot5 \times 3\cdot5 \times 3\cdot5 \times 3}{3\cdot5 \times 3\cdot5 \times 9}$$

$$= \frac{42\cdot0}{9} = \frac{14}{3} \text{ m} = 4\cdot67 \text{ m}$$

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(3\cdot5)^2 + (4\cdot67)^2}$$

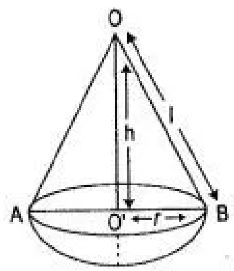
$$= \frac{35}{6} \text{ m}$$

Now surface area of buoy

= Surface area of right cone
+ surface area of hemisphere

$$= \pi rl + 2\pi r^2$$

$$= \pi r (l + 2r)$$



$$= \frac{22}{7} \times 3\cdot5\left(\frac{35}{6} + 2 \times 3\cdot5\right)$$

$$= 11 \times (5\cdot83 + 7)$$

$$= 11 \times 12\cdot83$$

$$= 141\cdot13 \text{ sq. m.}$$

Question 17. Water flows through a cylindrical pipe of internal diameter 7 cm at 36 km/hr. Calculate the time in minutes it would take to fill cylindrical tank, the radius of whose base is 35 cm and height is 1 m.

Solution :

Radius of pipe $= \frac{7}{2}$ cm

Rate of water flow $= 36$ km/h

$= 36 \times \frac{5}{18}$ m/s

$= 10$ m/s

$= 10 \times 100$ cm/s

$= 1000$ cm/s

$\therefore$ Volume of water flowing in 1 second $= \pi r^2 h$

$= \pi \cdot \frac{7}{2} \cdot \frac{7}{2} \cdot 1000 \text{ cm}^3$

$= \pi.7.7.250 \text{ cm}^3$ ...(1)

Radius of tank $(R) = 35$ cm

Height of tank $(H) = 1 \text{ m} = 100$ cm

$\therefore$ Volume of tank $= \pi r^2 H$

$= \pi.35.35.100 \text{ cm}^3$ ...(2)

$\therefore$ Time taken to fill the tank

$= \frac{\text{Volume of tank}}{\text{Volume of water flowing in 1 second}}$

$= \frac{\pi \times 35 \times 35 \times 100}{\pi \times 7 \times 7 \times 250}$ seconds

$= 10$ seconds

$= \frac{10}{60}$ minute

$= \frac{1}{6}$ minute. Ans.