Chapter 18. Trigonometry

Formulae

1. Trigonometrical ratios :

(i) $\sin\theta = \dfrac{\text{height}}{\text{hypotenuse}}$

(ii) $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$

(iii) $\tan\theta = \dfrac{\text{height}}{\text{base}}$

(iv) $\cot\theta = \dfrac{\text{base}}{\text{height}}$

(v) $\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}}$

(vi) $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{height}}$

2. Quotient relations :

(i) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(ii) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

3. Square relations :

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\,\theta$

4. Trigonometrical ratios of standard angles :

Angle → Ratio ↓	0°	30°	45°	60°	90°
$\sin\theta$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
$\cos\theta$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$
$\tan\theta$	$\sqrt{\frac{0}{4-0}}$	$\sqrt{\frac{1}{4-1}}$	$\sqrt{\frac{2}{4-2}}$	$\sqrt{\frac{3}{4-3}}$	not defined

5. Trigonometrical ratios of complementary angles :

(i) $\sin(90° - \theta) = \cos\theta$

(ii) $\cos(90° - \theta) = \sin\theta$

(iii) $\tan(90° - \theta) = \cot\theta$

(iv) $\cot(90° - \theta) = \tan\theta$

(v) $\sec(90° - \theta) = \text{cosec}\,\theta$

(v) $\text{cosec}(90° - \theta) = \sec\theta$

Determine the Following

Question 1. Solve : $2\cos^2\theta + \sin\theta - 2 = 0$.

Solution :

$$2\cos^2\theta + \sin\theta - 2 = 0$$
$$\Rightarrow 2(1-\sin^2\theta) + \sin\theta - 2 = 0$$
$$\Rightarrow 2 - 2\sin^2\theta + \sin\theta - 2 = 0$$
$$\Rightarrow -\sin\theta(2\sin\theta - 1) = 0$$
$$\Rightarrow \sin\theta(2\sin\theta - 1) = 0$$
$$\Rightarrow \sin\theta = 0 \text{ or } 2\sin\theta - 1 = 0$$
$$\Rightarrow \sin\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$$
$$\Rightarrow \theta = 30^\circ.$$

Question 2. Without using tables evaluate
$3\cos 80^\circ$. Cosec $10^\circ + 2\sin 59^\circ \sec 31^\circ$.

Solution : $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\sin 59^\circ \sec 31^\circ$

$$3\cos(90^\circ - 10^\circ)\cdot\frac{1}{\sin 10^\circ} + 2\sin(90^\circ - 31^\circ)\cdot\frac{1}{\cos 31^\circ}$$
$$= \frac{3\sin 10^\circ}{\sin 10^\circ} + \frac{2\cos 31^\circ}{\cos 31^\circ} = 3 + 2 = 5. \quad \text{Ans.}$$

Question 3. Given that

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}.$$

Find $(\theta_1 + \theta_2)$, when $\tan\theta_1 = \frac{1}{2}$, $\tan\theta_2 = \frac{1}{3}$.

Solution : We have,

$$\tan\theta_1 = \frac{1}{2}$$
and
$$\tan\theta_2 = \frac{1}{3}$$
$$\therefore \tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}$$
$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}}$$
$$\tan(\theta_1 + \theta_2) = \frac{\frac{5}{6}}{\frac{5}{6}}$$
$$\Rightarrow \tan(\theta_1 + \theta_2) = 1$$
$$\Rightarrow \theta_1 + \theta_2 = 45^\circ.$$

Question 4. Without using trigonometric tables, evaluate

$\sin^2 34° + \sin^2 56° + 2\tan 18° \tan 72° - \cot^2 30°$

Solution : $\sin^2 34° + \sin^2 56° + 2\tan 18° \tan 72° - \cot^2 30°$

$= \sin^2 34° + \cos^2(90° - 56°) + 2\tan 18° \cot(90° - 72°) - (\sqrt{3})^2$

$= (\sin^2 34° + \cos^2 34°) + 2\tan 18° \times \frac{1}{\tan 18°} - 3$

$= 1 + 2 - 3 = 3 - 3 = 0.$ Ans.

Question 5. Without using trigonometric tables, find the value of $(\sin 72° + \cos 18°)(\sin 72° - \cos 18°)$.

Solution : $(\sin 72° + \cos 18°)(\sin 72° - \cos 18°)$

$= [\sin(90° - 18°) + \cos 18°][\sin(90° - 18°) - \cos 18°]$

$= [\cos 18° + \cos 18°][\cos 18° - \cos 18°]$

$= 0.$ Ans.

Question 6. With out using table evaluate

$$\frac{2\tan 53°}{\cot 37°} - \frac{\cot 80°}{\tan 10°}$$

Solution : $\frac{2\tan 53°}{\cot 37°} - \frac{\cot 80°}{\tan 10°}$

$= \frac{2\tan 53°}{\cot(90° - 53°)} - \frac{\cot 80°}{\tan(90° - 80°)}$

$= \frac{2\tan 53°}{\tan 53°} - \frac{\cot 80°}{\cot 80°}$

$= 2 - 1$

$= 1$

Question 7. Solve : $\sin^2\theta - 3\sin\theta + 2 = 0$.

Solution : $\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow \sin^2\theta - 2\sin\theta - \sin\theta + 2 = 0$

$\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$

$(\sin\theta - 2)(\sin\theta - 1) = 0$

$\sin\theta - 2 = 0$

$\Rightarrow \sin\theta = 2$

$\sin\theta = 2$ has no solution for angle θ, as there is no any angle whose $\sin\theta$ is equal to 2.

$\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1$

$\theta = 90°.$ Ans.

Question 8. If $5 \tan \theta = 4$, find the value of

$$\frac{5 \sin \theta + 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}.$$

Solution :

$$5 \tan \theta = 4$$

$$\tan \theta = \frac{4}{5}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{4}{5}$$

$$\frac{5 \sin \theta + 3 \cos \theta}{5 \sin \theta + 2 \cos \theta} = \frac{\frac{5 \sin \theta}{\cos \theta} + 3\frac{\cos \theta}{\cos \theta}}{\frac{5 \sin \theta}{\cos \theta} + \frac{2 \cos \theta}{\cos \theta}}$$

[Nr., and Dr. Dividing by $\cos \theta$]

$$= \frac{5 \times \frac{4}{5} + 3}{5 \times \frac{4}{5} + 2} = \frac{4+3}{4+2} = \frac{7}{6}.$$ Ans.

Question 9. From trigonometric tables, write the values of:

(i) $\sin 37°19'$

(ii) $\cos 23°17'$

(iii) $\tan 45°48'$.

Solution :

(i) From the tables of natural sine, we have

$\sin 37°18' = 0{\cdot}6060$

Mean difference for $1' = 0{\cdot}0002$ (to be added)

So, $\sin 37°19' = 0{\cdot}6062$. Ans.

(ii) From the tables of natural cosines, we have

$\cos 23°12' = 0{\cdot}9191$

Mean difference for $5' = 0{\cdot}0006$ (to be subtracted)

$\cos 23°17' = 0{\cdot}9185$. Ans.

(iii) Similarly, $\tan 45°48' = 1{\cdot}0283$. Ans.

Question 10. The string of a kite is 150 m long and it makes an angle of 60° with the horizontal. Find the height of the kite from the ground.

Solution : Let h be the height of the kite. PB be the length of string such that PB = 150 m

In right angled ΔBPA,

$$\sin 60° = \frac{h}{150}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{h}{150}$$

$$\Rightarrow \quad h = \frac{150\sqrt{3}}{2}$$

$$= 75\sqrt{3}$$

$$h = 1{\cdot}732 \times 75$$

$$= 129{\cdot}9 \text{ m}$$

Hence, the height of kite above the ground

= 129·9 m. Ans.

Question 11. Solve the following equations:

(i) $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

(ii) $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1.$

Solution : (i) We have

$$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$$

$$\Rightarrow \quad \cos\theta\left\{\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}\right\} = 4$$

$$\Rightarrow \quad \cos\theta\left\{\frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}\right\} = 4$$

$$\Rightarrow \quad 2\cos\theta = 4(1-\sin\theta)(1+\sin\theta)$$

$$\Rightarrow \quad 2\cos\theta = 4(1-\sin^2\theta)$$

$$\Rightarrow \quad 2\cos\theta = 4\cos^2\theta$$

$$\Rightarrow \quad 4\cos^2\theta - 2\cos\theta = 0$$

$$\Rightarrow \quad 2\cos\theta(2\cos\theta - 1) = 0$$

$$\Rightarrow 2\cos\theta = 0 \text{ or } 2\cos\theta - 1 = 0$$

$$\Rightarrow \quad \cos\theta = 0 \text{ or } \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°, \text{ (since } 0 < \theta < 90°)$$

(ii) We have, $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$

$$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 2 = \sin^2\theta$$
$$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 2 - \sin^2\theta = 0$$
$$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 1 + \cos^2\theta = 0$$
$$\Rightarrow \quad 2\cos^2\theta - 3\cos\theta + 1 = 0$$
$$\Rightarrow \quad 2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$$
$$\Rightarrow 2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$$
$$\Rightarrow \quad (\cos\theta - 1)(2\cos\theta - 1) = 0$$
$$\Rightarrow \quad \cos\theta - 1 = 0 \text{ or } 2\cos\theta - 1 = 0$$
$$\Rightarrow \quad \cos\theta = 1 \text{ or } \cos\theta = \frac{1}{2}$$
$$\Rightarrow \quad \theta = 0° \text{ or } \theta = 60°$$

Since $0 < \theta < 90°$

So $\theta = 60°$ is the solution of the equation.

Question 12. Using trigonometric tables evaluate the following:

(i) $\tan 25°46' + \cot 45°25'$

(i) $\sin 64°42' + \cos 42°20'$

(iii) $\cos 64°42' - \sin 42°20'$

(iv) $\tan 78°55' - \tan 55°18'$.

Solution : (i) $\tan 25° 45' + \cot 45°25'$

$= 0{\cdot}482 + 0{\cdot}986 = 1{\cdot}468.$

(ii) $\sin 64°42' + \cos 42°20'$

$= 0{\cdot}9041 + 0{\cdot}7392$

$= 1{\cdot}6433.$

(iii) $\cos 64°42' - \sin 42°20'$

$= 0{\cdot}4274 - 0{\cdot}6734$

$= -0{\cdot}2460$

$= -0{\cdot}2460$

(iv) $\tan 78°55' - \tan 55°18'$

$= 5{\cdot}097 - 1{\cdot}444$

$= 3{\cdot}653.$

Question 13. (i) $\frac{\sin 40^\circ + \cos 50^\circ}{\tan 38^\circ 20'}$

(ii) $\frac{\sin 20^\circ 50' + \tan 67^\circ 40'}{\cos 32^\circ 20' - \sin 15^\circ 10'}$.

Solution : (i) $\frac{\sin 40^\circ + \cos 50^\circ}{\tan 38^\circ 20'}$

$$= \frac{0{\cdot}6428 + 0{\cdot}6428}{0{\cdot}7907}$$

$$= \frac{1{\cdot}2856}{0{\cdot}7907}$$

$$= 1{\cdot}6259.$$

(ii) $\frac{\sin 20^\circ 50' + \tan 67^\circ 40'}{\cos 32^\circ 20' - \sin 15^\circ 10'}$

$$= \frac{0{\cdot}3557 + 2{\cdot}4340}{0{\cdot}8450 - 0{\cdot}2616}$$

$$= \frac{2{\cdot}7897}{0{\cdot}5834}$$

$$= \frac{27897}{5834}$$

$$= 4{\cdot}7818.$$

Prove the Following

Question 1. If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$ and $z = r\cos\theta$, prove that $x^2 + y^2 + z^2 = r^2$.

Solution : We have

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Squaring and adding,

$x^2 + y^2 + z^2$

$= r^2 \sin^2\theta \cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$= r^2\sin^2\theta \times 1 + r^2\cos^2\theta$

$= r^2(\sin^2\theta + \cos^2\theta)$

$= r^2 \times 1 = r^2$.

Hence $x^2 + y^2 + z^2 = r^2$. Hence proved

Question 2. Prove that $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$.

Solution :

$$\begin{aligned}
\text{L.H.S.} &= \frac{\cos\theta}{1-\sin\theta} \\
&= \frac{\cos\theta\,(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \\
&= \frac{\cos\theta\,(1+\sin\theta)}{1-\sin^2\theta} \\
&= \frac{\cos\theta\,(1+\sin\theta)}{\cos^2\theta} \\
&= \frac{1+\sin\theta}{\cos\theta}. \qquad \text{Hence proved.}
\end{aligned}$$

Question 3. Prove that $\tan^2\phi + \cot^2\phi + 2 = \sec^2\phi.\text{cosec}^2\phi$.

Solution :

$$\begin{aligned}
\text{L.H.S.} &= \tan^2\phi + \cot^2\phi + 2 \\
&= \tan^2\phi + 1 + \cot^2\phi + 1 \\
&= \sec^2\phi + \text{cosec}^2\phi \\
&= \frac{1}{\cos^2\phi} + \frac{1}{\sin^2\phi} \\
&= \frac{\sin^2\phi + \cos^2\phi}{\sin^2\phi.\cos^2\phi} \\
&= \frac{1}{\sin^2\phi.\cos^2\phi} \\
&= \text{cosec}^2\phi.\sec^2\phi = \text{R.H.S.}
\end{aligned}$$

Hence proved.

Question 4. Prove that $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$.

Solution :

L.H.S. $= \sin^2\theta + \cos^4\theta$

$$\begin{aligned}
&= 1 - \cos^2\theta + \cos^4\theta \\
&= 1 - \cos^2\theta\,(1 - \cos^2\theta) \\
&= 1 - (1 - \sin^2\theta)\sin^2\theta \\
&= 1 - \sin^2\theta + \sin^4\theta \\
&= \cos^2\theta + \sin^4\theta = \text{R.H.S.}
\end{aligned}$$

Hence proved.

Question 5. Prove that

$$\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$$
$$= 2\sin^2\theta - 1$$
$$= 1 - 2\cos^2\theta.$$

Solution :

$$\begin{aligned}
\text{L.H.S.} &= \sin^4\theta - \cos^4\theta \\
&= (\sin^2\theta)^2 - (\cos^2\theta)^2 \\
&= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) \\
&= (\sin^2\theta - \cos^2\theta) \times 1 \\
&= \sin^2\theta - \cos^2\theta = \text{R.H.S} \\
&= \sin^2\theta - (1 - \sin^2\theta) \\
&= \sin^2\theta - 1 + \sin^2\theta \\
&= 2\sin^2\theta - 1 = \text{R.H.S.} \\
&= 2(1 - \cos^2\theta) - 1 \\
&= 2 - 2\cos^2\theta - 1 \\
&= 1 - 2\cos^2\theta = \text{R.H.S.}
\end{aligned}$$

Hence proved

Question 6. Prove that $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta.$

Solution :

$$\begin{aligned}
\text{L.H.S.} &= \sqrt{\frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}} \\
&= \sqrt{\frac{1 + \sin^2\theta - 2\sin\theta}{1 - \sin^2\theta}} \\
&= \sqrt{\frac{1 + \sin^2\theta - 2\sin\theta}{\cos^2\theta}} \\
&= \sqrt{\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} - \frac{2\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}} \\
&= \sqrt{\sec^2\theta + \tan^2\theta - 2\tan\theta.\sec\theta} \\
&= \sqrt{(\sec\theta - \tan\theta)^2} \\
&= \sec\theta - \tan\theta \\
&= \text{R.H.S.}
\end{aligned}$$

Hence proved

Question 7. $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\operatorname{cosec} A$

Solution : L. H. S. $=\dfrac{\sin^2 A+(1+\cos A)^2}{(1+\cos A)\sin A}$

$$=\frac{\sin^2 A+1+\cos^2 A+2\cos A}{(1+\cos A)\sin A}$$

$$=\frac{1+1+2\cos A}{(1+\cos A)\sin A}$$

$$=\frac{2(1+\cos A)}{(1+\cos A)\sin A}$$

$= 2\operatorname{cosec} A =$ R. H. S. Hence proved.

Question 8. Prove that

$$\sqrt{2+\tan^2\theta+\cot^2\theta}=\tan\theta+\cot\theta.$$

Solution :

L.H.S. $=\sqrt{2+\tan^2\theta+\cot^2\theta}$

$=\sqrt{\tan^2\theta+\cot^2\theta+2\tan\theta.\cot\theta}$

$[\because \tan\theta\cot\theta=1]$

$=\sqrt{(\tan\theta+\cot\theta)^2}$

$=\tan\theta+\cot\theta=$ R.H.S.

Hence proved.

Question 9. Prove that $\dfrac{\sec\theta-1}{\sec\theta+1}=\left(\dfrac{\sin\theta}{1+\cos\theta}\right)^2.$

Solution :

L.H.S. $=\dfrac{\sec\theta-1}{\sec\theta+1}$

$$=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}$$

$$=\frac{1-\cos\theta\times(1+\cos\theta)}{1+\cos\theta\times(1+\cos\theta)}$$

$$=\frac{1-\cos^2\theta}{(1+\cos\theta)^2}$$

$$=\frac{\sin^2\theta}{(1+\cos\theta)^2}$$

$$=\left(\frac{\sin\theta}{1+\cos\theta}\right)^2=\text{R.H.S.}$$

Question 10. Prove that

$$\frac{\sin\theta\tan\theta}{1-\cos\theta} = 1+\sec\theta.$$

Solution :

$$\text{L.H.S.} = \frac{\sin\theta\tan\theta}{1-\cos\theta} = \frac{\sin\theta.\dfrac{\sin\theta}{\cos\theta}}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\cos\theta\,(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)\,(1+\cos\theta)}{\cos\theta\,(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$$

$$= \sec\theta + 1$$

$= \text{R.H.S.}$ Hence proved.

Question 11. Prove that

$$\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta} = 1-2\sec\theta.\tan\theta+2\tan^2\theta.$$

Solution :

$$\text{L.H.S.} = \frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$$

$$= \frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta} \times \frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}$$

$$= \frac{(\sec\theta-\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$$

$$= \frac{\sec^2\theta+\tan^2\theta-2\sec\theta.\tan\theta}{1}$$

$$= 1+2\tan^2\theta-2\sec\theta.\tan\theta$$

$= \text{R.H.S.}$ Hence proved.

Question 12. Prove that $(1+\tan A)^2+(1-\tan A)^2 = 2\sec^2 A$

Solution :

$$\text{L.H.S.} = (1+\tan A)^2+(1-\tan A)^2$$

$$= 1+2\tan A+\tan^2 A + 1-2\tan A+\tan^2 A.$$

$$= 2\,(1+\tan^2 A)$$

$$= 2\sec^2 A = \text{R.H.S.}$$

Hence proved.

Question 13. If $x = a \sec\theta + b\tan\theta$ and $y = a \tan\theta + b\sec\theta$ prove that

$$x^2 - y^2 = a^2 - b^2$$

Solution : Here $x^2 = a^2\sec^2\theta + 2ab\sec\theta\tan\theta + b^2\tan^2\theta$

$$y^2 = a^2\tan^2\theta + 2ab\sec\theta\tan\theta + b^2\sec^2\theta$$

$$\Rightarrow \quad x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$$

$$= a^2 - b^2 \quad (\because \sec^2\theta - \tan^2\theta = 1)$$

Hence proved.

Question 14. Prove that

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\operatorname{cosec} A.\cot A.$$

Solution :

$$\text{L.H.S.} = \frac{1}{\sec A - 1} + \frac{1}{\sec A + 1}$$

$$= \frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1} = \frac{2\sec A}{\tan^2 A}$$

$$= 2\frac{1}{\cos A} \times \frac{1}{\frac{\sin^2 A}{\cos^2 A}} = 2\frac{1}{\cos A} \times \frac{\cos^2 A}{\sin^2 A}$$

$$= 2\operatorname{cosec} A.\cot A$$

$$= \text{R.H.S.}$$ Hence proved.

Question 15. Prove that

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$$

Solution : .L.H.S.

$$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^2\theta + \cos^2\theta)[\sin^4\theta + \cos^4\theta - \sin^2\theta.\cos^2\theta] - 3[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta] + 1$$

$$= 2 \times 1[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta - \sin^2\theta.\cos^2\theta] - 3[(1)^2 - 2\sin^2\theta\cos^2\theta] + 1$$

$$= 2[(1)^2 - 3\sin^2\theta\cos^2\theta] - 3[1 - 2\sin^2\theta.\cos^2\theta] + 1$$

$$= 2 - 6\sin^2\theta.\cos^2\theta - 3 + 6\sin^2\theta.\cos^2\theta + 1$$

$$= -1 + 1 = 0 = \text{R.H.S.}$$ Hence proved.

Question 16. Prove that

$$\frac{1+\sin\theta}{1-\sin\theta} = 1 + 2\frac{\tan\theta}{\cos\theta} + 2\tan^2\theta.$$

Solution :

$$\begin{aligned}
\text{R.H.S.} &= 1 + 2\frac{\tan\theta}{\cos\theta} + 2\tan^2\theta \\
&= 1 + 2\frac{\sin\theta}{\cos^2\theta} + 2\frac{\sin^2\theta}{\cos^2\theta} \\
&= \frac{\cos^2\theta + 2\sin\theta + 2\sin^2\theta}{\cos^2\theta} \\
&= \frac{1-\sin^2\theta + 2\sin\theta + 2\sin^2\theta}{1-\sin^2\theta} \\
&= \frac{1+\sin^2\theta + 2\sin\theta}{1-\sin^2\theta} \\
&= \frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} \\
&= \frac{1+\sin\theta}{1-\sin\theta} \\
&= \text{L.H.S.}
\end{aligned}$$

Hence proved.

Question 17. Prove that

$$1 - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$$

Solution :

$$\begin{aligned}
\text{L.H.S.} &= 1 - \frac{\cos^2\theta}{1+\sin\theta} = 1 - \frac{(1-\sin^2\theta)}{1+\sin\theta} \\
&= 1 - \frac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta} \\
&= 1 - (1-\sin\theta) = 1 - 1 + \sin\theta \\
&= \sin\theta = \text{R.H.S.}
\end{aligned}$$

Hence proved.

Question 18. If $A = 30°$, verify that

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

Solution :

$\text{L.H.S.} = \sin 2A$

Putting $A = 30°$ in L.H.S. and R.H.S., we get

$$\text{L.H.S.} = \sin 2 \times 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{R.H.S.} = \frac{2 \times \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2 \times 3}{\sqrt{3} \times 4} = \frac{\sqrt{3}}{2}$$

Hence,

L.H.S. = R.H.S. Hence proved.

Question 19. Prove that

$$\frac{\sin(90^\circ - \theta)\tan(90^\circ - \theta)\sec(90^\circ - \theta)}{\text{cosec}\,\theta.\cos\theta.\cot\theta} = 1.$$

Solution :

$$\text{L.H.S.} = \frac{\sin(90^\circ - \theta)\tan(90^\circ - \theta)\sec(90^\circ - \theta)}{\text{cosec}\,\theta.\cos\theta.\cot\theta}$$

$$= \frac{\cos\theta.\cot\theta.\text{cosec}\,\theta}{\text{cosce}\,\theta.\cos\theta.\cot\theta} = 1 = \text{R.H.S.}$$

Hence proved

Question 20. Prove that

$\cot\theta.\tan(90^\circ - \theta) - \sec(90^\circ - \theta).\text{cosec}\,\theta + 1 = 0.$

Solution :

$$\text{L.H.S.} = \cot\theta.\tan(90^\circ - \theta) - \sec(90^\circ - \theta).\text{cosec}\,\theta + 1$$

$$= \cot\theta.\cot\theta - \text{cosec}\,\theta.\text{cosec}\,\theta + 1$$

$$= (\cot^2\theta - \text{cosec}^2\theta) + 1$$

$$= -1 + 1 = 0$$

$$= \text{R.H.S.}$$

Hence proved.

Question 21. Prove that sec θ.cosec (90° – θ) – tan θ.cot (90° – θ) = 1.

Solution : L.H.S.

$= \sec\theta.\text{cosec}(90° - \theta) - \tan\theta.\cot(90° - \theta)$

$= \sec\theta \sec\theta - \tan\theta.\tan\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1 = $ R.H.S. Hence proved.

Question 22. Prove that

$\sec^2(90° - \theta) + \tan^2(90° - \theta) = 1 + 2\cot^2\theta$

Solution : L.H.S.

$= \sec^2(90° - \theta) + \tan^2(90° - \theta)$

$= \text{cosec}^2\theta + \cot^2\theta$

$= 1 + \cot^2\theta + \cot^2\theta$

$= 1 + 2\cot^2\theta$

$=$ R.H.S. Hence proved.

Question 23. Prove that $\text{cosec}^2(90° - \theta) + \cot^2(90° - \theta) = 1 + 2\tan^2\theta$.

Solution :

L.H.S. $= \text{cosec}^2(90° - \theta) + \cot^2(90° - \theta)$

$= \sec^2\theta + \tan^2\theta$

$= 1 + \tan^2\theta + \tan^2\theta$

$= 1 + 2\tan^2\theta$

$=$ R.H.S. Hence proved

Question 24. Prove that

$$\frac{\tan\theta}{\cot(90° - \theta)} + \frac{\sec(90° - \theta)\sin(90° - \theta)}{\cos\theta.\text{cosec}\,\theta} = 2$$

Solution :

$$\text{L.H.S.} = \frac{\tan\theta}{\cot(90° - \theta)} + \frac{\sec(90° - \theta)\sin(90° - \theta)}{\cos\theta.\text{cosec}\,\theta}$$

$$= \frac{\tan\theta}{\tan\theta} + \frac{\text{cosec}\,\theta.\cos\theta}{\cos\theta.\text{cosec}\,\theta}$$

$= 1 + 1 = 2$

$=$ R.H.S. Hence proved.

Question 25. Prove that

$$\sin(90° - \theta)\sin\theta\cot\theta = \cos^2\theta.$$

Solution :

$$\begin{aligned} \text{L.H.S.} &= \sin(90° - \theta)\sin\theta\cot\theta \\ &= \cos\theta \cdot \sin\theta \cdot \frac{\cos\theta}{\sin\theta} \\ &= \cos^2\theta = \text{R.H.S.} \quad \text{Hence proved.} \end{aligned}$$

Question 26. Prove that

$$\sin\theta\sin(90° - \theta) - \cos\theta\cos(90° - \theta) = 0.$$

Solution : L.H.S.

$= \sin\theta\sin(90° - \theta) - \cos\theta\cos(90° - \theta)$

$= \sin\theta.\cos\theta - \cos\theta.\sin\theta$

$= 0 = \text{R.H.S.}$ Hence proved.

Question 27. Prove that

$$\sin(90° - \theta)\cos(90° - \theta) = \tan\theta.\cos^2\theta.$$

Solution :

$$\begin{aligned} \text{L.H.S.} &= \sin(90° - \theta)\cos(90° - \theta) \\ &= \cos\theta.\sin\theta \\ \text{R.H.S.} &= \tan\theta.\cos^2\theta \\ &= \frac{\sin\theta}{\cos\theta} \times \cos^2\theta \\ &= \sin\theta.\cos\theta \\ \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved.} \end{aligned}$$

Question 28. Prove that

$$\frac{\sin(90° - \theta)}{\cos\theta} + \frac{\tan(90° - \theta)}{\cot\theta} + \frac{\operatorname{cosec}(90° - \theta)}{\sec\theta} = 3.$$

Solution : L.H.S.

$$= \frac{\sin(90° - \theta)}{\cos\theta} + \frac{\tan(90° - \theta)}{\cot\theta} + \frac{\operatorname{cosec}(90° - \theta)}{\sec\theta}$$

$$= \frac{\cos\theta}{\cos\theta} + \frac{\cot\theta}{\cot\theta} + \frac{\sec\theta}{\sec\theta}$$

$= 1 + 1 + 1 = 3 = \text{R.H.S.}$ Hence proved

Question 29. If A = 60°, B = 30° verify that

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A.\tan B}.$$

Solution : It is given that A = 60°, B = 30°. Putting A = 60° and B = 30° in the given equation, we get

$$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A.\tan B}$$

$$\Rightarrow \quad \tan(60°-30°) = \frac{\tan 60°-\tan 30°}{1+\tan 60°.\tan 30°}$$

$$\Rightarrow \quad \tan 30° = \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{2/\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \text{L.H.S.} = \text{R.H.S.}$$

Question 30. If 2 sin A – 1 = 0, show that

$$\sin 3A = 3\sin A - 4\sin^3 A.$$

Solution : If $2\sin A = 1$

i.e., $\sin A = \frac{1}{2} = \sin 30°$

$\Rightarrow$ $A = 30°$

and $\sin 3A = 3\sin A - 4\sin^3 A$

$\sin 3\times 30° = 3\sin 30° - 4\sin^3 30°$

Now L.H.S. $= \sin 90° = 1$

$$\text{R.H.S.} = 3\times\frac{1}{2}-4\times\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2}-\frac{4}{8}$$

$$= \frac{3}{2}-\frac{1}{2}$$

$$= 1$$

L.H.S. = R.H.S.

Question 31. If A, B, C are the interior angles of Δ ABC, prove that

$$\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$$

Solution : Here, $B + C = 180° - A$

$$\frac{B+C}{2} = 90° - \frac{A}{2}$$

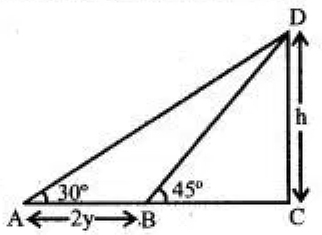
Taking tan on both sides, we get

$$\tan\frac{B+C}{2} = \tan\left(90° - \frac{A}{2}\right)$$

$$= \cot\frac{A}{2}.$$ Hence proved

Question 32. The length of a shadow of a tower standing on level plane is found to be 2y meters longer when the seen's altitude is 30° than when it was 45° prove that the height of the tower is $y(\sqrt{3}+1)$ meter.

Solution : In right angled Δ BCD.



$$\tan 45° = \frac{h}{BC}$$

$$h = BC \quad \text{... (1)}$$

In right angled Δ ACD

$$\tan 30° = \frac{h}{2y + BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{2y+h}$$

$$h(\sqrt{3}-1) = 2y$$

$$\Rightarrow \quad h = y(\sqrt{3}+1) \text{ m.}$$ Hence proved.

Question 33. Prove that :

(i) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

Solution :

(i) L.H.S. $= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \frac{1-\cos\theta}{\sqrt{1-\cos^2\theta}}$

$= \frac{1-\cos\theta}{\sqrt{\sin^2\theta}}$

$= \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta - \cot\theta$

= R.H.S. Hence proved.

(ii) L.H.S. $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta =$ R.H.S.

Hence proved

(ii) L.H.S. $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta =$ R.H.S.

Hence proved

Question 34. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then show that

$$m^2 - n^2 = 4\sqrt{mn}$$

Solution : Here

$$\begin{aligned} m^2 - n^2 &= (\tan A + \sin A)^2 - (\tan A - \sin A)^2 \\ &= (\tan A + \sin A + \tan - \sin A) \\ &\quad (\tan A + \sin A - \tan A + \sin A) \\ &= (2\tan A)(2\sin A) \\ &= 4\tan A \sin A \qquad \ldots (1) \end{aligned}$$

Also

$$\begin{aligned} 4\sqrt{mn} &= 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)} \\ &= 4\sqrt{\tan^2 A - \sin^2 A} \\ &= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A} \\ &= 4\sin A\sqrt{\frac{1-\cos^2 A}{\cos^2 A}} \\ &= 4\sin A\sqrt{\frac{\sin^2 A}{\cos^2 A}} \\ &= 4\sin A \cdot \frac{\sin A}{\cos A} \\ &= 4\sin A \,.\, \tan A \qquad \ldots (2) \end{aligned}$$

Using eq. (1) and eq. (2) we get the required conditions. Proved.

Question 35. If tan α = n tan β, sin α = m sin β, prove that $\cos^2 \alpha = \frac{m^2 - 1}{n^2 - 1}$.

Solution : We have,

$$\tan \alpha = n \tan \beta$$

$$\Rightarrow \quad \tan \beta = \frac{\tan \alpha}{n}$$

$$\Rightarrow \quad \cot \beta = \frac{n}{\tan \alpha}$$

$$\sin \alpha = m \sin \beta$$

$$\Rightarrow \quad \sin \beta = \frac{\sin \alpha}{m}$$

$$\Rightarrow \quad \text{cosec } \beta = \frac{m}{\sin \alpha}$$

Since

$$\text{cosec}^2 \beta - \cot^2 \beta = 1$$

$$\Rightarrow \quad \frac{m^2}{\sin^2 \alpha} - \frac{n^2}{\tan^2 \alpha} = 1$$

$$\Rightarrow \quad \frac{m^2}{\sin^2 \alpha} - \frac{n^2 \cos^2 \alpha}{\sin^2 \alpha} = 1$$

$$\Rightarrow \quad m^2 - n^2 \cos^2 \alpha = \sin^2 \alpha$$

$$\Rightarrow \quad m^2 - n^2 \cos^2 \alpha = 1 - \cos^2 \alpha$$

$$\Rightarrow \quad m^2 - 1 = (n^2 - 1) \cos^2 \alpha$$

$$\Rightarrow \quad \cos^2 \alpha = \frac{m^2 - 1}{n^2 - 1}.$$

Hence proved.

Question 36. If cos θ + sin θ = $\sqrt{2}$ cos θ, show that cos θ – sin θ = $\sqrt{2}$ sin θ.

Solution : We have

$$\cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

Squaring both side

$$(\cos \theta + \sin \theta)^2 = 2 \cos^2 \theta$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta = 2 \cos^2 \theta - \cos^2 \theta - \sin^2 \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta = \cos^2 \theta - \sin^2 \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta = (\cos \theta + \sin \theta)(\cos \theta - \sin \theta)$$

$$\Rightarrow 2 \sin \theta \cos \theta = (\cos \theta - \sin \theta) \times \sqrt{2} \cos \theta$$

(Given)

$$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta$$

$$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta.$$

Hence proved.

Question 37. Prove that (cosec A – sin A) (sec A – cos A) $\sec^2 A = \tan A$.

Solution :

$$\text{L. H. S.} = (\text{cosec } A - \sin A)(\sec A - \cos A) \cdot \sec^2 A$$

$$= \left(\frac{1}{\sin A} - \sin A\right) \cdot \left(\frac{1}{\cos A} - \cos A\right) \cdot \frac{1}{\cos^2 A}$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right) \times \left(\frac{1 - \cos^2 A}{\cos A}\right) \times \frac{1}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A}$$

$$\left[\begin{array}{l} \because (1 - \sin^2 A) = \cos^2 A \\ \because 1 - \cos^2 A = \sin^2 A \end{array}\right]$$

$$= \frac{\sin A}{\cos A} = \tan A$$

= R.H.S. Hence proved.

Question 38. Prove that : $\frac{\sin\theta \tan\theta}{1 - \cos\theta} = 1 + \sec\theta$

Solution :

$$\text{L.H.S.} = \frac{\sin\theta \cdot \tan\theta}{1 - \cos\theta}$$

$$= \frac{\sin\theta \cdot \frac{\sin\theta}{\cos\theta}}{1 - \cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(1 - \cos\theta)}$$

$$= \frac{1 - \cos^2\theta}{\cos\theta(1 - \cos\theta)}$$

$$= \frac{(1 + \cos\theta)(1 - \cos\theta)}{\cos\theta(1 - \cos\theta)}$$

$$= \frac{1 + \cos\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$$

$$= \sec\theta + 1 = \text{R.H.S.}$$

Hence proved.

Question 39. Prove that $\frac{\tan^2\theta}{(\sec\theta-1)^2}=\frac{1+\cos\theta}{1-\cos\theta}$.

Solution : L.H.S. $=\frac{\tan^2\theta}{(\sec\theta-1)^2}$

$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1}{\cos\theta}-1\right)^2}\left(\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right)$$

$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{(1-\cos\theta)^2}{\cos^2\theta}}\left(\because \sec\theta=\frac{1}{\cos\theta}\right)$$

$$=\frac{\sin^2\theta}{(1-\cos\theta)^2}\ [\because \sin^2\theta=1-\cos^2\theta]$$

$$=\frac{1-\cos^2\theta}{(1-\cos\theta)^2}$$

$$=\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)^2}\qquad\left(\because a^2-b^2=(a+b)(a-b)\right)$$

$$=\frac{1+\cos\theta}{1-\cos\theta}=\text{R.H.S.}$$

Question 40. Prove that

(i) $\frac{\cot A+\operatorname{cosec} A-1}{\cot A-\operatorname{cosec} A+1}=\frac{1+\cos A}{\sin A}$

(ii) $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$.

Solution : (i) L. H. S. $=\frac{\cot A+\operatorname{cosec} A-1}{\cot A-\operatorname{cosec} A+1}$

$$=\frac{(\cot A+\operatorname{cosec} A)-(\operatorname{cosec}^2 A-\cot^2 A)}{\cot A-\operatorname{cosec} A+1}$$

$$=\frac{(\cot A+\operatorname{cosec} A)[\{1-(\operatorname{cosec} A-\cot A)\}]}{\cot A-\operatorname{cosec} A+1}$$

$$=\cot A+\operatorname{cosec} A=\frac{\cos A}{\sin A}+\frac{1}{\sin A}$$

$$=\frac{\cos A+1}{\sin A}=\text{R.H.S.}$$ Hence proved.

(ii) L. H. S. $=\frac{\tan A+\sec A-1}{\tan A-\sec A+1}$

$$=\frac{(\tan A+\sec A)-(\sec^2 A-\tan^2 A)}{(\tan A-\sec A)+1}$$

$$=\frac{(\tan A+\sec A)(1-\sec A+\tan A)}{\tan A-\sec A+1}$$

$$=\tan A+\sec A$$

$$=\frac{\sin A}{\cos A}+\frac{1}{\cos A}=\frac{1+\sin A}{\cos A}=\text{R. H. S.}$$

Hence proved.

Question 41. Prove that

$$\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)=\frac{1}{\tan\theta+\cot\theta}.$$

Solution :

$$\text{L.H.S.} = \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$= \frac{\sin^2\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin\theta}$$

$$= \sin\theta.\cos\theta$$

$$\text{R.H.S.} = \frac{1}{\tan\theta+\cot\theta}$$

$$= \frac{1}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$$

$$= \frac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta}$$

$$= \sin\theta\cos\theta \qquad \text{Hence proved.}$$

Question 42. Prove that

$$\sqrt{\frac{1+\cos A}{1-\cos A}}=\frac{\tan A+\sin A}{\tan A\sin A}.$$

Solution :

$$\text{L.H.S.} = \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{1+\cos^2 A+2\cos A}{\sin^2 A}}$$

$$= \frac{1+\cos A}{\sin A}$$

$$\text{R.H.S.} = \frac{\tan A+\sin A}{\tan A\sin A}$$

$$= \frac{\sin A\left(\frac{1}{\cos A}+1\right)}{\frac{\sin A}{\cos A}\times\sin A}$$

$$= \frac{\sin A\,(1+\cos A)}{\cos A}\times\frac{\cos A}{\sin A\sin A}$$

$$= \frac{1+\cos A}{\sin A} \qquad \text{Hence proved.}$$

Question 43. It $\sec\theta + \tan\theta = P$ then prove that $\sin\theta = \frac{p^2-1}{p^2+1}$

Solution : $\sec\theta + \tan\theta = P$

$$\Rightarrow \quad \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = P$$

$$\Rightarrow \quad \frac{1+\sin\theta}{\cos\theta} = P$$

$$\Rightarrow \quad \frac{(1+\sin\theta)^2}{\cos^2\theta} = P^2,$$

[Squaring both sides]

$$\Rightarrow \quad \frac{1+\sin^2\theta+2\sin\theta}{\cos^2\theta} = P^2$$

$$\Rightarrow \quad \frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta} = \frac{P^2+1}{P^2-1},$$

[Applying componendo and dividendo]

$$\Rightarrow \quad \frac{1+1+2\sin\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta} = \frac{P^2+1}{P^2-1}$$

$$\Rightarrow \quad \frac{2(1+\sin\theta)}{2\sin\theta(1+\sin\theta)} = \frac{P^2+1}{P^2-1}$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{P^2+1}{P^2-1}$$

Taking reciprocals, we get

$$\Rightarrow \quad \sin\theta = \frac{P^2-1}{P^2+1}.$$

Hence proved

Question 44. Prove that

$$\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A} = \frac{2}{2\sin^2 A-1}.$$

Solution : L.H.S.

$$= \frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A}$$

$$= \frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$$

$$= \frac{\sin^2 A+\cos^2 A+2\sin A\cos A + \sin^2 A+\cos^2 A-2\sin A\cos A}{\sin^2 A-\cos^2 A}$$

$$= \frac{2(\sin^2 A+\cos^2 A)}{\sin^2 A-\cos^2 A} = \frac{2\times 1}{\sin^2 A-(1-\sin^2 A)}$$

$$= \frac{2}{\sin^2 A-1+\sin^2 A} = \frac{2}{2\sin^2 A-1} = \text{R.H.S.}$$

Hence proved

Question 45. Prove that

$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta.\cos^2\theta}-2.$$

Solution :

$$\text{L.H.S.} = \frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{\sin^4\theta+\cos^4\theta}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta.\cos^2\theta}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{(1)^2-2\sin^2\theta.\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta.\cos^2\theta}-\frac{2\sin^2\theta.\cos^2\theta}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\cos^2\theta}-2 = \text{R.H.S.}$$

Question 46. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then show that $x^2 + y^2 = 1$.

Solution : Given : $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow (x\sin\theta)\sin^2\theta + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$(\because y\cos\theta = x\sin\theta)$

$\Rightarrow x\sin\theta(\sin^2\theta+\cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x = \cos\theta \quad \ldots (1)$

Again $\quad x\sin\theta = y\cos\theta$

$\Rightarrow \quad \cos\theta\sin\theta = y\cos\theta$

$\Rightarrow \quad y = \sin\theta \quad \ldots (2)$

Squaring and adding (1) and (2) we get the required result. Hence proved.

Question 47. Prove that

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\sin A.\cos A.$$

Solution :

$$\text{L.H.S.} = \frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}$$

$$= \frac{\tan A}{(\sec^2 A)^2}+\frac{\cot A}{(\text{cosec}^2 A)^2}$$

$$= \frac{\sin A}{\cos A}\times\cos^2 A\times\cos^2 A$$

$$+\frac{\cos A}{\sin A}\times\sin^2 A\times\sin^2 A$$

$$= \sin A.\cos^3 A + \sin^3 A.\cos A$$

$$= \sin A\cos A(\cos^2 A+\sin^2 A)$$

$$= \sin A.\cos A\times 1 = \sin A.\cos A$$

$= \text{R.H.S.}$ Hence proved.

Question 48. Prove that

$$\frac{\sin A}{(\sec A+\tan A-1)}+\frac{\cos A}{(\operatorname{cosec} A+\cot A-1)}=1.$$

Solution : L.H.S.

$$=\frac{\sin A}{(\sec A+\tan A-1)}+\frac{\cos A}{(\operatorname{cosec} A+\cot A-1)}$$

$$=\frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1}+\frac{\cos A}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$$

$$=\frac{\sin A}{\frac{1+\sin A-\cos A}{\cos A}}+\frac{\cos A}{\frac{1+\cos A-\sin A}{\sin A}}$$

$$=\frac{\sin A.\cos A}{1+\sin A-\cos A}+\frac{\sin A.\cos A}{1+\cos A-\sin A}$$

$$=\frac{\sin A.\cos A\,(1+\cos A-\sin A+1+\sin A-\cos A)}{[1+(\sin A-\cos A)]\,[1-(\sin A-\cos A)]}$$

$$=\frac{2\sin A.\cos A}{(1)^2-(\sin A-\cos A)^2}$$

$$=\frac{2\sin A.\cos A}{1-(\sin^2 A+\cos^2 A-2\sin A\cos A)}$$

$$=\frac{2\sin A.\cos A}{1-1+2\sin A.\cos A}$$

$=\frac{2}{2}=1=$ R.H.S. **Hence proved.**

Question 49. Prove that

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta.\operatorname{cosec}\theta-2\sin\theta\cos\theta.$$

Solution :

$$\text{L.H.S.}=\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}$$

$$=\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\operatorname{cosec}^2\theta}$$

$$=\frac{\sin^3\theta}{\cos^3\theta}\times\cos^2\theta+\frac{\cos^3\theta}{\sin^3\theta}\times\sin^2\theta$$

$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}=\frac{\sin^4\theta+\cos^4\theta}{\sin\theta.\cos\theta}$$

$$=\frac{[\sin^2\theta+\cos^2\theta]^2-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$$

$$=\frac{(1)^2-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$$

$$=\frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$$

$$=\frac{1}{\sin\theta.\cos\theta}-\frac{2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$$

$$=\sec\theta.\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

$=$ R.H.S. **Hence proved.**

Question 50. Prove that

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Solution : L.H.S. $=\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2$

$$\Rightarrow \frac{1+\sin^2\theta+\cos^2\theta+2(\sin\theta-\cos\theta-\sin\theta.\cos\theta)}{1+\sin^2\theta+\cos^2\theta+2(\sin\theta+\cos\theta+\sin\theta.\cos\theta)}$$

$$=\frac{1+1+2(\sin\theta-\cos\theta-\sin\theta\cos\theta)}{1+1+2(\sin\theta+\cos\theta+\sin\theta\cos\theta)}$$

$$=\frac{2(1+\sin\theta-\cos\theta-\sin\theta\cos\theta)}{2(1+\sin\theta+\cos\theta+\sin\theta\cos\theta)}$$

$$=\frac{1+\sin\theta-\cos\theta(1+\sin\theta)}{1+\sin\theta+\cos\theta(1+\sin\theta)}$$

$$=\frac{(1+\sin\theta)(1-\cos\theta)}{(1+\sin\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}$$

= R.H.S. Hence proved.

Question 51. Prove that $\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=$ $\cos A+\sin A$.

Solution :

$$\text{L.H.S.}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$

$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}$$

$$=\frac{\cos^2 A-\sin^2 A}{(\cos A-\sin A)}$$

$$=\frac{(\cos A+\sin A)(\cos A-\sin A)}{(\cos A-\sin A)}$$

$$=\cos A+\sin A$$

= R.H.S. Hence proved.

Question 52. Prove the identity

$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$.

Solution :

$$\begin{aligned} \text{L.H.S.} &= (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) \\ &= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) \\ &= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right) \\ &= (\sin\theta + \cos\theta)\frac{1}{\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta} \\ &= \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \\ &= \sec\theta + \operatorname{cosec}\theta = \text{R.H.S.} \end{aligned}$$

Hence Proved.

Question 53. Prove that

(i) $\cos\theta\sin(90^\circ - \theta) + \sin\theta\cos(90^\circ - \theta) = 1$.

(ii) $\frac{\cos\theta}{\sin(90^\circ - \theta)} + \frac{\sin\theta}{\cos(90^\circ - \theta)} = 2$.

(iii) $\frac{\sin\theta\cos(90^\circ-\theta)\cos\theta}{\sin(90^\circ-\theta)} + \frac{\cos\theta\sin(90^\circ-\theta)\sin\theta}{\cos(90^\circ-\theta)} = 1$

(iv) $\text{cosec}^2(90^\circ-\theta) - \tan^2\theta = \cos^2(90^\circ-\theta) + \cos^2\theta.$

Solution :

(i) $$\begin{aligned}\text{L.H.S.} &= \cos\theta\sin(90^\circ-\theta) + \sin\theta\cos(90^\circ-\theta)\\ &= \cos\theta\cos\theta + \sin\theta\sin\theta\\ &= \cos^2\theta + \sin^2\theta\\ &= 1\\ &= \text{R.H.S.}\end{aligned}$$
Hence proved.

(ii) $$\begin{aligned}\text{L.H.S.} &= \frac{\cos\theta}{\sin(90^\circ-\theta)} + \frac{\sin\theta}{\cos(90^\circ-\theta)}\\ &= \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\sin\theta} = 1 + 1 = 2\\ &= \text{R.H.S.}\end{aligned}$$
Hence proved.

(iii) L.H.S.

$$= \frac{\sin\theta\cos(90^\circ-\theta)\cos\theta}{\sin(90^\circ-\theta)} + \frac{\cos\theta.\sin(90^\circ-\theta).\sin\theta}{\cos(90^\circ-\theta)}$$

$$= \frac{\sin\theta.\sin\theta.\cos\theta}{\cos\theta} + \frac{\cos\theta.\cos\theta.\sin\theta}{\sin\theta}$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{R.H.S.}$$

Hence proved.

(iv) $$\begin{aligned}\text{L.H.S.} &= \text{cosec}^2(90^\circ-\theta) - \tan^2\theta\\ &= \sec^2\theta - \tan^2\theta = 1\\ \text{R.H.S.} &= \cos^2(90^\circ-\theta) + \cos^2\theta\\ &= \sin^2\theta + \cos^2\theta = 1\end{aligned}$$

Hence, L.H.S. = R.H.S. Hence proved.

Question 54. Without using trigonometric table, prove that

(i) $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$.

(ii) $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 180^\circ = 0$.

Solution : (i) L.H.S.

$= \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$

$= \tan 10^\circ \tan 15^\circ \tan (90^\circ - 15^\circ) \tan (90^\circ - 10^\circ)$

$= \tan 10^\circ \tan 15^\circ \cot 15^\circ \cot 10^\circ$

$= \frac{1}{\cot 10^\circ} \times \frac{1}{\cot 15^\circ} \times \cot 15^\circ \times \cot 10^\circ$

$= 1 =$ R.H.S Hence proved.

(ii) L.H.S.

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 180^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \cos 90^\circ \ldots \cos 180^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \times 0 \times \cos 91^\circ \ldots \cos 180^\circ$

$= 0 =$ R.H.S. Hence proved.

Question 55. Without using trigonometric table, prove that

(i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ} = 2$

(ii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ = 0$.

Solution : (i) L.H.S. $= \cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$

$= \cos^2 26^\circ + \cos (90^\circ - 26^\circ) \sin 26^\circ + \frac{\tan 36^\circ}{\cot (90^\circ - 36^\circ)}$

$= \cos^2 26^\circ + \sin 26^\circ \cdot \sin 26^\circ + \frac{\tan 36^\circ}{\tan 36^\circ}$

$= \cos^2 26^\circ + \sin^2 26^\circ + 1$ $[\sin^2 \theta + \cos^2 \theta = 1]$

$= 1 + 1$

$= 2 =$ R.H.S. Hence proved.

(ii) We have, L.H.S.

$= \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$

$= \frac{\cos (90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos (90^\circ - 31^\circ)}{\sin 31^\circ} - 8 \times \left(\frac{1}{2}\right)^2$

$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 8 \times \frac{1}{4}$

$= 1 + 1 - 2 = 2 - 2 = 0 =$ R.H.S. Hence proved.

Question 56. Prove that

$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 = 1.$$

Solution : L.H.S. $= \left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2$

$$= \left(\frac{\sin 20^\circ . \sin 70^\circ}{\cos 20^\circ}\right)^2 + \left(\frac{\cos 20^\circ . \cos 70^\circ}{\sin 20^\circ}\right)^2$$

$$= \left[\frac{\sin 20^\circ . \sin (90^\circ - 20^\circ)}{\cos 20^\circ}\right]^2 + \left[\frac{\cos 20^\circ . \cos (90^\circ - 20^\circ)}{\sin 20^\circ}\right]^2$$

$$= \left[\frac{\sin 20^\circ . \cos 20^\circ}{\cos 20^\circ}\right]^2 + \left[\frac{\cos 20^\circ . \sin 20^\circ}{\sin 20^\circ}\right]^2$$

$= \sin^2 20^\circ + \cos^2 20^\circ$

$= 1 =$ R.H.S. Hence proved.

Question 57. Prove that

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \times \operatorname{cosec} 20^\circ = 0.$$

Solution : L.H.S.

$$= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \times \operatorname{cosec} 20^\circ$$

$$= \frac{\sin (90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\operatorname{cosec} (90^\circ - 70^\circ)}{\sec 70^\circ} - 2 \cos 70^\circ \times \operatorname{cosec} 20^\circ$$

$$= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \times \operatorname{cosec} 20^\circ$$

$= 1 + 1 - 2 \cos (90^\circ - 20^\circ) . \operatorname{cosec} 20^\circ$

$= 2 - 2 . \sin 20^\circ \cdot \frac{1}{\sin 20^\circ}$

$= 2 - 2 = 0 =$ R.H.S. Hence proved

Question 58. If x = h + a cos θ, y = k + b sin θ. Prove that

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1.$$

Solution : If is given that

$$x = h + a\cos\theta$$

and $$y = k + b\sin\theta$$

$$x - h = a\cos\theta \quad \text{...(i)}$$

$$y - k = b\sin\theta \quad \text{...(ii)}$$

The given equation is

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$

$$\therefore \text{L.H.S.} = \left(\frac{a\cos\theta}{a}\right)^2 + \left(\frac{b\sin\theta}{b}\right)^2,$$

[Putting the values of (i) and (ii)]

$$= \cos^2\theta + \sin^2\theta = 1 = \text{R.H.S.}$$

Hence proved.

Question 59. If A + B = 90°, show that

(i) $\sec^2 A + \sec^2 B = \sec^2 A \cdot \sec^2 B$.

(ii) $\frac{\sin B + \cos A}{\sin A} = 2\tan B + \tan A$.

Solution : (i) L. H. S. $= \sec^2 A + \sec^2 B$

$$= \frac{1}{\cos^2 A} + \frac{1}{\cos^2 B}$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\cos^2 (90^\circ - A)}$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$$

$$= \frac{1}{\sin^2 A \cos^2 A}$$

$$= \sec^2 A \operatorname{cosec}^2 A$$

$$= \sec^2 A \operatorname{cosec}^2 (90^\circ - B)$$

$$= \sec^2 A \sec^2 B = \text{R.H.S.}$$

Hence proved.

(ii) L. H. S. $= \frac{\sin B + \sec A}{\sin A}$

$$= \frac{\sin(90 - A) + \sec A}{\sin A}$$

$$= \frac{\cos A + \sec A}{\sin A}$$

$$= \frac{\cos^2 A + 1}{\sin A \cos A}$$

$$= \frac{2\cos^2 A + \sin^2 A}{\sin A \cos A}$$

$$= 2\cot A + \tan A$$

$$= 2\tan B + \tan A = \text{R.H.S.}$$

Hence proved.

Question 60. Prove the following identities :

(i) $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

(ii) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$

(iii) $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

(iv) $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

(v) $\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = \frac{2\sin\theta}{1 - 2\cos^2\theta}$

Solution : (i) L.H.S. $\frac{\cos\theta}{1+\sin\theta}$

Multiplying num., and deno., by $1 - \sin\theta$

$$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \text{R.H.S.}$$

Hence proved.

(ii) L.H.S. $= \sec^2\theta + \operatorname{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{1}{\cos^2\theta}$$

$$= \text{cosec}^2\,\theta\,.\,\sec^2\theta = \text{R.H.S.}$$

Hence proved.

(iii) L.H.S. $= \dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1+(\sin^2\theta+\cos^2\theta) + 2\sin\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1+1+2\sin\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{2\,(1+\sin\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= 2\sec\theta = \text{R.H.S.}$$

Hence proved.

(iv) L.H.S. $= \cot\theta - \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta} = \text{R.H.S.}$$

Hence proved.

(v) L.H.S. $= \frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta}$

$$= \frac{(\sin\theta - \cos\theta) + (\sin\theta + \cos\theta)}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{2\sin\theta}{(1-\cos^2\theta) - \cos^2\theta}$$

$$= \frac{2\sin\theta}{1 - 2\cos^2\theta} = \text{R.H.S.}$$

Hence proved.

Question 61. Prove the following identities :

(i) $\frac{1-\cos\theta}{1+\cos\theta} = (\cot\theta - \text{cosec}\,\theta)^2$

(ii) $\frac{1-\tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$

(iii) $\sec A\,(1 + \sin A)\,(\sec A - \tan A) = 1$

(iv) $\frac{1+\cos\theta}{1-\cos\theta} = \frac{\tan^2\theta}{(\sec\theta - 1)^2}$.

Solution :

(i) L.H.S. $= \frac{1-\cos\theta}{1+\cos\theta}$

$$= \frac{1-\cos\theta}{1+\cos\theta} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \left[\frac{1-\cos\theta}{\sin\theta}\right]^2$$

$$= \left[\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right]^2$$

$$= [\text{cosec}\,\theta - \cot\theta]^2$$

$$= [-(\cot\theta - \text{cosec}\,\theta)]^2$$

$$= (\cot\theta - \text{cosec}\,\theta)^2$$

$= \text{R.H.S.}$ Hence proved.

(ii) L.H.S. $= \frac{1-\tan^2\theta}{\cot^2\theta - 1} = \frac{1-\tan^2\theta}{\frac{1}{\tan^2\theta} - 1}$

$$= \frac{1-\tan^2\theta}{\frac{1-\tan^2\theta}{\tan^2\theta}}$$

$= \tan^2\theta = \text{R.H.S.}$ Hence proved

(iii) L.H.S. $= \sec A\ (1 + \sin A)\ (\sec A - \tan$
$A)$

$$= \frac{1}{\cos A}(1 + \sin A)\left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A}(1 + \sin A)\left(\frac{1 - \sin A}{\cos A}\right)$$

$$= \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A}$$

$$= 1 = \text{R.H.S.}$$ Hence proved.

(iv) R.H.S. $= \dfrac{\tan^2 \theta}{(\sec \theta - 1)^2} = \dfrac{\dfrac{\sin^2 \theta}{\cos^2 \theta}}{\left(\dfrac{1}{\cos \theta} - 1\right)^2}$

$$= \frac{\dfrac{\sin^2 \theta}{\cos^2 \theta}}{\dfrac{(1 - \cos \theta)^2}{\cos^2 \theta}} = \frac{1 - \cos^2 \theta}{(1 - \cos \theta)^2}$$

$$= \frac{(1 + \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)^2}$$

$$= \frac{1 + \cos \theta}{1 - \cos \theta} = \text{L.H.S.}$$

Hence proved.

Question 62. Prove that :

(i) $(\sec \theta - \tan \theta)^2 = \dfrac{1 - \sin \theta}{1 + \sin \theta}$

(ii) $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$

(iii) $(1 - \sin^2 A) \sec^2 A = 1$

(iv) $\cos^2 A + \dfrac{1}{1 + \cot^2 A} = 1$

(v) Prove that identity : $\dfrac{\sec A - 1}{\sec A + 1} = \dfrac{1 - \cos A}{1 + \cos A}$.

Solution :

(i) L.H.S. $= (\sec \theta - \tan \theta)^2$

$$= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}\right)^2$$

$$= \left(\frac{1 - \sin \theta}{\cos \theta}\right)^2$$

$$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{(1 + \sin \theta)(1 - \sin \theta)}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta} = \text{R.H.S.}$$

Hence proved.

(ii) L.H.S. $= (\sin^2\theta)^2 + (\cos^2\theta)^2$

$+ 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta = \text{R.H.S.}$

Hence proved.

(iii) L.H.S. $= (1 - \sin^2 A)\sec^2 A$

$= \cos^2 A \times \frac{1}{\cos^2 A}$

$= 1 = \text{R.H.S.}$ Hence proved.

(iv) L.H.S. $= \cos^2 A + \frac{1}{\text{cosec}^2 A}$

$= \cos^2 A + \sin^2 A$

$= 1 = \text{R.H.S.}$ Hence proved.

(v) L.H.S. $= \frac{\sec A - 1}{\sec A + 1}$

$= \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$

$= \frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}}$

$= \frac{1 - \cos A}{1 + \cos A}$

$=$ R.H.S. Hence proved.

Question 63. Prove that :

(i) $\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

(ii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(iii) $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$

(iv) Prove that identity : $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$.

Solution :

(i) L.H.S. $= \frac{1}{\sec\theta - \tan\theta}$

$= \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$

$= \frac{\cos\theta \times (1 + \sin\theta)}{(1 - \sin\theta) \times (1 + \sin\theta)}$

$= \frac{\cos\theta\,(1 + \sin\theta)}{1 - \sin^2\theta}$

$= \frac{\cos\theta\,(1 + \sin\theta)}{\cos^2\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta = \text{R.H.S.}$

Hence proved.

(ii) L.H.S. $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$

$= \dfrac{\tan\theta\,(1 - 2\,(1 - \cos^2\theta))}{2\cos^2\theta - 1}$

$= \dfrac{\tan\theta\,(1 - 2 + 2\cos^2\theta)}{2\cos^2\theta - 1}$

$= \dfrac{\tan\theta\,(2\cos^2\theta - 1)}{(2\cos^2\theta - 1)}$

$= \tan\theta =$ R.H.S. Hence proved.

(iii) L.H.S. $= \dfrac{\dfrac{\sin\theta}{\cos\theta} + \sin\theta}{\dfrac{\sin\theta}{\cos\theta} - \sin\theta} = \dfrac{\sin\theta\left(\dfrac{1}{\cos\theta} + 1\right)}{\sin\theta\left(\dfrac{1}{\cos\theta} - 1\right)}$

$= \dfrac{\sec\theta + 1}{\sec\theta - 1} =$ R.H.S.

Hence proved.

(iv) L.H.S. $= \dfrac{\sec A - 1}{\sec A + 1} = \dfrac{\dfrac{1}{\cos A} - 1}{\dfrac{1}{\cos A} + 1}$

$= \dfrac{\dfrac{1 - \cos A}{\cos A}}{\dfrac{1 + \cos A}{\cos A}} = \dfrac{1 - \cos A}{1 + \cos A}$

$=$ R.H.S. Hence proved.

Question 64. Prove that :

(i) $1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} = \operatorname{cosec} \theta$

(ii) $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}$

(iii) $\frac{(1 + \tan^2 A) \cot A}{\operatorname{cosec}^2 A} = \tan A$

(iv) $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta.$

Solution :

(i) $$\text{L.H.S.} = 1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta}$$

$$= \frac{1 + \operatorname{cosec} \theta + \operatorname{cosec}^2 \theta - 1}{1 + \operatorname{cosec} \theta}$$

$$= \frac{\operatorname{cosec} \theta (1 + \operatorname{cosec} \theta)}{(1 + \operatorname{cosec} \theta)}$$

$$= \operatorname{cosec} \theta = \text{R.H.S.}$$

Hence proved.

(ii) $$\text{L.H.S.} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$$

$$\frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - \cos^2 A} = \text{R.H.S.}$$

Hence proved.

(iii) $$\text{L.H.S.} = \frac{(1 + \tan^2 A) \cot A}{\operatorname{cosec}^2 A}$$

$$= \frac{\sec^2 A \cot A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A} \cdot \cot A}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} \cdot \frac{\cos A}{\sin A}$$

$$= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$$

Hence proved.

(iv) L.H.S.

$= (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$
$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta.\sec\theta$
$= (\sin^2\theta + \cos^2\theta) + 1 + \cot^2\theta + 2\sin\theta \times \frac{1}{\sin\theta} + 1 + \tan^2\theta + 2\cos\theta \cdot \frac{1}{\cos\theta}$
$= 1 + 1 + 1 + 2 + 2 + \tan^2\theta + \cot^2\theta$
$= 7 + \tan^2\theta + \cot^2\theta$ = R.H.S. Hence proved.

Question 65. Prove that :

(i) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$

(ii) $\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}$

(iii) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

(iv) $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2.$

Solution :

(i) L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1+\cos A}{1+\cos A}}$

$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$

$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$

$= \frac{\sin A}{1+\cos A}$ = R.H.S.

Hence proved.

(ii) $\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}$

or $\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A} = \frac{2}{\sin A}$

L.H.S. $= \frac{(\text{cosec}\,A + \cot A) + (\text{cosec}\,A - \cot A)}{(\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot A)}$

$= \frac{2\,\text{cosec}\,A}{\text{cosec}^2 A - \cot^2 A} = \frac{2\,\text{cosec}\,A}{1}$

$= \frac{2}{\sin A}$ = R.H.S. Hence proved.

(iii) L.H.S. $= \sin^6\theta + \cos^6\theta$
$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= 1.\{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta$

$- 3\sin^2\theta\cos^2\theta\}$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta =$ R.H.S.

(iv) L.H.S. $= \dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \cos\theta\sin\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$= 1 - \sin\theta\cos\theta + 1 + \sin\theta\cos\theta$

$= 2 =$ R.H.S. Hence proved.

Question 66. Prove that $\sin^2 5° + \sin^2 10°$ ...... $\sin^2 85° + \sin^2 90° = 9\frac{1}{2}$.

Solution : L.H.S.

$= \sin^2 5° + \sin^2 10° + \sin^2 15° + \sin^2 20° + \sin^2 25° + \sin^2 30° + \sin^2 35° + \sin^2 40° + \sin^2 45° + \sin^2 50° + \sin^2 55° + \sin^2 60° + \sin^2 65° + \sin^2 70° + \sin^2 75° + \sin^2 80° + \sin^2 85° + \sin^2 90°$.

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + (\sin^2 15° + \sin^2 75°) + (\sin^2 20° + \sin^2 70°) + (\sin^2 25° + \sin^2 65°) + (\sin^2 30° + \sin^2 60°) + (\sin^2 40° + \sin^2 50°) + (\sin^2 35° + \sin^2 55°) + \sin^2 45° + \sin^2 90°$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°) + (\sin^2 15° + \cos^2 15°) + (\sin^2 20° + \cos^2 20°) + (\sin^2 25° + \cos^2 25°) + (\sin^2 30° + \cos^2 30°) + (\sin^2 40° + \cos^2 40°) + (\sin^2 35° + \cos^2 35°) + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2$

$\left[\begin{array}{l} \because \sin(90° - \theta) = \cos\theta \\ \because \sin 90° = 1 \text{ and } \sin 45° = \frac{1}{\sqrt{2}} \end{array}\right]$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \frac{1}{2} + 1$

$= 9\frac{1}{2} =$ R.H.S. Hence proved.

Question 67. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$, prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$.

Solution : It is given that :

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta = 1 \quad \text{...(A)}$$

and

$$\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta = 1 \quad \text{...(B)}$$

On Squaring equation (A), we get

$$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2 = (1)^2$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+2\frac{x}{a}\cdot\frac{y}{b}\sin\theta.\cos\theta=1 \quad \text{...(C)}$$

On squaring equation (B), we get

$$\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)^2 = (1)^2$$

$$\Rightarrow \frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta-2\frac{x}{a}\cdot\frac{y}{b}\sin\theta.\cos\theta=1 \quad \text{...(D)}$$

Adding (C) and (D), we get

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+2\frac{x}{a}\cdot\frac{y}{b}\cdot\sin\theta.\cos\theta + \frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\cdot\frac{y}{b}\cdot\sin\theta.\cos\theta = 1+1$$

$$\Rightarrow \frac{x^2}{a^2}(\sin^2\theta+\cos^2\theta)+\frac{y^2}{b^2}(\sin^2\theta+\cos^2\theta)=2$$

$$\Rightarrow \frac{x^2}{a^2}\times 1+\frac{y^2}{b^2}\times 1 = 2$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2} = 2.$$

Hence proved.

Question 68. Prove that

$$\left[\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right]\sin^2\theta.\cos^2\theta = \frac{1-\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}.$$

Solution : L.H.S.

$$=\left[\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right]\sin^2\theta.\cos^2\theta$$

$$=\left[\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta}+\frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$=\left[\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}}+\frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right]\sin^2\theta.\cos^2\theta$$

$$=\left[\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$=\left[\frac{\cos^2\theta(1-\sin^4\theta)+\sin^2\theta(1-\cos^4\theta)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right]\sin^2\theta.\cos^2\theta$$

$$=\left[\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$=\frac{\cos^4\theta+\cos^4\theta.\sin^2\theta+\sin^4\theta+\sin^4\theta.\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta.\cos^2\theta}$$

$$=\frac{(\cos^4\theta+\sin^4\theta)+\sin^2\theta.\cos^2\theta(\sin^2\theta+\cos^2\theta)}{1+1+\sin^2\theta.\cos^2\theta}$$

$$=\frac{(\cos^2\theta+\sin^2\theta)^2-2\sin^2\theta.\cos^2\theta+\sin^2\theta.\cos^2\theta\times1}{2+\sin^2\theta.\cos^2\theta}$$

$$=\frac{(1)^2-2\sin^2\theta.\cos^2\theta+\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}$$

$$=\frac{1-\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}=\text{R.H.S.}$$

Hence proved.

Question 69. Prove that

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=\sec A.\operatorname{cosec} A+1.$$

Solution : L.H.S.

$$=\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$$

$$=\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$$

$$=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}-\frac{\cos^2 A}{\sin A(\sin A-\cos A)}$$

$$=\frac{\sin^3 A-\cos^3 A}{\sin A.\cos A.(\sin A-\cos A)}$$

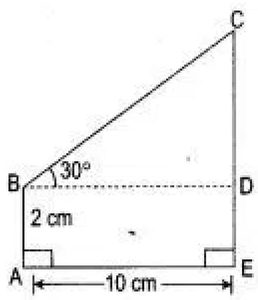
$$=\frac{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A.\cos A)}{\sin A.\cos A.(\sin A-\cos A)}$$

$$=\frac{1+\sin A.\cos A}{\sin A.\cos A}=\frac{1}{\sin A.\cos A}+\frac{\sin A.\cos A}{\sin A.\cos A}$$

$=\operatorname{cosec} A.\sec A+1=$ R.H.S. Hence proved.

Figure Based Questions

Question 1. In figures, find the length CF.



Solution : $\because$ $BD = AF$

$BD = 10$ cm

In BCD, we have

$$\tan 30° = \frac{CD}{BD}$$

$$\frac{1}{3} = \frac{CD}{10}$$

$$CD = \frac{10\ 3}{3}\text{ cm}$$

$$CF = CD + DF = \frac{10\ 3}{3}+2\text{ cm}$$

$$= \frac{10\ 3+6}{3}\text{ cm.}$$ Ans.

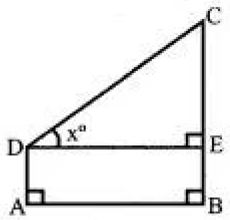
Question 2. With reference to the figure given alongside, a man stands on the ground at a point A, which is on the same horizontal plane as B, the foot of a vertical pole BC. The height of the pole is 10 m. The man's eye is 2 m above the ground. He observes the angle of elevation at C, the top of the pole as x°, where tan x° = 2/5.

Calculate :

(i) The distance AB in m;

(ii) The angle of elevation of the top when he is standing 15 m from the pole.

Give your answer to the nearest degree. See the figure alongside.



Solution : (i) In right $\triangle$ CDE, we have

$$\frac{DE}{EC} = \cot x^\circ$$

$$\frac{AB}{BC - BE} = \frac{5}{2}$$

$$\frac{AB}{10 - 2} = \frac{5}{2}$$

$$AB = \frac{5}{2} \quad 8 \ \ m = 20 \text{ m.}$$

(ii) When $AB = 15$ m, then $DE = 15$

In right $\triangle$ CDE, we have

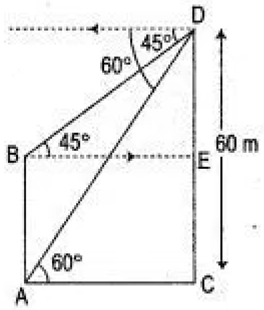
$$\tan \quad EDC = \frac{EC}{DE} = \frac{8}{15} = 0{\cdot}5333$$

From tables of natural tangents, we have

$$EDC = 28°2' \text{ nearest } \quad 28°$$

(nearest degree)

Question 3. From the top of a tower 60 m high, the angles of depression of the top and bottom of pole are observed to be 45° and 60° respectively. Find the height of the pole.



Solution : From the adjoining figure, in right-angled BED,

$$\frac{DE}{BE} = \tan 45°$$

$$DE = BE \qquad \ldots (1)$$

In right-angled ACD,

$$\frac{CD}{AC} = \tan 60°$$

$$\frac{60}{AC} = \quad 3$$

$$AC = 20 \quad .3$$

From (1), $DE = BE = AC = 20 \quad 3$

Now $AB = CD - DE$

$$= (60 - 20 \quad 3) \text{ m}$$

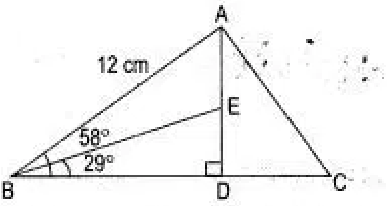
$$= 20\,(3 - \quad 3) \text{ m}. \qquad \text{Ans.}$$

Question 4. In triangle ABC, AB = 12 cm, LB = 58°, the perpendicular from A to BC meets it at D. The bisector of angle ABC meets AD at E. Calculate:

(i) The length of BD;

(ii) The length of ED.

Give your answers correct to one decimal place.



Solution : (i) In right angled ABD,

$$\frac{BD}{BA} = \cos 58°$$

$$BD = BA \cos 58°$$

$$= 12 \quad (0{\cdot}5299) \text{ cm}$$

$$= 6{\cdot}3588 \text{ cm}$$

(ii) In right angled EBD,

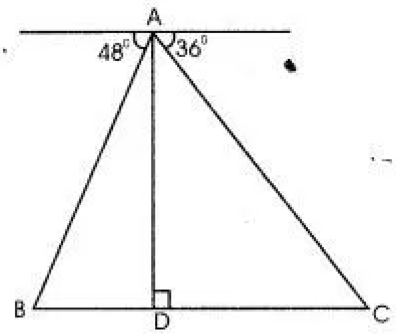
$$\frac{ED}{BD} = \tan 29°$$

$$ED = BD \tan 29°$$

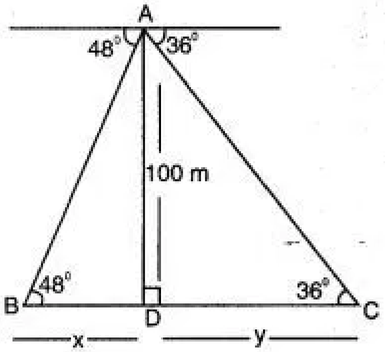
$$= (6{\cdot}3588)(0{\cdot}5543) \text{ cm}$$

$$= 3{\cdot}52 \text{ cm. (approx.)}$$

Question 5. From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.



Solution : From rt. angle ADC,



$$\frac{AD}{CD} = \tan 36°$$

$$\frac{100}{y} = \tan 36°$$

$$y = \frac{100}{\tan 36°}$$

$$= \frac{100}{0{\cdot}7265}$$

$$y = 137{\cdot}646 \text{ m}$$

From rt. angle ADB,

$$\frac{100}{x} = \tan 48°$$

$$x = \frac{100}{1{\cdot}1106}$$

$$= 90{\cdot}04 \text{ m.}$$

Distance between the ships

$= x + y$

$= 137{\cdot}638 + 90{\cdot}04$

$= 227{\cdot}678$ m.

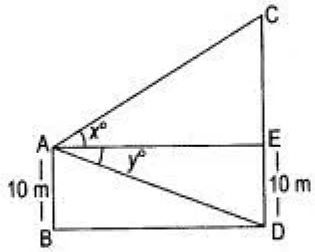
$= 228$ m. (appro.)

Concept Based Questions

Question 1. From a window A, 10 m above the ground angle of elevation of the top C of a tower is x°, where $\tan x = \frac{5}{2}$ and the angle of depression of the foot Δ of the tower is y°, where $\tan y° = \frac{1}{4}$, calculate the height CD of the tower in metres.

Solution : Let h be the height of the tower.

Also $AB = ED = 10$ m



In ΔDAE,

$$\tan y° = \frac{DE}{AE}$$

$$\frac{1}{4} = \frac{10}{AE}$$

$$\Rightarrow \quad AE = 40 \text{ m}$$

Now in ΔCAE,

$$\tan x° = \frac{CE}{AE}$$

$$\Rightarrow \quad \frac{5}{2} = \frac{CE}{40}$$

$$\Rightarrow \quad CE = \frac{40 \times 5}{2}$$

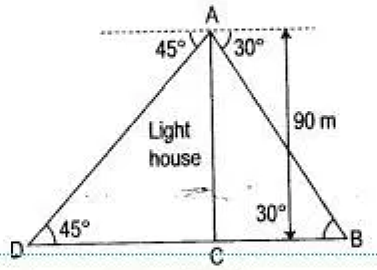
$$= 100 \text{ m}$$

Height of the tower $h = CE + ED$

$$= 100 + 10 = 110 \text{ m.}$$

Question 2. From a light house, the angles of depression of two ships on opposite sides of the light house were observed to be 30° and 45°. If the height of the light house is 90 metres and the line joining the two ships passes through the foot of the light house, find the distance between the two ships, correct to two decimal places.

Solution : Let AB is the light house, C and D are the position of two ships.



From right angled ΔABC,

$$\tan 30° = \frac{AC}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{90 \text{ m}}{BC}$$

$$\Rightarrow \quad BC = [90 \times \sqrt{3}] \text{ m}$$

$$\therefore \quad BC = 155{\cdot}88 \text{ m}$$

Again from right angled ΔACD,

$$\tan 45° = \frac{AC}{CD}$$

$$\Rightarrow \quad 1 = \frac{90 \text{ m}}{CD}$$

$$\Rightarrow \quad CD = 90 \text{ m}$$

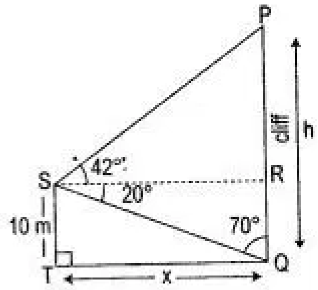
Hence, the distance between the two ships

$$= BC + CD = (155{\cdot}88 + 90) \text{ m}$$

$$= 245{\cdot}88 \text{ m.}$$ Ans.

Question 3. A man on the deck of a ship is 10 m above water level. He observes that the angle of elevation of the top of a cliff is 42° and the angle of depression of the base is 20°. Calculate the distance of the cliff from the ship and the height of the cliff.

Solution : Let the height of the cliff be h meters and the distance of the cliff from the ship be x meters.



In right angled ΔQRS,

$$QR = ST = 10 \text{ m}, TQ = RS = x \text{ m}$$

$$\therefore \quad \tan 70° = \frac{RS}{QR}$$

$$\Rightarrow \quad 2{\cdot}747 = \frac{x}{10 \text{ m}}$$

$$\therefore \quad x = 27{\cdot}47 \text{ m}$$

Hence, the distance of the cliff from the ship

$= 27{\cdot}47$ m. Ans.

Again in right angled ΔPRS,

$$\tan 42° = \frac{PR}{RS}$$

$$\Rightarrow \quad 0{\cdot}9004 = \frac{PR}{27{\cdot}47}$$

$$\Rightarrow \quad PR = [0{\cdot}9004 \times 27{\cdot}47] \text{ m}$$

$$= 24{\cdot}73 \text{ m}$$

$$\therefore \quad PQ = PR + RQ$$

$$= [24{\cdot}73 + 10] \text{ m}$$

$$= 34{\cdot}73 \text{ m.}$$

Hence the height of the cliff

$= 34{\cdot}73$m. Ans.

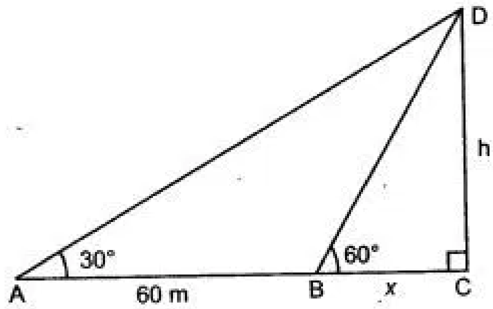
Question 4. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.

Solution : Let the height be h

In Δ BCD, $\frac{h}{x} = \tan 60°$

$$\frac{h}{x} = \sqrt{3}$$

$$h = \sqrt{3}\,x$$



In Δ ACD, $\frac{h}{x+60} = \tan 30°$

$$\frac{h}{x+60} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = x + 60$$

$$3x - x = 60$$

$$2x = 60$$

$$x = 30$$

Now, $h = \sqrt{3}\,x$

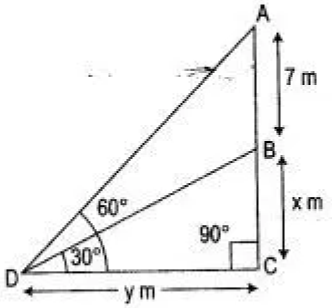
$$h = 30 \times \sqrt{3}$$

$$= 30 \times 1{\cdot}732$$

$$\text{Height} = 51{\cdot}96 \text{ m}$$

Question 5. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 7 metres. At a point in a plane the angle of elevation of the bottom and the top of the flagstaff are respectively 30° and 60°. Find the height of the tower.

Solution : Let the height of the tower be x m and distance $DC = y$ m



$\therefore \quad AB$ = height of flagstaff = 7 m

Now in rt $\angle d, \Delta BCD$,

$$\frac{BC}{CD} = \tan 30°$$

$$\therefore \quad \frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad y = \sqrt{3}x \quad \ldots(i)$$

Also in right $\angle d, \Delta ACD$,

$$\frac{AC}{CD} = \tan 60°$$

$$\Rightarrow \quad \frac{x+7}{y} = \sqrt{3}$$

$$\Rightarrow \quad x + 7 = \sqrt{3}y$$

$$x + 7 = \sqrt{3}(\sqrt{3}x) \quad \text{[from (i)]}$$

$$\Rightarrow \quad x + 7 = 3x$$

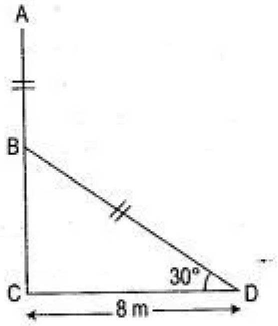
$$\Rightarrow \quad 2x = 7$$

$$\Rightarrow \quad x = \frac{7}{2} = 3\cdot5 \text{ m.} \quad \text{Ans.}$$

Question 6. A pole being broken by the wind the top struck the ground at an angle of 30° and at a distance of 8m from the foot of the pole. Find the whole height of the pole.

Solution : Let ABC be the pole. When broken at B by the wind, let its top A strike the ground such that

$$\angle CAB = 30°$$
$$AC = 8 \text{ m}$$



In ΔACB, $\tan 30° = \frac{BC}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$BC = \frac{8}{\sqrt{3}}$$

Again In Δ ACB,

$$\cos 30° = \frac{AC}{AB}$$

$$\frac{\sqrt{3}}{2} = \frac{8}{AB}$$

$$AB = \frac{16}{\sqrt{3}}$$

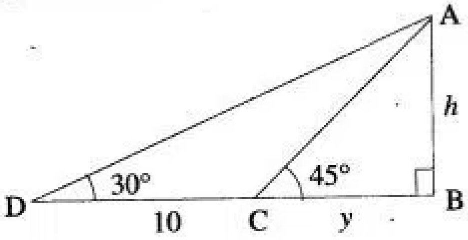
Height of the pole $= AC = AB + BC$

$$= \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}$$

$\Rightarrow 8\sqrt{3}$ m or 13·86 m.

Question 7. The shadow of a vertical tower on a level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower, correct to two decimal places.

Solution : Let the height of tower be h meter and length of shadow y meter initially.



In Δ ABC, $\tan 45° = \frac{AB}{BC}$

$$1 = \frac{h}{y}$$

$$y = h \quad \ldots (1)$$

In Δ ABD, $\tan 30° = \frac{AB}{DB}$

$$\frac{1}{\sqrt{3}} = \frac{h}{y+10}$$

$$y + 10 = h\sqrt{3} \quad \ldots (2)$$

Put $y = h$ in eqn. (ii),

$$h + 10 = h\sqrt{3}$$

$$h(\sqrt{3} - 1) = 10$$

$$h = \frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{10}{(3-1)}(\sqrt{3}+1)$$

$$= \frac{10}{2}(\sqrt{3}+1)$$

$$= 5(1{\cdot}732 + 1)$$

$$= 5 \times 2{\cdot}732$$

$$= 13{\cdot}66 \text{ meter}$$

Question 8. A man on the top of vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower ? (Give your answer correct to nearest seconds).

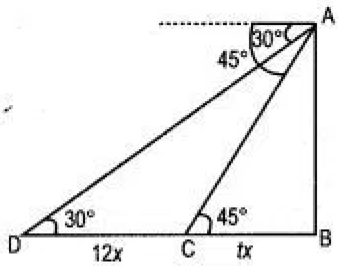
Solution : Here, $\angle ACB = 30°$ and $\angle ADB = 45°$. Let C denote the initial position of the car and D be its position after 12 minutes. Let the speed of the car be x metre/minute, then

$$CD = 12x \text{ metres}$$

(∵ Distance = Speed × Time)

Let the car take t minutes to reach the tower from D. Then,

$$DB = tx \text{ metres}$$



Now in the right-angled triangles ACB,

$$\tan 30° = \frac{AB}{CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{CD + DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{12x + tx}$$

$$\Rightarrow \quad AB = \frac{12x + tx}{\sqrt{3}} \qquad \dots (1)$$

Also, in the right-angled traingle ADB,

$$\tan 45° = \frac{AB}{DB}$$

$$\Rightarrow \quad 1 = \frac{AB}{DB}$$

$$\Rightarrow \quad AB = DB = tx \qquad \dots (2)$$

From(1) and (2), we have

$$t = \frac{12}{\sqrt{3} - 1} = \frac{12\left(\sqrt{3} + 1\right)}{2}$$

$$= 6(\sqrt{3} + 1)$$

$$= 16.39$$

$$\therefore \quad \text{Time} = 16.39 \text{ minutes}$$

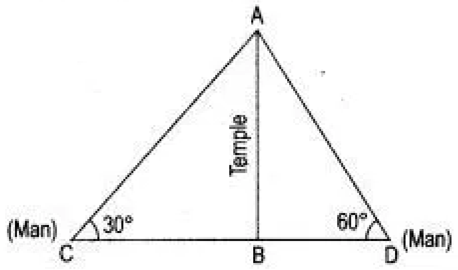
$$= 16 \text{ minutes } 23 \text{ seconds.}$$

Ans.

Question 9. Two men on either side of a temple 75 m high observed the angle of elevation of the top of the temple to be 30° and 60° respectively. Find te distance between the two men.

Solution : Given height of the temple $AB = 75$ m

Now in right $\angle d\ \Delta ABC$,



$$\frac{BC}{AB} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \quad \frac{BC}{75} = \sqrt{3}$$

$$\Rightarrow \quad BC = 75\sqrt{3} \quad \text{...(i)}$$

Also in right $\angle d\ \Delta ABD$,

$$\frac{BD}{AB} = \cot 60°$$

$$\Rightarrow \quad \frac{BD}{75} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 25\sqrt{3} \quad \text{...(ii)}$$

Now the distance between the two men $= CD$

$$= BC + BD = 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3}$$

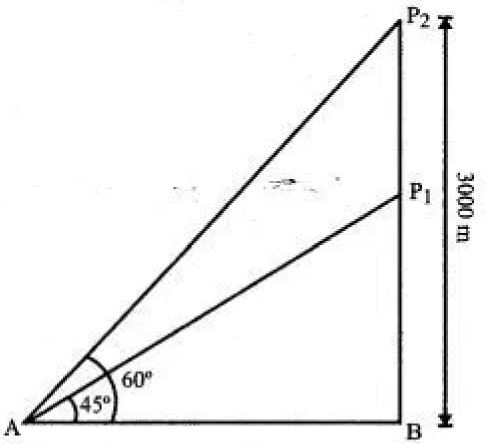
Hence, the distance between two men

$$= 100\sqrt{3} \text{ m.} = 173{\cdot}2 \text{ m} \quad \text{Ans.}$$

Question 10. An aeroplane when 3,000 meters high passes vertically above another aeroplane at an instance when their angles of elevation at the some observation point are 60° and 45° respectively. How many meters higher is the one than the other.

Solution : Let P_1 and P_2 denote the positions of the two planes. Then in right-angled ΔP_1AB,

$$\frac{P_1B}{AB} = \tan 45° \Rightarrow P_1B = AB$$



In right-angled ΔP_2AB,

$$\frac{P_2B}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad AB = \frac{P_2B}{\sqrt{3}}$$

$$= \frac{3,000}{\sqrt{3}}$$

$$= 1,000\sqrt{3}$$

∴ Vertical distance between the two planes is

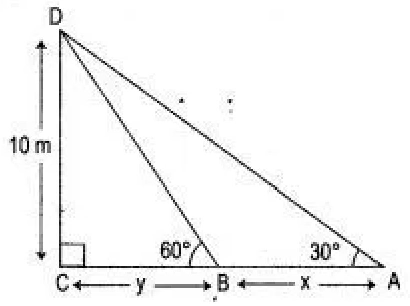
$$P_1P_2 = P_2B - P_1B = 3000 - 1000\sqrt{3}$$

$$= 1,000\,(3 - \sqrt{3})\text{ m.} \qquad \text{Ans.}$$

Question 11. From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10 m, find the distance between A and B correct to two decimal places.

Solution :

Let CD is the building A and B are two given point using horizontally on the same side of building.



In Δ DBC,

$$\tan 60° = \frac{DC}{CB}$$

$$\sqrt{3} = \frac{10}{y} \quad ...(1)$$

In Δ DCA, $\tan 30° = \frac{DC}{CA}$

$$\frac{1}{\sqrt{3}} = \frac{10}{x+y} \quad ...(2)$$

From (1), put $y = \frac{10}{\sqrt{3}}$ in (2), we get

$$\frac{1}{\sqrt{3}} = \frac{10}{x + \frac{10}{\sqrt{3}}}$$

$$\frac{1}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}x + 10}$$

$$30 = \sqrt{3}x + 10$$

$$x = \frac{20}{\sqrt{3}}$$

$$x = 11{\cdot}55 \text{ m.}$$

Hence, distance between two points A and B is 11·55 m. Ans.

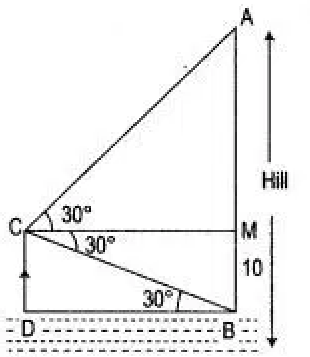
Question 12. A man is standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of hill.

Solution : Let AB be the height of the hill.

In rt ∠d, Δ BCD,

$$\frac{CD}{DB} = \tan 30°$$

$$\Rightarrow \quad DB = 10\sqrt{3} \text{ m}$$



In rt ∠d Δ AMC,

$$\frac{AM}{CM} = \tan 60°$$

$$\Rightarrow \quad AM = \sqrt{3}\,CM$$

$$\Rightarrow \quad AM = \sqrt{3}\,DB$$

$$= \sqrt{3} \times 10\sqrt{3} = 30 \text{ m}$$

Thus, $AB = AM + MB$

$= (30 + 10)\text{ m} = 40\text{ m.}$

$\therefore$ Height of the hill be 40 m. Ans.

Question 13. A round balloon of radius 'a' subtends an angle θ at the eye of the observer while the angle of elevation of its centre is ϕ. Prove that the height of the centre of the balloon is a $\sin\phi \operatorname{cosec}\frac{\theta}{2}$.

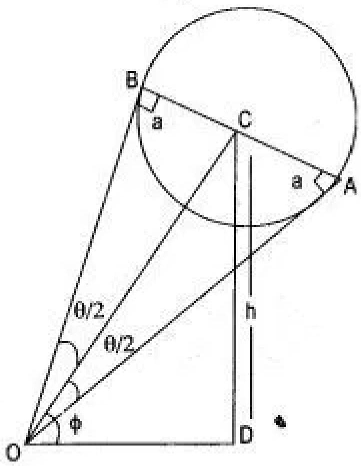
Solution : Let C be the centre of the balloon, O be the position of man's eye.

Let h be the height of the centre of the balloon

then $\angle AOB = \theta$

so $\angle BOC = \angle COA$

$= \theta/2$



In ΔOAC, $\sin\theta/2 = \frac{a}{OC}$

$\Rightarrow \quad OC = a \operatorname{cosec}\frac{\theta}{2}$

In ΔCOD, $\sin\phi = \frac{h}{OC}$

$\Rightarrow \quad h = OC \sin\phi$

$h = a \operatorname{cosec}\frac{\theta}{2} \cdot \sin\phi.$

$\Rightarrow \quad h = a \sin\phi \operatorname{cosec}\frac{\theta}{2}.$

Question 14. Vertical tower is 20m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower ?

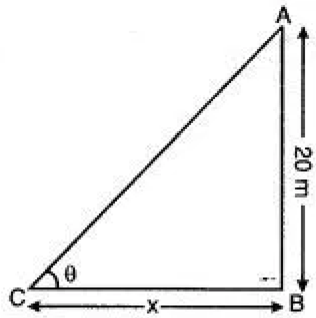
Solution :

Given, $\cos\theta = 0{\cdot}53$

Let the man is standing at a distance of 'x' m from the foot of the tower

$$\cos\theta = \frac{BC}{AC} = \frac{x}{\sqrt{x^2+400}}$$

$$0{\cdot}53 = \frac{x}{\sqrt{x^2+400}}$$



$$\Rightarrow \quad (0{\cdot}53)^2 = \frac{x^2}{x^2+400}$$

$$\Rightarrow 0{\cdot}2809\,x^2 + 112{\cdot}36 = x^2$$

$$\Rightarrow x^2 - 0{\cdot}2809\,x^2 = 112{\cdot}36$$

$$\Rightarrow \quad x^2 = \frac{112{\cdot}36}{0{\cdot}7191}$$

$$\Rightarrow \quad x^2 = 156{\cdot}25$$

$$x = 12{\cdot}5 \text{ metres.}$$

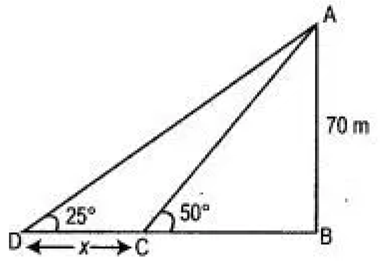
$\therefore$ The man is standing from the foot of the tower be 12·5 meter. Ans.

Question 15. Two person standing on the same side of a tower in a straight line with it measure the angle of elevation of the top of the tower as 25° and 50° respectively. If the height of the tower is 70 m find the distance between the two person.

Solution : Let CD be the distance between the two persons

In ΔABC,

$$\cot 50° = \frac{BC}{AB}$$



$$\cot(90° - 40°) = \frac{BC}{70}$$

$$\tan 40° = \frac{BC}{70}$$

$$BC = 70 \tan 40°$$

$$= 70 \times 0.8391 = 58.74 \text{ m}$$

In ΔABD,

$$\cot 25° = \frac{BD}{AB}$$

$$\cot(90° - 65°) = \frac{BD}{70}$$

$$\tan 65° = \frac{BD}{70}$$

$$BD = 70 \tan 65°$$

$$= 70 \times 2.11451$$

$$= 150.12 \text{ m}$$

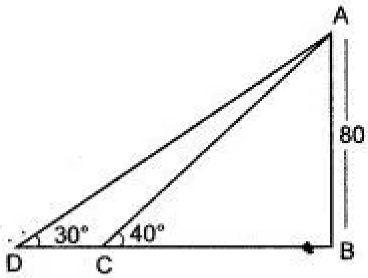
$$CD = 150.12 - 58.74$$

$$= 91.38 \text{ m}$$

$\therefore$ The distance between the two person be 91·38m. Ans.

Question 16. As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre.

Solution : In fig. AB is 80 m tall light house, the two ships are C and D.



In Δ ABC,

$$\tan 40° = \frac{AB}{BC}$$

$$\Rightarrow \quad BC = \frac{AB}{\tan 40°}$$

$$BC = \frac{80}{0{\cdot}8391} = 95{\cdot}34 \text{ m}$$

In Δ ABD, $\tan 30° = \frac{AB}{BD}$

$$\Rightarrow \quad BD = \frac{AB}{\tan 30°} = \frac{80}{0{\cdot}5774}$$

$$= 138{\cdot}55 \text{ m}$$

Distance between two ships

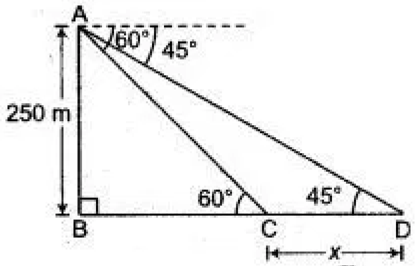
$$DC = BD - BC$$

$$= 138{\cdot}55 - 95{\cdot}34$$

$$= 43{\cdot}21 \text{ m} = 43 \text{ m.} \quad \text{Ans.}$$

Question 17. An aeroplane at an altitude of 250 m observes the angle of depression of two Boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number.

Solution: Let the width of the river CD be x,



In Δ ABC, $\tan 60° = \frac{AB}{BC}$

$$\sqrt{3} = \frac{250}{BC}$$

$$BC = \frac{250}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \left(\frac{250}{3}\right)\sqrt{3} \qquad \ldots(i)$$

In Δ ABD, $\tan 45° = \frac{AB}{BD}$

$\Rightarrow$ AB = BD = 250 ...(ii)

$\therefore$ BD = BC + CD

$$250 = \left(\frac{250}{3}\right)\sqrt{3} + x$$

[using (i) and (ii)]

$$\therefore \quad x = 250 - \left(\frac{250}{3}\right) \times 1{\cdot}732$$

$= 250 - 83{\cdot}33 \times 1{\cdot}732$

$= 250 - 144{\cdot}33$

$= 105{\cdot}67$ m

$= 106$ m

$= 106$ m (to the nearest whole numbers)

Thus, width of the river is 106 m. Ans.

Question 18. The angles of elevation of the top of a tower from two points A and B at distance of a and b respectively from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$.

Solution : Let the height of the tower 'OT' = h

Let O be the base of tower.

Let A and B be two points on the same line through the base such that

$$OA = a, OB = b$$

$\because$ The angles at A and B are complementary

$\therefore \quad \angle TAO = \alpha$

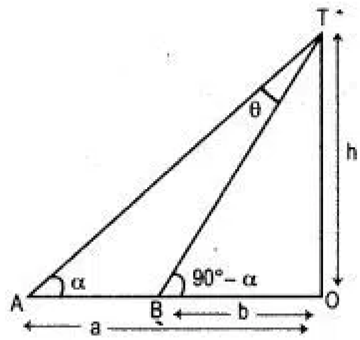
then $\quad \angle TBO = 90° - \alpha$

In rt $\angle d \, \Delta \, OAT$,

$$\tan \alpha = \frac{OT}{OA} = \frac{h}{a} \quad \ldots(i)$$

In rt $\angle d \, \Delta \, OBT$,

$$\tan (90° - \alpha) = \frac{OT}{OB} = \frac{h}{b}$$



$$\Rightarrow \quad \cot \alpha = \frac{h}{b} \quad \ldots(ii)$$

Multiplying (i) and (ii) we have

$$\tan \alpha \cot \alpha = \frac{h}{a} \times \frac{h}{b} = \frac{h^2}{ab}$$

$$1 = \frac{h^2}{ab}$$

$$\Rightarrow \quad h^2 = ab$$

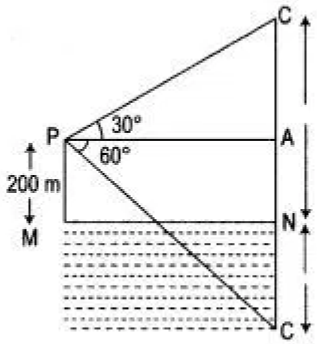
$$\Rightarrow \quad h = \sqrt{ab}$$

Hence, the height of the tower = $\sqrt{ab}$. Ans.

Question 19. (i) The angle of elevation of a cloud from a point 200 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.
(ii) If the angle of elevation of a cloud from a point h meters above a lake is a*and the angle of depression of its reflection in the lake is |i. Prove that the height of the cloud is

$$\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$$

Solution : Let P be the point of observation and C, the position of cloud. $CN \perp$ from C on the surface of the lake and C' be the reflection of the cloud in the lake so that



$$CN = NC' = x \text{ (say)}$$

Then, $PM = 200$ m

$\therefore$ $AN = MP = 200$ m

$$CA = CN - AN = (x - 200) \text{ m}$$

$$C'A = NC' + AN = (x + 200) \text{ m}$$

Let $PA = y$ m

Then in right $\angle d\ \Delta PAC$,

$$\frac{CA}{PA} = \tan 30^\circ$$

$$\Rightarrow \quad \frac{x - 200}{y} = \frac{1}{\sqrt{3}}$$

$$y = \sqrt{3}(x - 200) \quad \ldots\text{(i)}$$

Also in right $\angle d\ \Delta C'AP$,

$$\frac{C'A}{PA} = \tan 60^\circ$$

$$\frac{x + 200}{y} = \sqrt{3}$$

$$\Rightarrow \quad x + 200 = \sqrt{3}y$$

$$\Rightarrow \quad y = \frac{x + 200}{\sqrt{3}} \quad \ldots\text{(ii)}$$

From (i) and (ii),

$$\frac{x + 200}{\sqrt{3}} = \sqrt{3}(x - 200)$$

$$\Rightarrow \quad x + 200 = 3(x - 200)$$

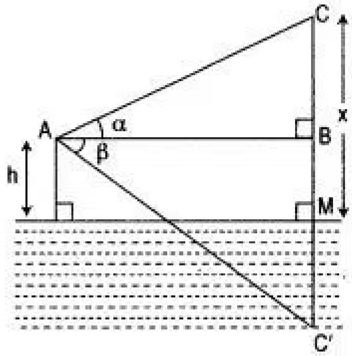
$$\Rightarrow \quad x + 200 = 3x - 600$$

$$\Rightarrow \quad 2x = 800$$

$$\Rightarrow \quad x = 400 \text{ m}$$

Hence, the height of the cloud = 400 m. Ans.

(ii) Let LM be the upper surface of the lake and A be a point such that $AL = h$.



Let C be the position of the cloud and C' be its reflection in the lake.

$$CM = MC' = x \text{ (let)}$$

$$\angle BAC = \alpha \text{ and } \angle BAC' = \beta$$

Now in Δ CBA,

$$\tan\alpha = \frac{CB}{AB}$$

$$\tan\alpha = \frac{x-h}{AB}$$

$$AB = \frac{x-h}{\tan\alpha} \quad \text{...(i)}$$

In Δ C'BA, $\tan\beta = \frac{C'B}{AB}$

$$\tan\beta = \frac{x+h}{AB}$$

$$AB = \frac{x+h}{\tan\beta} \quad \text{...(ii)}$$

From (i) and (ii),

$$\frac{x-h}{\tan\alpha} = \frac{x+h}{\tan\beta}$$

or $$\frac{x+h}{x-h} = \frac{\tan\beta}{\tan\alpha}$$

App. componendo and tividendo

$$\frac{x+h+x-h}{x+h-x+h} = \frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$$

$$\frac{2x}{2h} = \frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$$

$$x = \frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$$

$\therefore$ Height of the cloud is

$$x = \frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}.$$

Hence proved.

Question 20. From the top of a hill, the angles of depression of two consecutive kilometer stones, due east are found to be 30° and 45° respectively. Find the distance of the two stones from the foot of the hill.

Solution :

Let AB be hill of which B is foot of hill and D and C are two consecutive km stones.

$\therefore \quad DC = 1 \text{ km} = 1000 \text{ m}$

In right angled $\Delta ABC, \tan 45° = \frac{AB}{BC}$

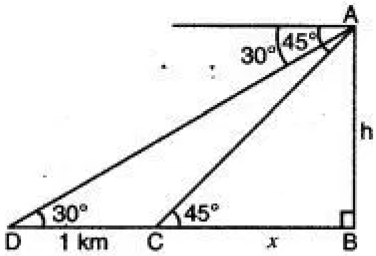
$$1 = \frac{h}{x}$$

$$x = h \quad \text{...(i)}$$

In right angled $\Delta ABD, \tan 30° = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + 1000}$$

$$x + 1000 = h\sqrt{3} \quad \text{...(ii)}$$



But from equation (i), $x = h$,

$$\therefore \quad x + 1000 = x\sqrt{3}$$

$$x(\sqrt{3} - 1) = 1000$$

$$x = \frac{1000}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{1000(\sqrt{3} + 1)}{2}$$

$$= 500(\sqrt{3} + 1)$$

$$= 500 \times 2{\cdot}732$$

$$= 1366 \text{ metre}$$

$$= 1{\cdot}366 \text{ km}$$

$\therefore$ Ist km stone is 1·366 km and IInd km stone is 2·366 km from foot of hill. Ans.

Question 21. A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m., away from the bank, he finds the angle of elevation to be 30°. Calculate:

(i) The width of the river and

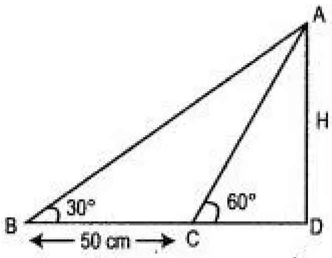
(ii) The height of the tree

Let H be height of tree

Solution : (i) Let height of the tree be H meter.

In rt $\angle d\ \Delta$ ACD, $\tan 60° = \dfrac{H}{CD}$

$$\sqrt{3} = \frac{H}{CD}$$



$$\therefore \qquad CD = \frac{H}{\sqrt{3}} \qquad \ldots(i)$$

In rt $\angle d\ \Delta$ABD

$$\tan 30° = \frac{H}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{BD}$$

$$\therefore \qquad BD = \sqrt{3}\,H \qquad \ldots(ii)$$

$$BD - CD = 50$$

$$\frac{\sqrt{3}\,H}{1} - \frac{H}{\sqrt{3}} = 50 \qquad \text{[Using (i) and (ii)]}$$

$$\therefore \qquad \frac{3H - H}{\sqrt{3}} = 50$$

$$\therefore \qquad 2H = 50\sqrt{3}$$

Or $\qquad H = \dfrac{50\sqrt{3}}{2} = 25\sqrt{3}$

$$H = 43{\cdot}3 \text{ m}$$

(i) The width of the river CD $= \dfrac{25\sqrt{3}}{\sqrt{3}} = 25$ m

(ii) The height of the tree H = 43·3 m Ans.