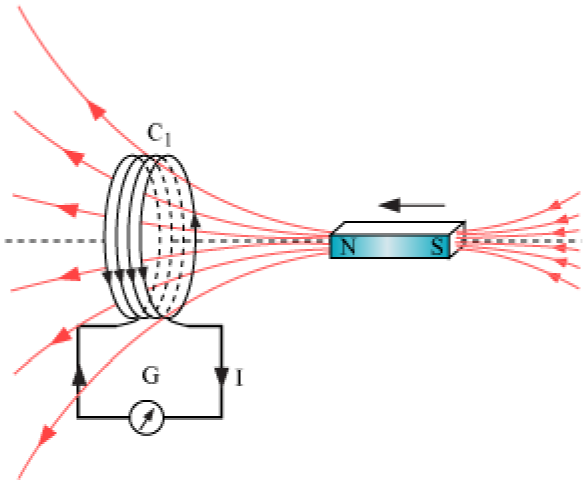
Electromagnetic Induction

Experiments of Faraday and Henry; Magnetic Flux; Faraday's Law of Induction

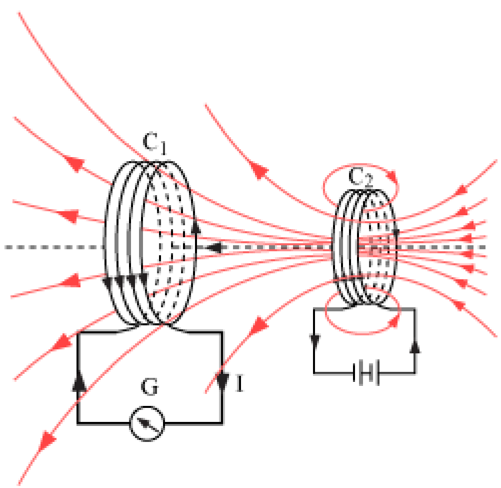
Faraday's Experiments

Experiment 1 – Current induced by a magnet:



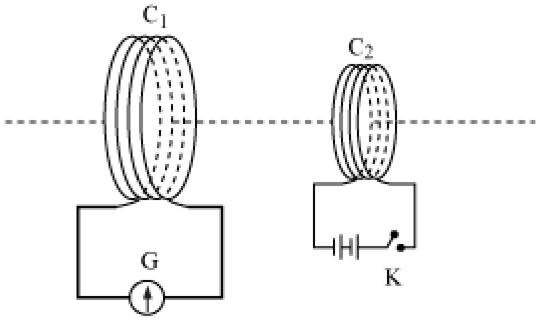
The relative motion between the magnet and the coil generates electric current in the coil. The current so generated is called induced current.

Experiment 2 – Current induced by a current:



When the bar magnet is replaced by a second coil C_2 , connected to a battery, the steady current in coil C_2 produces a steady magnetic field. As coil C_2 is moved towards coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coil C_1. When C_2 is moved away, the galvanometer shows a deflection again, but this time in the opposite direction. The deflection lasts as long as coil C_2 is in motion.

Experiment 3 – Current induced by changing current



The figure shows two coils C_1 and C_2 held stationary. Coil C_1 is connected to galvanometer G while the second coil C_2 is connected to a battery through a tapping key k.

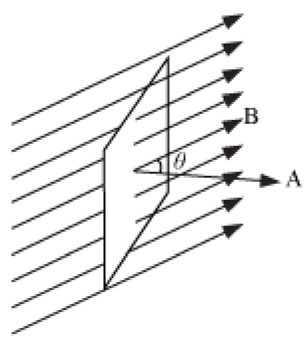
It is observed that the galvanometer shows a momentary deflection when the tapping key k is pressed. If the key is pressed continuously, there is no deflection in the galvanometer. When the key is released, a momentary deflection is observed again, but in the opposite direction.

Magnetic Flux

The magnetic flux Φ through any surface held in a magnetic field $\vec{B}$ is measured by the total number of magnetic lines of force crossing the surface.

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

Here, θ is the smaller angle between $\vec{B}$ and $\vec{A}$, which the normal to the surface area makes with $\vec{B}$



- **Unit of magnetic flux –** The SI unit of magnetic flux is Weber (Wb). One Weber is the amount of magnetic flux over an area of 1 m^2 held normal to a uniform magnetic field of one tesla.

  1 Weber = 1 tesla × 1 m^2

  The c.g.s unit of Φ is Maxwell (Mx).

  1 Weber = 10^8 Maxwell

Faraday's Laws of Electromagnetic Induction

First Law – Whenever the amount of magnetic flux linked with a circuit changes, an *emf* is induced in the circuit. The induced *emf* lasts as long as the change in magnetic flux continues.

Second Law – The magnitude of *emf* induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

According to Faraday's Second Law, induced *emf*,

$$e \propto \frac{(\phi_2 - \phi_1)}{t}$$

$$e = \frac{k(\phi_2 - \phi_1)}{t}$$

Here, *k* is the constant of proportionality.

$k = 1$

$$\therefore e = \frac{\phi_2 - \phi_1}{t}$$

If $d\Phi$ is a small change in magnetic flux in a small time dt,

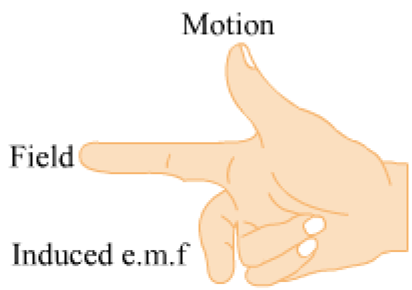
$$\boxed{e = \frac{-d\phi}{dt}}$$

For *N* turns,

$$\boxed{e = -N\frac{d\phi}{dt}}$$

Fleming's Right Hand Rule:
It is used to determine the direction of induced emf or current in a straight conductor when it is moved perpendicular to the magnetic field.

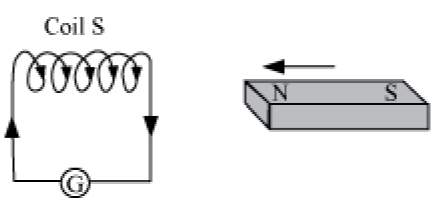
Statement:
The thumb, the first finger and the middle finger of the right hand are stretched, such that they are mutually perpendicular to each other. If the first finger is along the direction of the magnetic field and the thumb is along the direction of the motion of the conductor, the middle finger represents the direction of induced emf or current in the conductor.



Lenz Law and Conservation of Energy

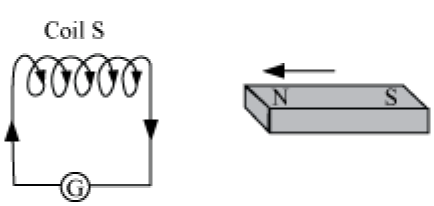
According to Lenz's Law, the polarity of the induced *emf* is such that it opposes the change in magnetic flux responsible for its production.

Experiment:



When the bar magnet with its N-pole is moved towards the coil, the induced current produced in coil S opposes the motion of the magnet. This will happen if the induced current in the coil S produces magnetic field lines from left to right, i.e. if the induced current flows through the coil S in the clockwise direction (when seen from left).

Lenz's Law and the Principle of Conservation of Energy

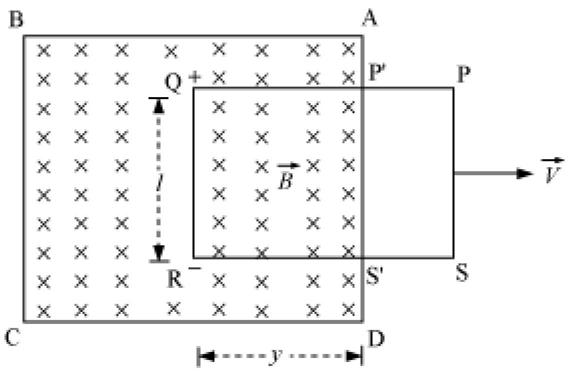


Lenz's Law is in accordance with the law of conservation of energy. In the above experiment, when the N-pole of the magnet is moved towards the coil, the right face of the coil acquires a north polarity. Thus, work has to be done against the force of repulsion in bringing the magnet closer to the coil.

When the N-pole of the magnet is moved away, the S-pole develops on the right face of the coil. Therefore, work has to be done against the force of attraction in taking the magnet away from the coil.

This mechanical work in moving the magnet with respect to the coil changes into electrical energy, producing induced current. Hence, energy transformation takes place.

Motional Electromotive Force



Consider that at any time *t*, the part $P'Q = S'R = y$ of the coil is inside the magnetic field. Let *l* be the length of the arm of the coil.

Area of the coil inside the magnetic field at time *t*,

$$\Delta S = QR \times RS' = ly$$

∴Magnetic flux linked with the coil at any time *t*,

$\Phi = B\Delta S = Bly$

The rate of change of magnetic flux linked with the coil is given by,

$$\frac{d\phi}{dt} = \frac{d}{dt}(Bly) = Bl\frac{dy}{dt} = Blv$$

Where,

v→Velocity with the coil pulled out of the magnetic field

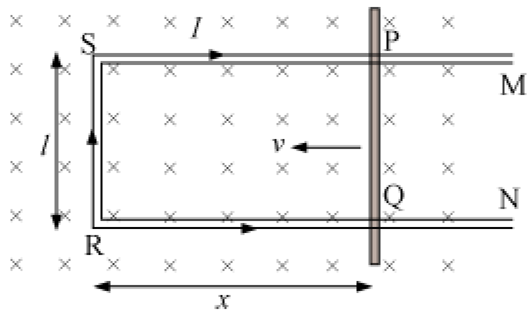
If *e* is the induced *em*f, then according to Faraday's law,

$$e = -\frac{d\phi}{dt}$$

$$\therefore \boxed{e = -Blv}$$

From Fleming's Right hand rule, the current due to induced *emf* will flow from the end R to Q i.e., along QPSR in the coil.

Energy Consideration



Let '*r*' be the resistance of movable arm PQ of the rectangular conductor. We assume that the remaining arms QR, RS, and SP have negligible resistances compared to *r*. Thus, the overall resistance of the rectangular loop is *r* and this does not change as PQ is moved.

Current *I* in the loop is,

$$I = \frac{\varepsilon}{r} = \frac{Blv}{r} \qquad \text{...(i)}$$

Due to the presence of the magnetic field, there is a force on the arm PQ. This force is directed outwards in the direction opposite to the velocity of the rod. The magnitude of this force is,

$$F = IlB = \frac{B^2 l^2 v}{r}$$

Alternatively, the arm PQ is being pushed with a constant speed *v*. The power required to do this is,

$$P = Fv = \frac{B^2 l^2 v^2}{r} \qquad \text{...(ii)}$$

The agent that does this work is mechanical. This mechanical energy is dissipated as joule heat and is given by,

$$P_J = I^2 r = \left(\frac{Blv}{r}\right)^2 r = \frac{B^2 l^2 v^2}{r}$$

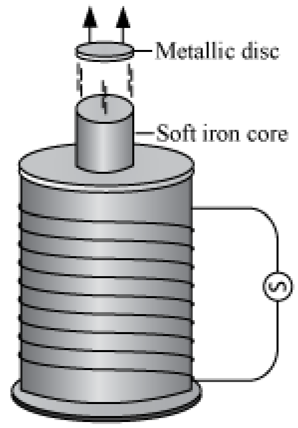
This is identical to equation (ii).

Thus, mechanical energy, which was required to move the arm PQ, is converted into electrical energy and then to thermal energy.

Eddy Currents

Eddy currents are the currents induced in a conductor when placed in a changing magnetic field.

- **Experiment to explain the origin of eddy currents**



Experiment – Introduce a soft iron core inside a solenoid and connect it to a source of alternating *emf*. Place a metallic disc over the cross-sectional face of the soft iron core. As the circuit is switched on, the metallic disc is thrown up into the air.

Explanation – When the circuit is switched on, the current starts growing through the solenoid. As the current grows, the magnetic field lines along the axis of the solenoid, and hence the magnetic flux through the disc, also increases from zero to some finite value. Due to this, induced currents are produced in the disc. According to Lenz law, the induced current set up in the disc acts in such a manner that it opposes the increase of magnetic flux through it. As a result, the disc is thrown up.

Applications of Eddy Currents

- Electromagnetic damping – Some galvanometers have a fixed core, which is made of non-magnetic metallic materials. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

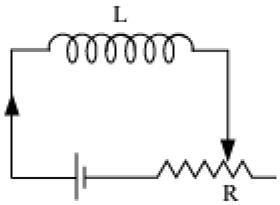
- Induction furnace – In an induction furnace, very high temperature can be produced by producing large eddy currents.

- Magnetic braking in trains – Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

Inductance

Self-induction – Property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an *emf* in itself

Self-induction is also called the inertia of electricity.



- Coefficient of self-induction:

  Suppose I = Current flowing in the coil at any time

  Φ = Amount of magnetic flux linked

  It is found that $\Phi \propto I$

  $\boxed{\phi = LI}$

  Where,

  L is the constant of proportionality and is called coefficient of self induction of the coil

  The *emf* induced in the coil is given by,

  $$e = -\frac{d\phi}{dt} = \frac{-d}{dt}(LI)$$

  $$\boxed{e = -L\frac{dI}{dt}}$$

  If $\frac{dI}{dt} = 1$, then

  $$e = -L \times 1$$

  $$\boxed{L = -e}$$

- SI unit of self-inductance is Henry.

- 1 Henry = $\frac{1\,\text{Volt}}{1\ \text{Ampere/Sec}}$ = 1 Weber/Ampere

Self-Inductance of a Long Solenoid

A long solenoid is one whose length is very large as compared to its radius of cross-section. The magnetic field B at any point inside such a solenoid is practically constant and is given by,

$$B = \frac{\mu_0 NI}{l} \qquad ...(i)$$

Where,

μ_0 = Absolute magnetic permeability of free space

N = Total number of turns in the solenoid

∴ Magnetic flux through each turn of the solenoid coil = B × Area of each turn

$$B = \left(\mu_0 \frac{N}{l} I\right) A$$

Where,

A = Area of each turn of the solenoid

Total magnetic flux linked with the solenoid

$$\phi = \mu_0 \frac{N}{l} IA \times N \qquad ...(ii)$$

If L is coefficient of self-inductance of the solenoid, then

$\therefore \Phi = LI$... (iii)

Equating (ii) and (iii),

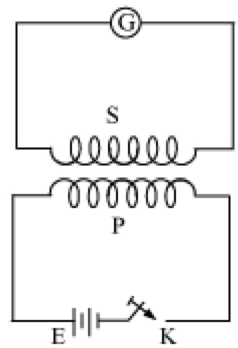
$$LI = \mu_0 \frac{N}{l} IA \times N$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

If core is of any other magnetic material, μ_0 is replaced by $\boxed{\mu = \mu_0 \mu_r}$

Mutual Induction

The phenomenon according to which an opposing *emf* is produced in a coil as a result of change in current, hence, the magnetic flux linked with a neighbouring coil is called mutual induction.



- Coefficient of mutual induction − Consider two coils P and S. Suppose that a current I is flowing through the coil P at any instant i.e.,

  $\Phi \propto I$

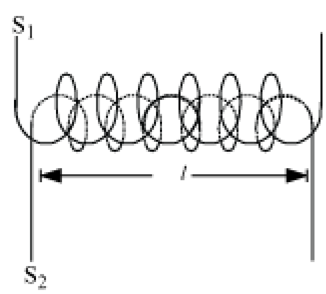
  $\Phi = MI$… (i)

  Where, M is called coefficient of mutual induction

  If 'e' is the induced *emf* produced in the S-coil, then

  $$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

  Mutual Inductance of Two Long Solenoids



Consider two long solenoids S_1 and S_2 of same length l, such that solenoid S_2 surrounds solenoid S_1 completely.

Let

n_1 – Number of turns per unit length of S_1

n_2 – Number of turns per unit length of S_2

I_1 – Current passed through solenoid S_1

Φ_{21} – Flux linked with S_2 due to current flowing through S_1

$\Phi_{21} \propto I_1$

$\Phi_{21} = M_{21}I_1$

Where, M_{21} is the coefficient of mutual induction of the two solenoids

When current is passed through solenoid S_1, an *emf* is induced in solenoid S_2.

Magnetic field produced inside solenoid S_1 on passing current through it,

$B_1 = \mu_0 n_1 I_1$

Magnetic flux linked with each turn of solenoid S_2 will be equal to B_1 times the area of cross-section of solenoid S_1.

Magnetic flux linked with each turn of the solenoid $S_2 = B_1 A$

Therefore, total magnetic flux linked with the solenoid S_2,

$\Phi_{21} = B_1 A \times n_2 l = \mu_0 n_1 I_1 \times A \times n_2 l$

$\Phi_{21} = \mu_0 n_1 n_2 A I_1$

$\therefore M_{21} = \mu_0 n_1 n_2 A l$

Similarly, the mutual inductance between the two solenoids, when current is passed through solenoid S_2 and induced *emf* is produced in solenoid S_1, is given by

$M_{12} = \mu_0 n_1 n_2 A l$

$\therefore M_{12} = M_{21} = M$ (say)

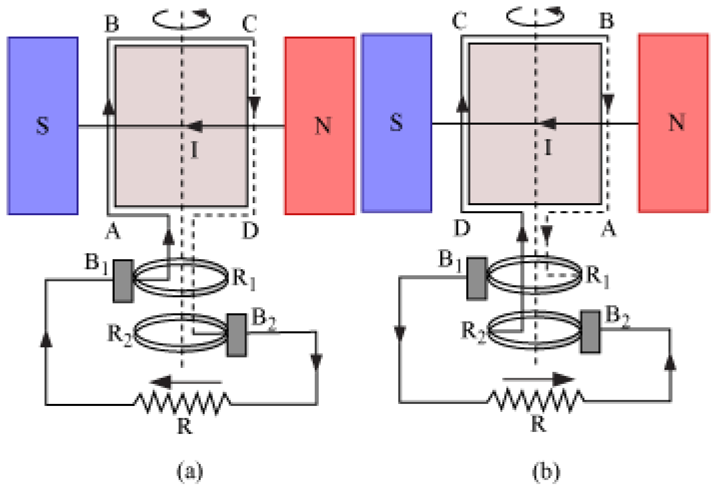
Hence, coefficient of mutual induction between the two long solenoids

$$\boxed{M = \mu_0 n_1 n_2 A l}$$

AC Generator

Principle – Based on the phenomenon of electromagnetic induction

Construction



Main parts of an ac generator:

- Armature – Rectangular coil ABCD
- Filed Magnets – Two pole pieces of a strong electromagnet
- Slip Rings – The ends of coil ABCD are connected to two hollow metallic rings R_1 and R_2.
- Brushes – B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in tight contact with R_1 and R_2 respectively.

Theory and Working – As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes. An *emf* is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD. In the external circuit, current flows from B_2 to B_1.

To calculate the magnitude of *emf* induced:

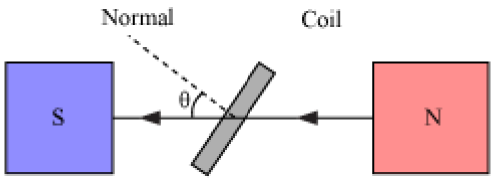
Suppose

$A \rightarrow$ Area of each turn of the coil

$N \rightarrow$ Number of turns in the coil

$\vec{B} \rightarrow$ Strength of magnetic field

$\theta \rightarrow$ Angle which normal to the coil makes with $\vec{B}$ at any instant t



∴ Magnetic flux linked with the coil in this position:

$\phi = N(\vec{B} \cdot \vec{A})$ = NBA cosθ= NBA cosωt …(i)

Where, 'ω' is angular velocity of the coil

As the coil rotates, angle θchanges. Therefore, magnetic flux Φ linked with the coil changes and hence, an *emf* is induced in the coil. At this instant t, if e is the *emf* induced in the coil, then

$$e = -\frac{d\theta}{dt} = -\frac{d}{dt}(NAB\ \cos\omega t)$$
$$= -NAB\frac{d}{dt}(\cos\omega t)$$
$$= -NAB(-\sin\omega t)\omega$$

∴e = NAB ω sinωt