Current Electricity

Electric Current & Ohm's Law

Electric Current (I)

- It is the rate of flow of electric charge flowing though any section of wire.
- $I = \frac{\text{Total charge flowing}}{\text{Time taken}}$
- Unit of current is Ampere (A).
- One ampere of current flows in a wire when one coulomb of charge flows through the wire in one second.

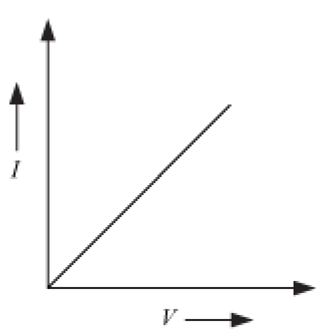
Electric Current in a Conductor

- The electrons in a conductor move due to thermal motion during which they collide with the fixed ions.
- The direction of its velocity after the collision is completely random. At a given time, there is no preferential direction for the velocities of the electrons. Therefore, there will be no net current.
- Electrons in a conductor move under the action of the electric field applied across its two ends.

Ohm's Law

- Electric current flowing through a conductor is directly proportional to the potential difference across the two ends of the conductor; physical quantities such as temperature, mechanical strain, etc. remaining constant.

$V \propto I$

- 

$V - I$ graph for an ohmic conductor

$V = RI$

Where, R is resistance of the conductor

- SI unit of resistance is ohm.
- Resistance of a conductor depends on:
- Length of the conductor (l)
- Area of cross-section (A) of the conductor

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

ρ is the electrical resistivity of the conductor.

- Using Ohm's law,

$$V = IR$$

$$= I\rho \frac{l}{A} \quad \ldots\ldots \ (i)$$

$\frac{I}{A}$ is current per unit area, also called current density (J).

$$\therefore J = \frac{I}{A} \quad \ldots\ldots \quad \text{(ii)}$$

Let E be applied electric field across the conductor.

$\therefore V = E\,l$ ……. (iii)

From equations (i), (ii), and (iii),

$El = J\rho l$

$E = J\rho$

$$J = \left(\frac{1}{\rho}\right)E$$

Where, $\frac{1}{\rho}$ is called the conductivity (σ)

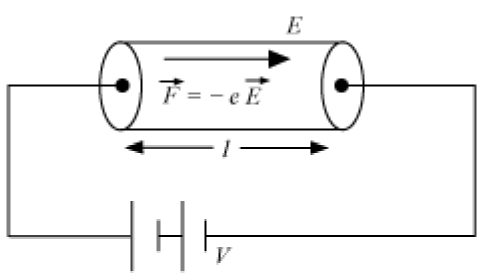
$$\therefore \sigma = \frac{1}{\rho}$$

$$\boxed{\therefore J = \sigma E}$$

Drift of Electrons & Limitations of Ohm's Law

- Free electrons are in continuous random motion. They undergo change in direction at each collision and the thermal velocities are randomly distributed in all directions.

∴ Average thermal velocity, $u = \frac{u_1 + u_2 + \ldots + u_n}{n}$ is zero … (1)

- 

The electric field E exerts an electrostatic force '$-Ee$'.

Acceleration of each electron is, $\vec{a} = \frac{-e\vec{E}}{m}$ ...(2)

Where,

m → Mass of an electron

e → Charge on an electron

- Drift velocity – It is the velocity with which free electrons get drifted towards the positive terminal under the effect of the applied electric field.

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \ldots\ldots\ldots + \vec{v}_n}{n}$$

$$\vec{v}_d = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \ldots\ldots\ldots + (\vec{u}_n + \vec{a}\tau_n)}{n}$$

Where,

$\vec{u}_1, \vec{u}_2 \rightarrow$ Thermal velocities of the electrons

$\vec{a}\tau_1, \vec{a}\tau_2 \rightarrow$ Velocity acquired by electrons

τ_1, τ_2 → Time elapsed after the collision

$$\vec{v}_d = \frac{(\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_n)}{n} + \frac{\vec{a}(\tau_1 + \tau_2 + \ldots + \tau_n)}{n}$$

Since $\frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_n}{n}$ = 0,

∴ $v_d = a\,\tau$

Where, $\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \ldots\ldots + \tau_n}{n}$ is the average time elapsed

Substituting for *a* from equation (2),

$$\vec{v}_d = \frac{-e\vec{E}}{m}\tau \qquad \ldots(4)$$

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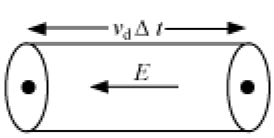


Electron drift to a small distance in a time $\Delta t = V_d \Delta t$

Amount of charge passing through the area A in time Δt, $q = I\Delta t$

$I\Delta t = neAv_d\Delta t$

Where,

$n\rightarrow$ Number of free electrons per unit volume

From equation (4),

$$I\Delta t = \frac{e^2 A}{m}\tau n\Delta t\left|E\right|$$

Current density (J) $= \frac{I}{A}$

$$\therefore \left|J\right| = \frac{ne^2}{m}\tau\left|E\right|$$

We know,

J = σ E

$$\boxed{\therefore \sigma = \frac{ne^2}{m}\tau}$$

Mobility (μ)

- It is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{V_d}{E}$$

- Unit of mobility is m^2/Vs.

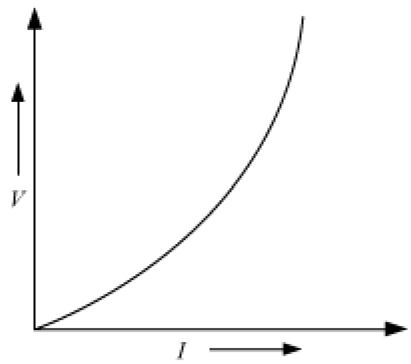
- $$\because V_d = \frac{e\tau E}{m}$$

$$\boxed{\therefore \mu = \frac{e\tau}{m}}$$

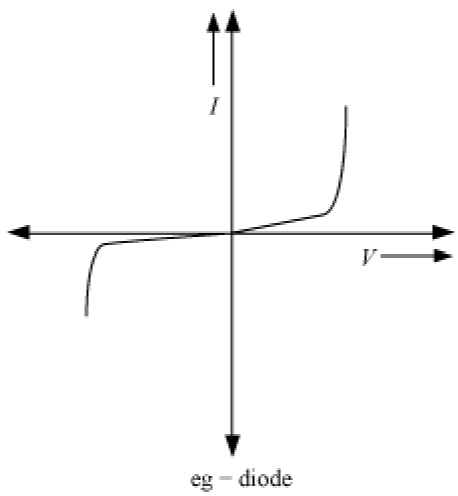
Limitations of Ohm's Law

There are several materials and devices for which the proportionality of V and I are as follows:

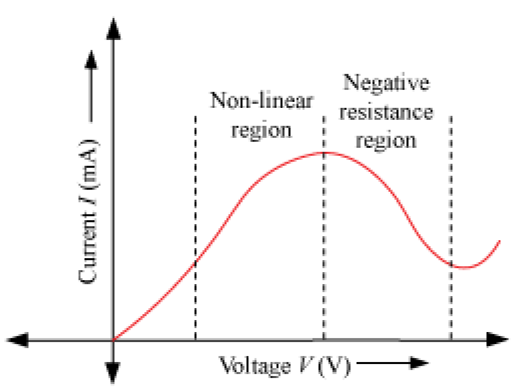
- *V* ceases to be proportional to *I*.



- Sign of V affects the relation between *V* and *I*.



- There is more than one value of *V* for the same current.



Example – GaAs

Resistance and Resistivity

Do you know why it is advisable to use thick conducting wires in wiring?

This is related to the amount of resistance offered by wires or resistors used in circuits. We know that resistance is a natural tendency of the conductors to oppose the flow of electric charge from it. Hence, it causes a loss of electricity and should be minimised. This can be achieved by using thick conducting wires. There are two types of resistors:

- Wire bound resistor
- Carbon resistor
- Carbon resistors are very small in size. Therefore, their values are given using a colour code.

TABLE 3.2 RESISTOR COLOUR CODES			
Colour	**Number**	**Multiplier**	**Tolerance (%)**
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	

Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

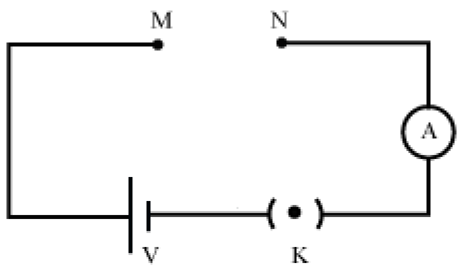
- Here, first two bands from one end indicate the first two significant figures of resistance in ohms.
- Third band indicates the decimal multiplier.
- The last band stands for tolerance. Its absence indicates a tolerance of 20%.

Now, we will discuss the different factors on which resistance of a conductor depends.

Factors that affect resistance

Construct an open circuit with open ends M and N (as shown in the figure). Take six pieces of wire with dimensions as given below.

Case (i) Copper wire → of length MN of the given cross-section.
Case (ii) Copper wire → of length greater than MN having the same cross-section.
Case (iii) Copper wire → of length less than MN having the same cross section.
Case (iv) Copper wire → of length MN but a comparatively thicker cross-section.
Case (v) Copper wire → of length MN but comparatively a thinner cross-section.
Case (vi) Chromium wire → of length MN and of same cross-section as in case (i).



Connect copper wire of case (i) between the open points M and N, and note the readings in the ammeter. The reading will give you the amount of current flowing through the copper wire.

Similarly, connect wire of case (ii) and note the reading in the ammeter. Repeat the process with the remaining four wires and make a table of your readings in each case. Scrutinize the readings and compare them with the dimensions of the wire. **Does the current depend on the length, cross-section, and nature of the material used?**

You will notice that the amount of current that flows through wire in case (i) is greater than that flowing through wire in case (ii). **What does this mean?** This means that a long wire offers greater resistance in comparison to a short wire. Also, resistance decreases with an increase in the cross-sectional area. Similarly, if you allow the current to flow in each wire for a relatively longer period, you will find that there is a decrease in current. Hence, from this activity we can say that resistance of a conductor depends on the following:

I. Length of the conductor

II. Cross-sectional area of the conductor

III. Temperature of the conductor

IV. Nature of the material used

Symbol of resistor

I. length of the conductor

Resistance is directly proportional to the length of the conductor i.e.

$R \propto l$

Where, l → length of the conductor

II. Cross-section of the conductor

Resistance is inversely proportional to the area of cross-section of the conductor i.e.

$R \propto 1/A$

A → area of cross-section

- Since conductors have a circular cross-section, the area of cross-section is directly proportional to the square of the radius of the cross-section i.e.

$R \propto 1/r^2$

r → radius of the cross-section

When the diameter of a conductor is made double, its resistance becomes one fourth.

Thus, we can write,

$R \propto l/A$

Or,

$R = \rho * l/A$

Where, ρ is the proportionality constant, called the **electrical resistivity** of the material of the conductor. It is also known as specific resistance.

The resistivity or specific resistance of a substance is equal to its resistance, if it has a unit length and unit cross-sectional area.

The SI unit of resistivity is Ω m (Ohm-meter).

- Resistivity is the characteristic property of a material. It only depends on the nature of the material and not its dimensions. This is one of the major differences between resistance and resistivity. But like resistance, resistivity also varies with temperature.

  Conductivity

  The reciprocal of resistivity is known as conductivity. Its SI unit is $ohm^{-1}metre^{-1}$ or $\Omega^{-1}m^{-1}$ or siemen $metre^{-1}$

  $\sigma = 1/\rho = l/Ra$

- Metals have low resistivities. It is in the range of 10^{-8} Ω m to 10^{-6} Ω m. Insulators have resistivities 10^{18} times greater than metals. Semi-conductors lie in between them. The following table shows the resistivity of some materials at 20 °C.

Category	**Material**	**Resistivity (in Ω m)**
Conductors	Silver	1.60×10^{-8}
	Copper	1.62×10^{-8}
	Aluminium	2.63×10^{-8}
	Tungsten	5.20×10^{-8}
	Nickel	6.48×10^{-8}
	Iron	10.0×10^{-8}

	Chromium	12.9×10^{-8}
	Mercury	94.0×10^{-8}
	Manganese	184×10^{-8}
Alloys	Manganin (Cu – Mn – Ni)	44×10^{-8}
	Constantan (Cu – Ni)	49×10^{-8}
	Nichrome (Ni – Cr – Mn – Fe)	100×10^{-8}
Semiconductors	Germanium	0.6
	Silicon	2300
Insulators	Glass	$10^{10} - 10^{14}$
	Paper	10^{12}
	Hard rubber	$10^{13} - 10^{16}$

	Diamond	$10^{12} - 10^{13}$
	Ebonite	$10^{15} - 10^{17}$

Temperature Dependence of Resistivity

For Metallic Conductor

In terms of relaxation time, the resistivity of the material of a conductor is given by

$$\rho = \frac{m}{ne^2\tau}$$

Here, the letters have their usual meanings.

If the temperature increases, the amplitude of the vibrations of the +ve ions in the conductor also increases. Due to this, the free electrons collide more frequently with the vibrating ions; as a result, the average relaxation time decreases. Because $\rho \propto 1/\tau$, the resistivity of a metallic conductor increases with increase in temperature.

- Resistivity of a metallic conductor is given by

  $\rho_T = \rho_0 [1 + \alpha (T - T_0)]$

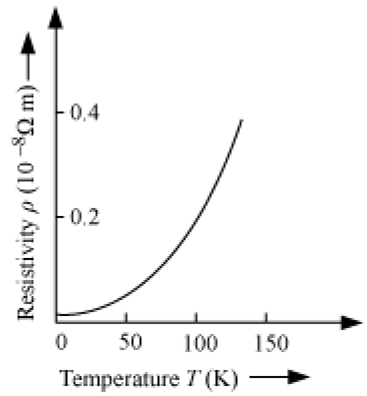
  Here,

  ρ_T = Resistivity at temperature T

  ρ_0 = Resistivity at reference temperature T_0

  α = Temperature coefficient of resistivity

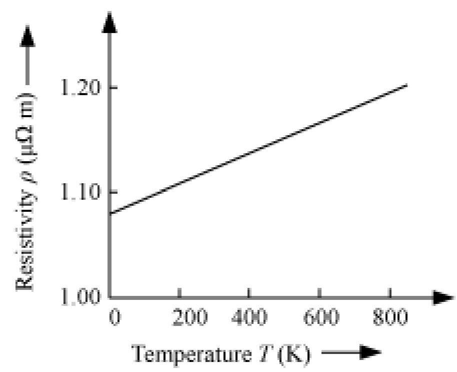
- α is + ve for metals.
- Graph of ρ_T plotted against T should be a straight line. At temperature lower than 0°C, the graph deviates from a straight line.



For Alloys

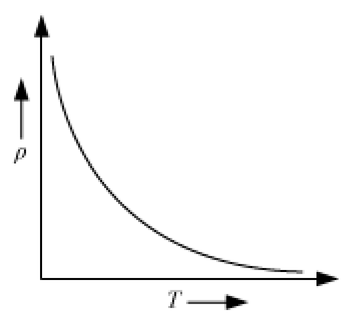
In case of an alloy, the resistivity is very large and it has a very weak temperature dependence.

- Nichrome (an alloy) exhibits weak dependence of resistivity with temperature.



For Semiconductors

- Resistivity of a semiconductor decreases with temperature.



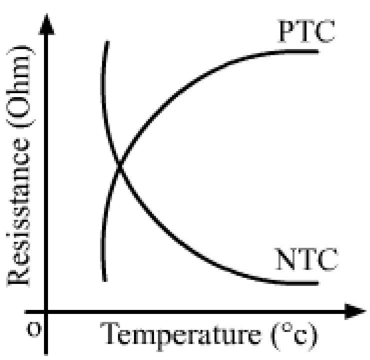
Superconductivity

- Superconductivity is the phenomenon in which a material loses its resistivity completely at a particular temperature.

- The temperature at which this process takes place is called **critical temperature**.
- **Uses of superconductivity:**
- Superconducting cables can be used for power distribution without loss.
- Speed of a computer can be increased by superconducting wires.
- It helps in producing a very strong magnetic field without power loss.
- **Limitation:** Superconductivity exists at subzero temperature, which hinders its use at normal temperature.

Thermistor

- Thermistor is a heat-sensitive semiconductor device whose resistance changes very rapidly and non-linearly with change in temperature.The temperature coefficient of resistance of a thermistor is very high.
- Thermistors are prepared from oxides of metals like iron, nickel, cobalt and copper. They are available in various shapes.
- A thermistor can measure low temperatures (even 0.001° C).
- Thermistors are used for remote sensing, voltage stabilisation, temperature control, etc.
- Temperature dependence of resistance of a thermistor is shown by

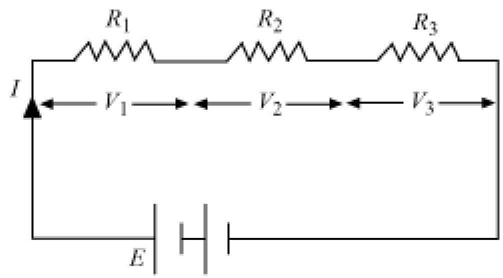


Combination of Resistors; Electric Energy and Power

Combination of Resistors

Resistors in Series

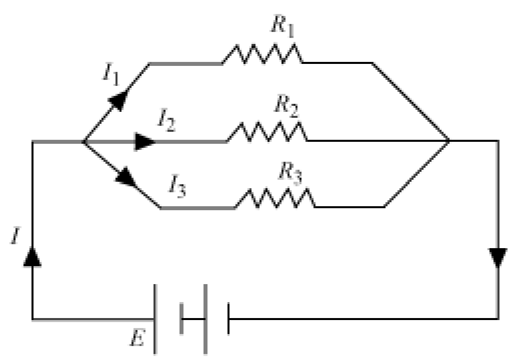
- Two or more resistors are said to be connected in series, if same current passes through each of them, when a potential difference is applied across them.



Equivalent resistance, $R_S = R_1 + R_2 + R_3$

Resistors in Parallel

- Two or more resistors are said to be connected in parallel, if potential difference across each of them is equal to the applied potential difference.

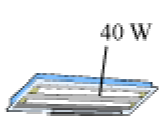


- Equivalent resistance (R_P)

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Electric Energy and Power

We use electricity to run various electrical appliances such as bulbs, tube lights, refrigerators, electric heaters etc. in our homes. **Do you think all these home appliances consume an equal amount of electricity at a given time?**



No, the amount of electricity consumed by an electrical appliance depends on the power rated on that appliance. For example, for 220 V potential supply, a tube light of rated power 40 W draws 0.18 A of current, whereas a bulb of rated power 100 W draws 0.45 A of current.

How can you determine the rate of consumption of energy by a given appliance?

Electric power

Electric power is defined as the rate of consumption of energy or simply the rate of doing work.

i.e., power (P) = $\frac{\text{work done } (W)}{\text{Time } (t)}$ ……………. (i)

The work done by current (I) when it flows in a potential (V) for time (t) can be given by

$W = VIt$ ………………… (ii)

⇒ Power, (P) = $\frac{VIt}{t}$ = VI

∴ Electric power or $P = VI$

The SI unit of power is watts (W).

Also, the energy dissipated or consumed by an electric appliance per unit time is given by

$Ht=I^2R$

It is also an equation for electric power.

i.e., $P = I^2R$

Now, according to Ohm's law

⇒ $V = IR$

⇒I=VRI=VR

If we substitute the value of I in the equation of power, we get

P=(VR)2R=V2RP=VR2R=V2R

Hence, we get the expression for electric power as

$P = VI=I^2R=V^2/R$

Where, 1 W = 1 V × 1 A = 1 V A

1 watt is defined as the power consumed by an electrical circuit that carries a current of 1 ampere, when it is operated at a potential difference of 1 volt.

- Since Watt is a very small unit, we define a larger unit of power as **kilowatt** (kW). Thus,

$$1 \text{ kW} = 1000 \text{ W}$$

For practical purposes, we define kilowatt hour (kWh) as 'unit' where,

1 unit = 1 kWh = 1000 W × 1 hour

= 1000 W × 3600 s

= 36×10^5 Ws

= 3.6×10^6 J

1 kWh is also the commercial unit of electric energy.

Note that electricity is a flow of electrons and nothing else. Hence, power stations only make the electrons flow through conducting wires for which they charge us. They do not create or generate the electrons.

Cell, Emf, Internal Resistance, Cells in Series and Parallel

Electric Cell

An electric cell is a device used for maintaining permanent potential difference with the simplest arrangement between the conductors.

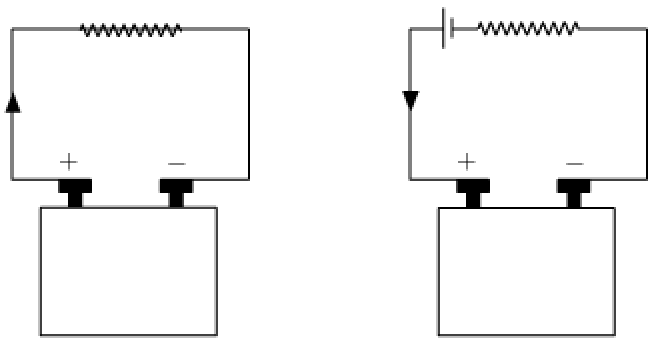
There are two types of electric cells:

- Primary Cell − In a primary cell, chemical energy is directly converted to electrical energy. They cannot be reused once completely discharged.

  For example, simple voltaic cell, Leclanche cell

- Secondary Cell− In a secondary cell also, chemical energy is directly converted to electrical energy. But the basic difference between primary and secondary cells is that secondary cells are reusable as they can be charged using external sources. Thus, when secondary cells are charged, electric energy gets converted into chemical energy.
- In a secondary cell, when current leaves the cell at the positive terminal and enters the cell at negative terminal, the cell is said to be getting discharged (fig a) i.e. it is providing electric energy to the outer circuit. In this phase, the chemical energy stored in the cell gets converted to electrical energy.

- When the cell is connected to some external source of emf, the current then enters the cell at the positive terminal and leaves the cell at negative terminal. In this phase, the cell is said to be getting charged (fig b) i.e. here the electric energy of the source is converted to chemical energy.



Discharging (a) Charging (b)

Examples of secondary cells: lead acid cell, Ni−Cd cell

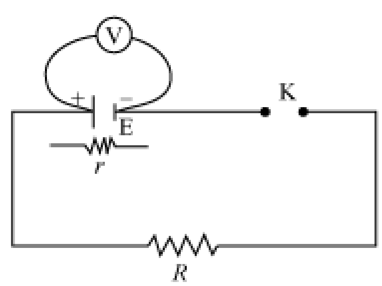
Emf

- Potential difference between the two poles of the cell in an open circuit is called *emf* of the cell.

SI unit is volt (V).

Internal resistance (*r*) of cell

- Resistance offered by the electrolyte of the cell when the electric current flows through it



E – *emf* of cell

r – Internal resistance of the cell

R – External resistance

K – Key

V – Voltmeter

- The key K is closed and a current I flows in the circuit.

  According to Ohm's law,

  $$I = \frac{E}{R+r} \qquad ...(1)$$

- Let V be the terminal potential difference. The terminal potential difference V is less than *emf* E of the cell by an equal amount, which is equal to potential drop across external resistance R i.e.,

  $\therefore V = E - Ir$

  Also, terminal potential difference is equal to potential differences across external resistance.

  $V = IR$

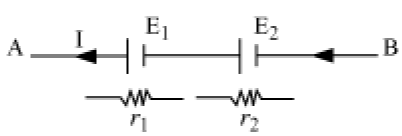
  From equation (1),

  $$\therefore V = \frac{E}{r+R}R$$

  $$\boxed{r = \left(\frac{E}{V} - 1\right)R}$$

Combination of cells

Cells in Series



E_1 E_2 – *emf* of two cells

r_1, r_2 – Internal resistance of two cells

I – Current in the circuit

Terminal potential difference across the first cell, $V_1 = E_1 - Ir_1$

Terminal potential difference across the second cell, $V_2 = E_2 - Ir_2$

Potential difference between the points A and B,

$V = V_1 + V_2 = (E_1 - Ir_1) + (E_2 - Ir_2)$

$= (E_1 + E_2) - I\,(r_1 + r_2)$

Let

E − Effective *emf*

r − Effective internal resistance

$V = E - Ir$

$\therefore E = E_1 + E_2$

$r = r_1 + r_2$

- Current in the circuit,

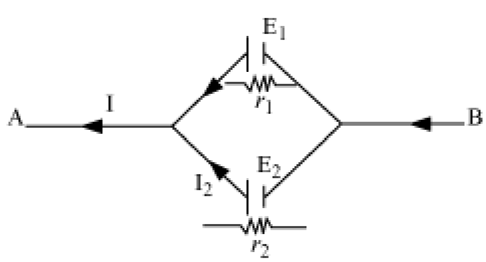
$$I = \frac{E_1 + E_2}{R + (r_1 + r_2)}$$

- If the two cells are connected in opposite direction, then

$E = E_1 - E_2$

$$\therefore I = \frac{E_1 - E_2}{R + (r_1 + r_2)}$$

Cell in Parallel



E_1, E_2 − *emf* of two cells

r_1, r_2 − Internal resistances of cell

I_1, I_2 – Current due to the two cells

Terminal potential difference across the first cell,

$V = E_1 - I_1 r_1$

$$I_1 = \frac{E_1 - V}{r_1}$$

For the second cell,

$$I_2 = \frac{E_2 - V}{r_2}$$

$$\therefore I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \times V$$

$$V = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

Let E be effective *emf* and r is effective internal resistance.

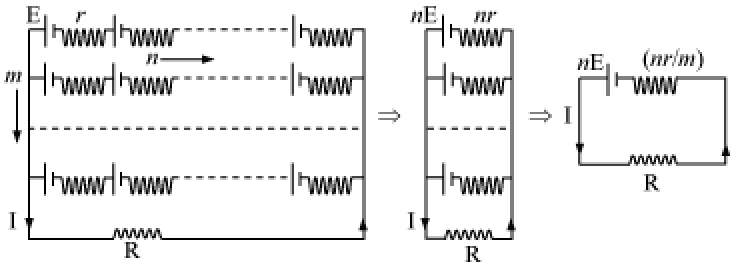
$V = E - Ir$

$$\therefore E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

And, $r = \frac{r_1 r_2}{r_1 + r_2}$

Mixed Grouping of Cells

Suppose ‘n’ cells of emf ‘E’ and internal resistance ‘r’ are connected in series in each branch; and there are ‘*m*’ such branches. ‘R’ is the external resistance connected to the circuit.



In each branch, the net emf is *n*E and the net internal resistance is *n*r. The equivalent resistance of all the internal resistances in parallel is,

$$\frac{1}{r_{eq}} = \frac{1}{nr} + \frac{1}{nr} + \frac{1}{nr} \text{..................upto m}$$

$$\Rightarrow \frac{1}{r_{eq}} = \frac{m}{nr}$$

$$\Rightarrow r_{eq} = \frac{nr}{m}$$

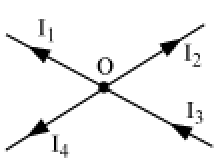
We finally have a circuit with a cell of emf *n*E, internal resistance $\frac{nr}{m}$ connected to a resistance R. The current through the circuit is,

$$I = \frac{nE}{R + \frac{nr}{m}}$$

Kirchhoff's Rules, Wheatstone Bridge & Meter Bridge

Kirchhoff's First Law – Junction Rule

- The algebraic sum of the currents meeting at a point in an electrical circuit is always zero.



I_1, I_2 I_3, and I_4

Convention:

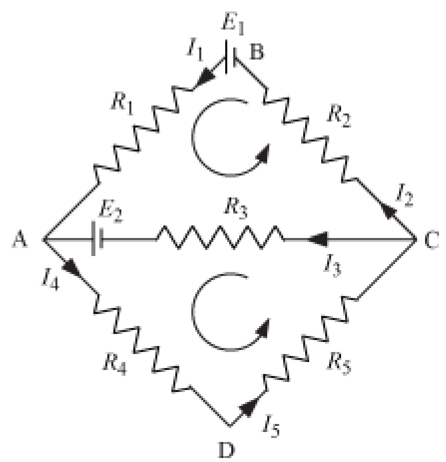
Current towards the junction – positive

Current away from the junction − negative

i.e., $I_3 + (-I_1) + (-I_2) + (-I_4) = 0$

Kirchhoff's Second Law – Loop Rule

- In a closed loop, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and the current flowing through them.



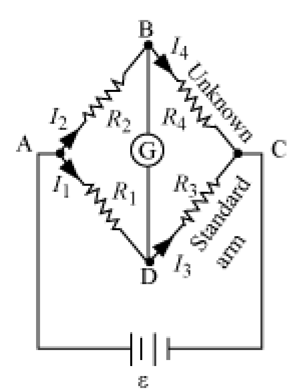
For the closed part BACB, we have:

$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$

For the closed part CADC, we have:

$E_2 = I_3R_3 + I_4R_4 + I_5R_5$

Wheatstone Bridge



- R_1, R_2, R_3, and R_4 are the four resistances.

- Galvanometer (G) has a current I_g flowing through it at a balanced condition, $I_g = 0$.
- Applying the junction rule at B, we have:

  $I_2 = I_4$

- Similarly, at D, we have $I_1 = I_3$
- Applying the loop rule to the closed loop ADBA, we have:

$$-I_1R_1 + 0 + I_2R_2 = 0$$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad \text{...(1)}$$

- Applying the loop rule to the closed loop CBDC, we have:

$$I_2R_4 + 0 - I_1R_3 = 0 \qquad \because I_3 = I_1,\ I_4 = I_2$$

$$\therefore \frac{I_1}{I_2} = \frac{R_4}{R_3} \qquad \text{...(2)}$$

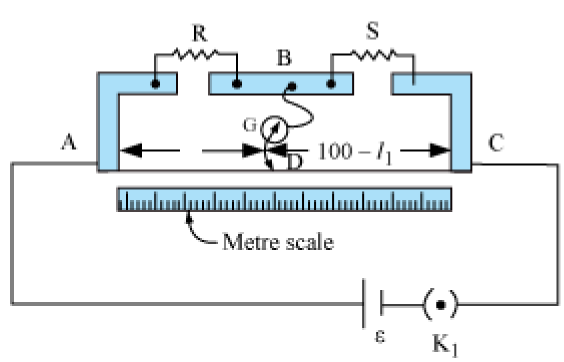
From equations (1) and (2), we get:

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$ (Balanced condition)

- For a balanced bridge, the unknown resistance can be determined as follows:

$$\boxed{R_4 = R_3\frac{R_2}{R_1}}$$

Metre Bridge



- It consists of a 1 m long wire of uniform cross-section
- The construction of a metre bridge is shown in the above figure.
- Suppose that

  R – Unknown resistance

  S – Standard resistance

  l_1 – Distance from A

  R_{cm} – Resistance of the wire per unit centimetre

  $R_{cm}l_1$ – Resistance of length AD

  R_{cm} $(100 - l_1)$ – Resistance of length DC

- From the figure, the balance condition gives

$$\frac{R}{S} = \frac{R_{cm} l_1}{R_{cm}(100 - l_1)}$$

$$\boxed{\therefore R = \frac{S l_1}{100 - l_1}}$$

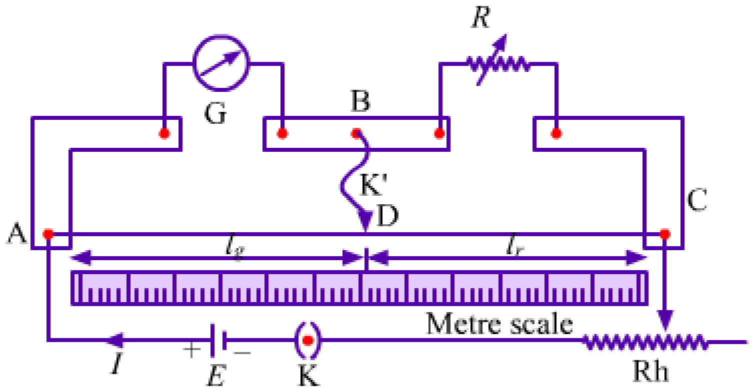
Probable errors while using a metre bridge

- If the wire used in a metre bridge is not uniform, then the resistance of the wire will not be proportional to its length. Thus, there will be error in the value of unknown resistance.
- The contact resistances may developed at the ends of the wire of a metre bridge. They affect null deflection point and introduce an error in the calculation of the unknown resistance.
- The ends A and C of the wire may not coincide with the 0 and 100 cm marks of the metre scale.

Minimisation of the errors in metre bridge

- A wire of uniform thickness must be used.
- The values of the known resistances should be chosen so that the null point comes in or near the middle of the wire of the metre bridge.
- The experiment should be repeated by interchanging the positions of the known and the unknown resistances, in order to minimise contact resistances.

Kelvin's Method to Determine Resistance of a Galvanometer



- Here, a galvanometer of resistance (G) is connected to one gap of the metre bridge and a resistance box is connected to the other gap.
- A cell of emf *E*, key K and rheostat Rh is connected in series to the bridge wire.
- The junction B between the galvanometer and the resistance box is connected to a jockey that can slide along the bridge wire.
- A suitable resistance *R* is taken in the resistance box. The deflection of the galvanometer is noted without touching the jockey with the bridge wire.
- The jockey is slid along the bridge wire and the point is found where the galvanometer shows the same deflection as found while the jockey was not touching the bridge wire.

Suppose that D = point where the galvanometer shows the same deflection;
l_g = distance of point D from end A of wire;
l_r = distance of point D from end C of wire
and
k = resistance per unit length of wire AC

According to the balancing conditions of the Wheatstone bridge, we have:
GR = klg/klr = (lg/lr)
⇒ G = R × (lg/lr) = R × (lg100 − lg)

The resistance of the galvanometer G can be determined if the values of *R* and l_g are known.

Potentiometer

POTENTIOMETER:

Principle

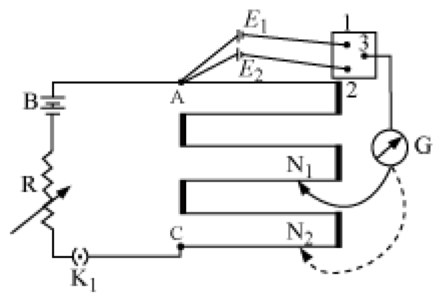
When a constant current is passed through a wire of uniform cross-sectional area, the potential drop across any portion of the wire is directly proportional to the length of that portion.

Construction

- It consists of a number of segments of wire of uniform cross-sectional area.
- The small vertical portions, connecting the various sections of wire, are made of thick metal stripes.
- A rheostat is connected to the circuit, which can vary the amount of current flowing in the wire.

Applications of Potentiometer

- **Comparison of emfs of two cells**



In the above figure you see that a current I flows through the wire which can be varied by a variable resistance (rheostat, R) in the circuit. Since the wire is uniform, the potential difference between A and any point at a distance l from A is $\varepsilon(l)= \varphi l$, where φ is the potential drop per unit length.

When we connect 1 and 3 the cell with the emf E_1 gets connected to galvanometer G. Let the null point be N_1 at length l_1.

potential drop across the length = φl_1

Applying the loop rule to AN1G31A, we get:

$$\Phi\, l1 + 0 - E1 = 0 \qquad ...(1)$$

[Loop rule: The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. Null point means there is no current in the AE_1N_1]

The emf of the cell will be equal to potential drop across the length.

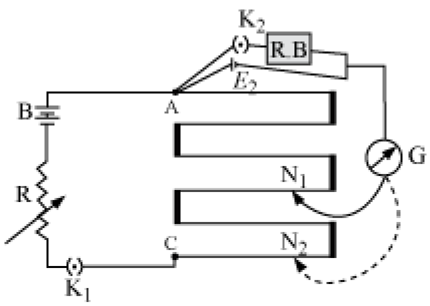
Similarly, when we connect 1 and 2 cell with emf E_2 comes in play.

$$\Phi\, l_2 + 0 - E_2 = 0 \qquad ...(2)$$

From equations (1) and (2), we get:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

- **Measurement of internal resistance of a cell**



- A cell of emf E (internal resistance r) is connected across a resistance box (R) through key K_2.
- When K_2 is opened, the balance length is obtained at length $AN_1 = l_1$.

$E = \Phi\, l_1$ ...(3)

- When K_2 is closed, the balance length is obtained at $AN_2 = l_2$.

Let V be the terminal potential difference of the cell.

Then $V = \Phi\, l_2$ ...(4)

From equations (3) and (4), we get:

$$\frac{E}{V} = \frac{l_1}{l_2} \qquad (5)$$

- $E = I(r + R)$ and $V = IR$

$$\therefore \frac{E}{V} = \frac{r + R}{R} \qquad (6)$$

From (5) and (6), we get:

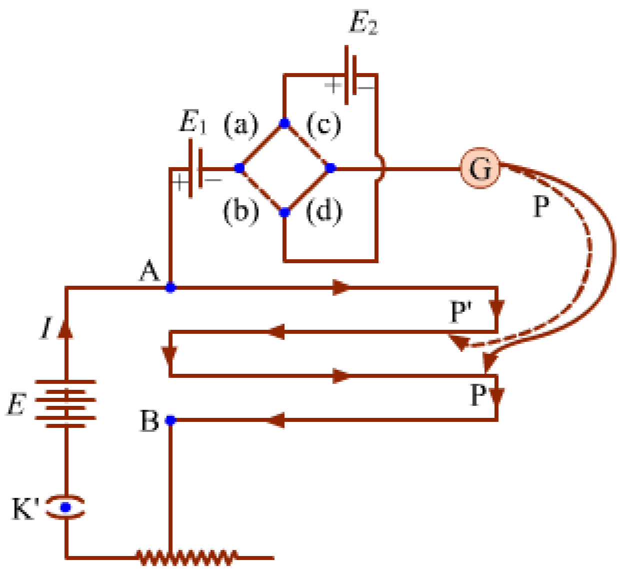
$$\frac{R+r}{R}=\frac{l_1}{l_2}$$

Therefore, we have $\frac{E}{V}=\frac{l_1}{l_2}$

$$\because r=R\left(\frac{E}{V}-1\right)$$

$$\boxed{\therefore r=R\left(\frac{l_1}{l_2}-1\right)}$$

Comparison of emfs of two batteries by combination method



When the keys (a) and (b) are closed and keys (c) and (d) are opened, the cells are connected as the total emf of the combination becomes E_1+E_2.
At null point P, the galvanometer does not show any deflection. Let the length AP be l_1 (distance of null point from end A of wire AB).
$\therefore E_1+E_2 = Ikl_1$ (k is the resistance per unit length of the wire)
$\Rightarrow E_1+E_2 = \Phi l_1$ ...(1) (Φ is the potential gradient for wire AB)

Keeping the same potential gradient, the keys (a) and (b) are opened and (c) and (d) are closed. The total emf of the combination of the two batteries now becomes E_1-E_2.
When the jockey touches point P' of wire AB, there is no deflection in the galvanometer. Let AP' = l_2 be the

balancing length.

$\therefore E_1 - E_2 = Ikl_2$

$\Rightarrow E_1 - E_2 = \Phi l2$ ...(2)

Dividing (1) and (2), we get:

$$\frac{E_1+E_2}{E_1-E_2} = \frac{I\Phi l_1}{I\Phi l_2} = \frac{l_1}{l_2}$$

That can be simplified as follows:

$$\frac{E_1}{E_2} = \frac{l_1+l_2}{l_1-l_2}$$

The emfs can be compared by the putting the values of the balancing lengths l_1 and l_2

Sensitivity of Potentiometer

The smallest ptential difference that can be measured with a potentiometer is known as its sensitivity. The sensitivity of a potentiometer can be increased by decreasing its potential gradient (*E/l*). The potential gradient can be decreased by following ways:

- Increasing the length of potentiometer
- Decreasing the current in the potentiometer wire circuit if the wire is of fixed length

Precautions While Using a Potentiometer

- The potentiometer wire should have a uniform thickness.
- The emf of the auxiliary battery must be greater than the individual emfs of the batteries connected for the accurate comparison of the emfs.
- The positive terminal of the batteries must be connected to the potentiometer's end where the positive terminal of the auxiliarry battery is connected.
- The potentiometer wire should have high resistance.

Advantages of a Potentiometer over a Voltmeter

- A voltmeter can be used to measure the terminal potential difference of a cell, whereas a potentiometer is used to measure the terminal potential difference as well as the emf of a cell.
- A potentiometer can measure the internal resistance of the cell, which a voltmeter cannot measure.
- A potentiometer can accurately measure small potential differences, which a voltmeter cannot.
- The accuracy of a potentiometer can also be increased by increasing the length of the wire, whereas the accuracy of the voltmeter is fixed.

Disadvantages of a Potentiometer

- A voltmeter can directly indicate the value of the potential difference, whereas a potentiometer cannot.
- A voltmeter is portable, but a potentiometer is not.