Magnetism And Matter

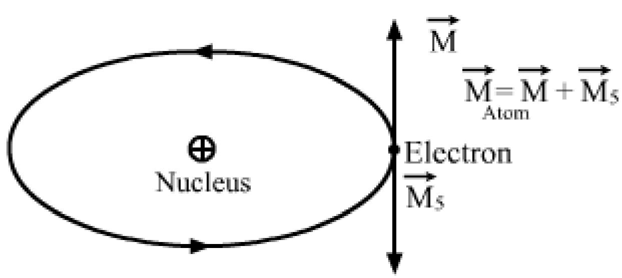
The Bar Magnet

Bar Magnet

- A bar magnet has two poles similar to the positive and negative charges of an electric dipole.
- One pole is designated as the North Pole and the other pole as the South Pole.
- When suspended freely, these poles point (approximately) towards the geographic North and South Poles.
- Like poles repel each other and unlike poles attract each other.
- The poles of a magnet can never be separated.
- Magnetic mono pole does not exist.

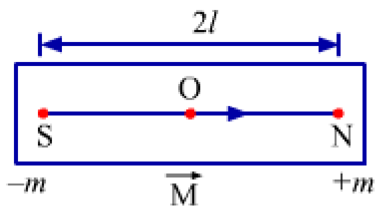
Some Definitions of Magnet
Axis - The line passing through both poles (North and South) of a magnet is called the axis of magnet.
Equator - The plane passing through the centre of a magnet and perpendicular to the magnetic axis is called equator.
Magnetic length - Magnetic length is the distance between two poles of a magnet.

Origin of Magnetism due to Moving Charge



An atom consists of a positively charged nucleus around which a negatively charged electron revolves. The circular motion of an electron can be regarded as a tiny loop of current. It possesses some orbital magnetic moment at the centre. In addition to orbital motion, the electron also performs spin motion around its axis. Another magnetic moment called spin magnetic moment is also associated with the electron. Thus, the net magnetic moment of an atom is the vector sum of orbital magnetic moment and spin magnetic moment.
Normally, the atomic magnetic moment of a substance orients randomly; this result into zero magnetic moment. Hence, a substance does not show magnetic property. If magnetic moments are oriented in a particular direction, then the substance shows magnetic properties.

Magnetic Dipole Moment



Magnetic dipole moment is the product of either pole strength (*m*) and magnetic length (2 l') of a magnet.

Magnetic dipole moment, ($\vec{M}$) = Strength of either pole (*m*) × Magnetic length (2 $\vec{l}$)

= $m(2\vec{l})$

Magnetic dipole moment is a vector quantity. The SI unit of magnetic moment is joule/tesla or ampere metre^2.

Equivalence between Magnetic Dipole and Current-Carrying Coil

(1) Both the current-carrying coil and magnetic dipole produce a magnetic field around them.

(2) Because a current-carrying coil produces a magnetic field, it behaves like a magnetic dipole whose one face represents the North Pole and the other face represents the South Pole. When the current in the loop is in clockwise direction, it behaves as the South Pole; and when the current in the loop is in anticlockwise direction, it behaves as the North Pole.

(3) A magnetic dipole experiences a torque when placed in an external magnetic field. The torque is given by

$\tau = M B \sin\theta$

Similarly, when a current-carrying coil is placed in a magnetic field, it experiences a torque, which is given by

$\tau = nI A B \sin\theta$

(4) Magnetic induction at a point on the axis is given by

$$B_{axis} = \frac{\mu_0}{4\pi}\frac{2M}{r^3} \quad \ldots(1)$$

Magnetic induction at a point on the axis at a large distance from the centre is given by

$$B_{axis} = \frac{\mu_0}{4\pi}\frac{2nIA}{r^3} \quad \ldots(2)$$

From the above two equations, we can say:

$M = nIA$ (Magnetic moment of the current-carrying coil)

Bar Magnet as Equivalent Solenoid

We know that a current loop acts as a magnetic dipole. Magnetic field lines of a bar magnet and a current-carrying solenoid resemble each other. Therefore, a bar magnet can be thought of as a large number of circulating currents in analogy with a solenoid.

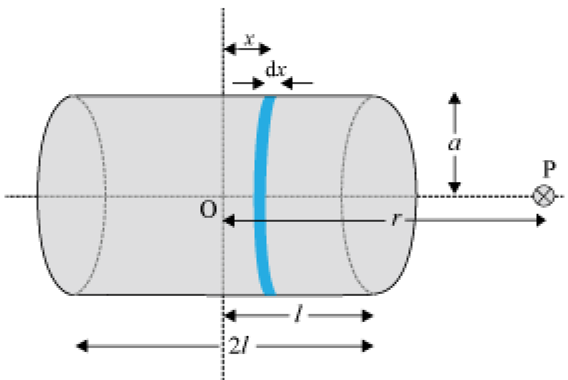
Let:

i = Current passing through the solenoid

a = Radius of the solenoid

$2l$ = Length of the solenoid

n = Number of turns per unit length of the solenoid



Now, let P be the point at which magnetic field is to be calculated. Consider a small element of thickness dx of the solenoid at a distance x from O.

Number of turns in the element = ndx

The magnitude of the field at point P due to the circular element is given by

$$dB = \frac{\mu_0 i a^2 (ndx)}{2\left[(r-x)^2 + a^2\right]^{\frac{3}{2}}} \quad \text{...(i)}$$

If P lies at a very large distance from O, i.e., $r >> a$ and $r >> x$, then

$$\left[(r-x)^2 + a^2\right]^{\frac{3}{2}} \approx r^3$$

$$dB = \frac{\mu_0 i a^2 ndx}{2r^3} \quad \text{...(ii)}$$

Total magnetic field at P due to the current-carrying solenoid:

$$B = \frac{\mu_0 nia^2}{2r^3} \int_{-l}^{+l} dx$$

$$\text{or } B = \frac{\mu_0 nia^2}{2r^3} [x]_{-l}^{l}$$

$$\Rightarrow B = \frac{\mu_0 nia^2}{2r^3} (2l)$$

$$B = \frac{\mu_0}{4\pi} \frac{2n(2l)i\pi a^2}{r^3} \qquad \text{...(iii)}$$

If M is the magnetic moment of the solenoid, then M = Total number of turns × Current × Area of cross-section.

$$M = n(2l) \times i \times \pi a^2$$

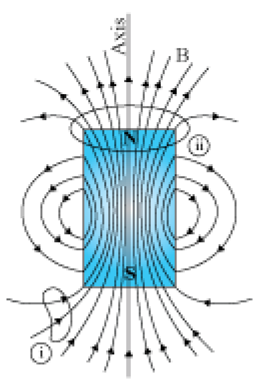
$$\therefore \boxed{B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}}$$

This is the expression for the magnetic field on the axial line of a short bar magnet.

Thus, the magnetic moment of a bar magnet is equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

Magnetic Field Lines

- A magnetic field line is an imaginary curve the tangent to which at any point gives the direction of the magnetic field $\vec{B}$ at that point.
- The magnetic field lines of a magnet (or of a solenoid carrying current) form closed continuous loops.

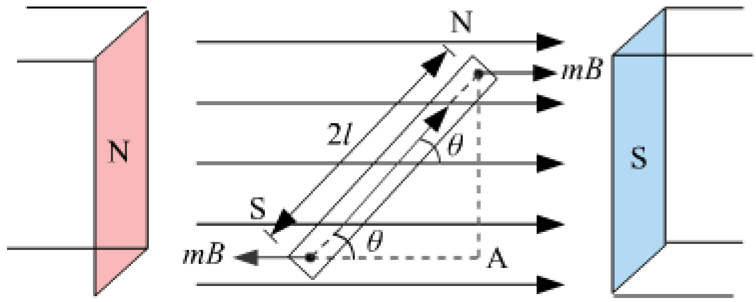


Bar magnet

- Outside the body of a magnet, the direction of magnetic field lines is from the North Pole to the South Pole.

- No two magnetic field lines can intersect each other. This is because we can draw two tangents at the point of intersection. Having two directions of a magnetic field at the same point is not possible.

Dipole in a Uniform Magnetic Field; The Electrostatic Analogue



Force on N-pole = mB, along $\vec{B}$.

Force on S-pole = mB, opposite $\vec{B}$.

Here,

$m \rightarrow$ Strength of each pole

$B \rightarrow$ Strength of the magnetic field

These equal and unlike forces form a couple, which tends to rotate the magnet clockwise so as to align it along $\vec{B}$.

Torque acting on the bar magnet,

τ = Force × Perpendicular distance

= mB × NA

In ΔNAS, $\sin\theta = \frac{NA}{NS} = \frac{NA}{2l}$

$\therefore$ NA = $2l \sin\theta$.

Now,

$\tau = mB \times 2l \sin\theta$

$\Rightarrow \tau = B \times (m2l) \sin\theta$

$\tau = MB \sin\theta$

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The Electrostatic Analogue

- The magnetic dipole moment of a bar magnet, $\vec{M} = m(2\vec{l})$.

  Here,

  m → Strength of each pole

  $2l$ → Length of the dipole

  The magnetic dipole is analogous to an electric dipole consisting of two equal charges of opposite sign (± q) separated by a certain distance $(2\vec{a})$. It has an electric dipole moment,

  $$\vec{p} = q(2\vec{a})$$

- The equations for magnetic field $\vec{B}$ due to a magnetic dipole can be obtained from the equations of electric field $\vec{E}$ due to an electric dipole by making the following changes:

  $$\vec{E} \to \vec{B}$$
  $$\vec{p} \to \vec{M}$$
  $$\frac{1}{4\pi\varepsilon_0} \to \frac{\mu_0}{4\pi}$$

  Thus, for any point on the axial line of a bar magnet at a distance d from the centre of the magnet,

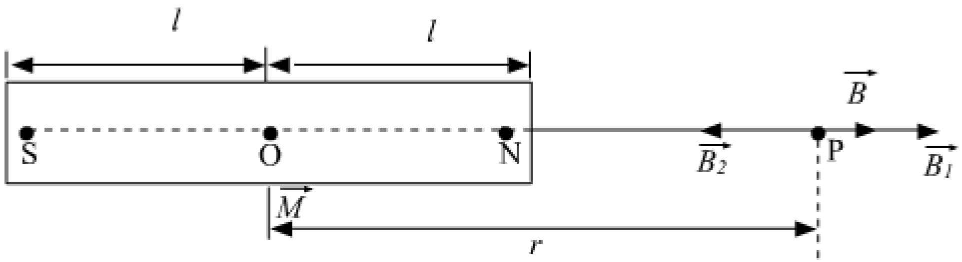
  $$B_A = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

  Similarly, the equatorial field B_E of a bar magnet at a distance d, for $d >> l$,

  $$B_E = \frac{\mu_0 M}{4\pi d^3}$$

Magnetic induction due to a bar magnet at a point along the axis of the bar magnet

Consider a bar magnet of dipole length 2l and magnet dipole moment M−→M→, as shown.



We need to find the magnetic induction at P.
Let *r* be the distance of point P from the centre of the magnet (O), and OS = ON = *l.*
∴ ∴ NP = (*r*−-*l*) and SP = (*r* + *l*).

$$B_1 = \frac{\mu_0}{4\pi}\frac{(+m)}{(\mathrm{NP})^2} = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2} \quad \ldots\ldots(1)$$

This field will be along NP.

Magnetic induction at P due to the S pole of pole strength (-*m*),

$$B_2 = \frac{\mu_0}{4\pi}\frac{(-m)}{(\mathrm{SP})^2} = \frac{\mu_0}{4\pi}\frac{(-m)}{(r+l)^2} \quad \ldots\ldots(2)$$

This field will be along PS.

As $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are acting along the same line, magnitude of *B* is given by,

$$B = \overrightarrow{B_1} + \overrightarrow{B_2}$$

$$\therefore\ B = \frac{\mu_0}{4\pi}\ \frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}$$

$$\therefore\ B = \frac{\mu_0\ m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$\therefore\ B = \frac{\mu_0\ m}{4\pi}\left[\frac{(r+l)^2 - (r-l)^2}{\left(r^2-l^2\right)^2}\right]$$

$$\therefore\ B = \frac{\mu_0 m.4d}{4\pi\left(r^2-l^2\right)^2}$$

$$\therefore\ B = \frac{\mu_0\ 2Md}{4\pi\left(r^2-1^2\right)^2}$$

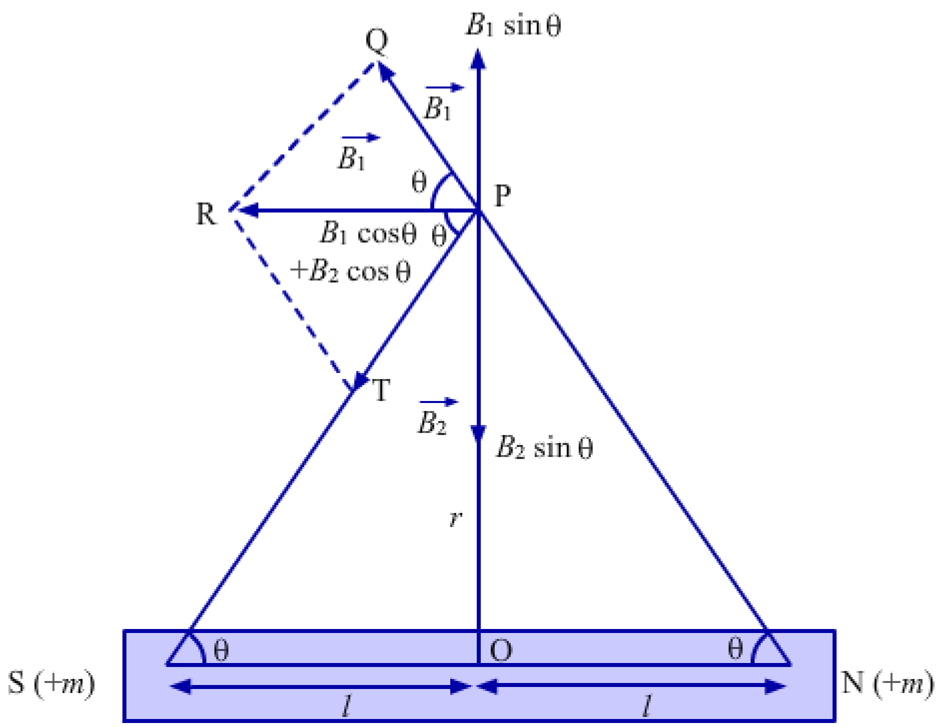
If (l^2 << d^2),

$$B = \frac{\mu_0\ 2M}{4\pi\ r^3}$$

This is directed along the axis of the magnet along S-N, i.e in the direction of the magnetic moment.

Magnetic induction at a point along the equator of a bar magnet.

Let P be the position on the equatorial line of the bar magnet and OP = r .
OS = ON = l
$NP^2 = SP^2 = r^2 + l^2$



If NP and SP make an angle θ with the axis of dipole.
Magnetic induction at P due to the N pole of the magnet,

$$B_1 = \frac{\mu_0}{4\pi}\frac{(+m)}{(\mathrm{NP})^2} = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)} \quad \ldots\ldots(1)$$

This field is along PQ.

Magnetic induction at P due to the S pole of the magnet,

$$B_2 = \frac{\mu_0}{4\pi}\frac{(-m)}{(SP)^2} = \frac{\mu_0}{4\pi}\frac{(-m)}{(r^2+l^2)} \quad \ldots\ldots(2)$$

This field is along PS.

In magnitude, $|B_1| = |B_2|$.

Components $B_1\cos\theta$ and $B_2\cos\theta$ are acting along the same direction and get added, whereas components $B_1\sin\theta$ and $B_2\sin\theta$ are acting in opposite directions. Hence, they will cancel each other. The resultant magnetic field at ,

$$B_{eq} = B_1 \cos\theta + B_2 \cos\theta$$
$$\therefore B_{eq} = 2B_1 \cos\theta \qquad (\because B_1 = B_2)$$
$$\therefore B_{eq} = 2\frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)} \times \frac{l}{\sqrt{r^2+l^2}}$$
$$\therefore B_{eq} = \frac{\mu_0}{4\pi}\frac{m\times 2l}{(r^2+l^2)^{\frac{3}{2}}}$$
$$\therefore B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{\frac{3}{2}}}$$

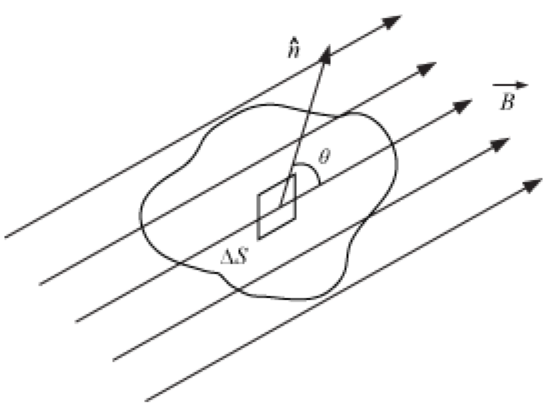
(If $l^2 << r^2$),

$$B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

This is directed along the axis of the magnet along N-S, i.e opposite the direction of magnetic moment.

Magnetism and Gauss Law

- **Statement –** The net magnetic flux (Φ_B) through any closed surface is always zero.
- This law suggests that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it.



- Suppose a closed surface S is held in a uniform magnetic field $\vec{B}$.

Consider a small vector area element $\Delta\vec{S}$ of this surface.

Magnetic flux through this area element is defined as $\Delta\phi_B = \vec{B}\cdot\Delta\vec{S}$

Considering all small area elements of the surface, we obtain net magnetic flux through the surface as:

$$\boxed{\phi_B = \sum_{\text{all}} \Delta\phi_B = \sum_{\text{all}} \vec{B}\cdot\Delta\vec{S} = 0}$$

Compare this with the Gauss law of electrostatics.

$$\sum \vec{E}\cdot\Delta\vec{S} = \frac{q}{\varepsilon_0}$$

The difference between the Gauss law of magnetism and electrostatics is that magnetic monopole doesn't exist.

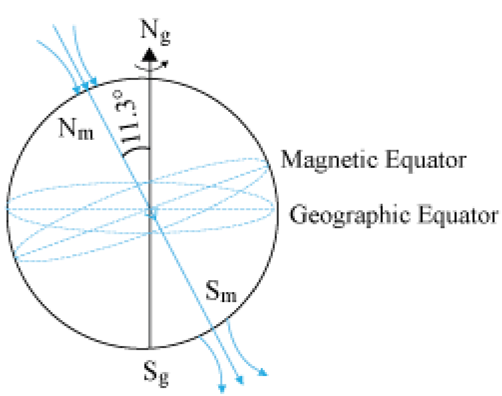
Earth's Magnetism

Earth's Magnetism

- **Dynamo effect –** The magnetic field of earth has arisen due to electrical currents produced by convective motion of metallic fluids in the outer core of the earth. This is known as the dynamo effect.

- The magnetic field lines of the earth resemble that of a magnetic dipole located at the centre of the earth. The axis of the dipole is presently tilted by approximately 11.3° with respect to the axis of rotation of earth.

- The North magnetic pole is located at latitude of 79.74° N and a longitude of 71.8° W, a place somewhere in North Canada. The magnetic South Pole is at 79.74° S and 108.22° E in Antarctica.



The pole near the geographic North Pole of the earth is called the North magnetic pole and the pole near the geographic South Pole is called the South magnetic pole.

- **Geographic meridian** – The vertical plane passing through the geographic North –South direction is called geographic meridian.

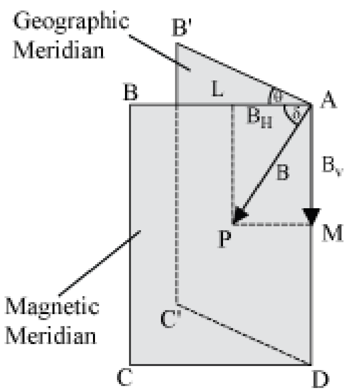
- **Magnetic Meridian** – The vertical plane passing through N – S line of a freely suspended magnet is called magnetic meridian.

Magnetic Declination and Dip (Or Magnetic Elements)

- **Magnetic elements** – The physical quantities, which determine the intensity of earth's total magnetic field completely (both in magnitude and direction), are called magnetic elements.

- There are three magnetic elements of earth:

- **Magnetic declination** – Declination at a place is the angle between the geographic meridian and magnetic meridian. It is denoted by θ.



- **Magnetic inclination or dip** – Dip at a place is defined as the angle made by the direction of the earth's total magnetic field with the horizontal direction. It is denoted by δ.

- **Horizontal component of earth's magnetic field** – It is the component of earth's magnetic field along the horizontal direction. It is denoted by B_H.

In the figure, for right-angled triangle ALP,

$$\cos\delta = \frac{AL}{AP} = \frac{B_H}{B}$$

$B_H = B\cos\delta$… (i)

Also, $\sin\delta = \frac{LP}{AP} = \frac{AM}{AP} = \frac{B_V}{B}$

$B_V = B\sin\delta$… (ii)

Squaring and adding the equations (i) and (ii), we obtain

$$B_H^2 + B_V^2 = B^2\cos^2\delta + B^2\sin^2\delta$$
$$B^2(\cos^2\delta + \sin^2\delta) = B_H{}^2 + B_V^2$$
$$B^2 = B_H^2 + B_V^2$$
$$B = \sqrt{B_H^2 + B_v^2} \qquad \text{...(iii)}$$

Dividing equation (ii) by (i), we obtain

$$\frac{B\sin\delta}{B\cos\delta} = \frac{B_V}{B_H}$$
$$\boxed{\tan\delta = \frac{B_V}{B_H}}$$

Magnetisation and Magnetic Intensity

- **Magnetic intensity –** It is given by the relation,

$$H = \frac{B_0}{\mu_0} \qquad \text{...(i)}$$

Where,

B_0 → Magnetic field inside vacuum

μ_0 → $4\pi \times 10^{-7}$ TmA^{-1}

SI unit → Am^{-1}

- **Intensity of magnetisation –** It is defined as the magnetic moment developed per unit volume when a magnetic specimen is subjected to magnetising field. It is denoted by *I*.

$$I = \frac{M}{V}$$

SI unit → Am^{-1}

- **Magnetic Induction –** It is defined as the number of magnetic lines of induction crossing per unit area through the magnetic substance. It is denoted by *B*.

Magnetic induction B is the sum of the magnetic field B_0 and the magnetic field $\mu_0 I$ produced due to the magnetisation of the substance. Thus,

$B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I$

$B = \mu_0 (H + I)$ ...(ii)

- **Magnetic susceptibility** – The magnetic susceptibility of a magnetic substance is defined as the ratio of the intensity of magnetisation to the magnetic intensity. It is denoted by χ_m.

$$\therefore \chi_m = \frac{I}{H} \qquad \text{...(iii)}$$

- **Magnetic permeability** – The magnetic permeability of a magnetic substance is defined as the ratio of the magnetic induction to the magnetic intensity. It is denoted by μ.

$$\therefore \mu = \frac{B}{H}$$

Now, on dividing both sides of the equation (ii) by H, we obtain

$$\frac{B}{H} = \mu_0 \left(1 + \frac{I}{H}\right)$$

From equation (iii), we have

$$\frac{B}{H} = \mu_0 (1 + \chi_m)$$

However, $\frac{B}{H} = \mu$

$$\therefore \boxed{\mu = \mu_0 (1 + \chi_m)}$$

Diamagnetic	**Paramagnetic**	**Ferromagnetic**
$-1 \leq \chi \leq 0$	$0 < \chi < \varepsilon$	$\chi >> 1$

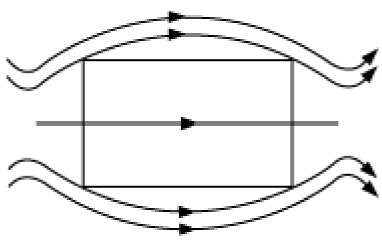
$0 \leq \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Magnetic Properties of Materials

- Materials can be classified as diamagnetic, paramagnetic, or ferromagnetic on the basis of susceptibility (χ).

  Here, ε is a small positive number introduced to quantify paramagnetic materials.

- **Diamagnetism** – Diamagnetic substances are those which have a tendency to move from stronger to the weaker part of the external magnetic field.



- When a bar of diamagnetic material is placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced.

- Explanation of Diamagnetism – Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero.

  When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. This happens due to induced current in accordance with Lenz law.

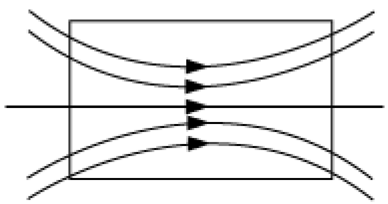
  Thus, the substance develops a net magnetic moment in direction opposite to that of the applied field and hence, repels.

- Examples of Diamagnetic materials – Bismuth copper, lead, silicon, nitrogen (at STP), water, and sodium chloride

- Meissner effect – Superconductors exhibit perfect diamagnetism. A superconductor repels a magnet and is repelled by the magnet. This phenomenon of perfect diamagnetism in superconductors is called the Meissner effect.

- **Paramagnetism** – Substances that have the tendency to move from a region of weak magnetic field to strong magnetic field i.e, they get weakly attracted to a magnet

- Explanation of paramagnetism – The atoms of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random motion of the atoms, no net magnetisation is seen. In the presence of an external field B_0, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0.

  The field lines get concentrated inside the material and the field inside is enhanced.

- Examples of paramagnetic materials – Aluminium, sodium, calcium, oxygen (at STP), and copper chloride

- Curie's law – Magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T.

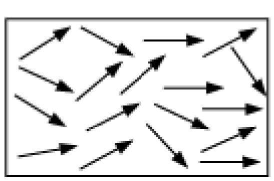
  $$M = C\frac{B_0}{T}$$

  $$\chi = C\frac{\mu_0}{T}$$

  Where, C is called Curie's constant

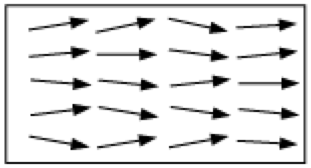
- **Ferromagnetism** – Substances which get strongly magnetised when placed in an external magnetic field

- Explanation of Ferromagnetism – The atoms in a ferromagnetic material possess a dipole moment aligned in a common direction over a macroscopic volume called domain. Each domain has a net magnetisation.

  Randomly oriented domains

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domain grows in size.

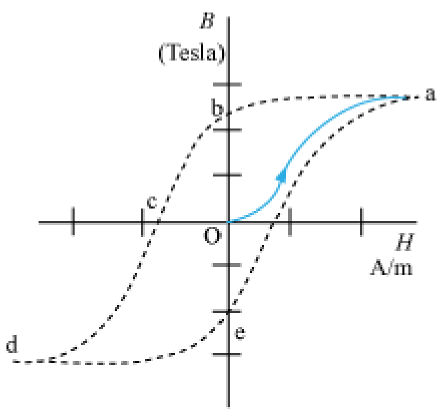


Aligned domains

- The ferromagnetic property depends on temperature. At high temperature, a ferromagnet becomes a paramagnet. The temperature of transition from ferromagnetic to paramagnetic is called the Curie temperature (T_C). The susceptibility in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - T_C} (T > T_C)$$

- Hysterisis



The above graph shows the behaviour of the material as we take it through one cycle of magnetisation.

An unmagnetised sample is placed in a solenoid and current through the solenoid is increased. The magnetic field *B* in the material rises and saturates as depicted in the curve *Oa*. Next, *H* is decreased and reduced to zero.

At $H = 0$, $B \neq 0$ (curve *ab*)

The value of *B* at $H = 0$ is called retentivity.

Now, the current in the solenoid is reversed and slowly increased. Certain domains are flipped until the net field inside stands nullified. This is represented by the curve *bc*.

The value of H at c is called coercivity. As the reversed current is increased in magnitude, we once again obtain saturation (curve cd).

Now, the current is reduced (curve de) and reversed (curve ea). The cycle repeats itself. For a given value of H, B is not unique, but depends on previous history of the sample. This phenomenon is called hysterisis.

Permanent Magnets and Electromagnets

Permanent Magnets

- Those substances which, at room temperature, retain their ferromagnetic property for a long period of time are called permanent magnets.
- The making of permanent magnets is an old art. One can hold an iron rod in the North−South direction and hammer it repeatedly.
- One can also hold a steel rod and strike its one end with a bar magnet continuously to make it a permanent magnet.
- An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and then pass current. The magnetic field of the solenoid magnetises the rod.
- The material used for making permanent magnets should have high retentivity—to make the magnet strong and highly coercive—so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage.
- Some suitable materials for making permanent magnets are alnico, cobalt steel and ticonal.

Electromagnets

- The core of an electromagnet is made of ferromagnetic materials, which have high permeability and low retentivity.
- The magnetism of an iron rod can be increased thousandfold by placing a soft iron rod in a solenoid and passing current.

Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery and bulk quantities of iron and steel.

Factors on which magnetic strength of electromagnet depends:

- Current passing through the solenoid
- Number of turns of the solenoid

- Permeability of the core of solenoid