

Proportion

EXERCISE 12(A)

Question 1.

In each of the following, check whether or not the given ratios form a proportion :

(i) $8 : 16$ and $12 : 15$

(ii) $16 : 28$ and $24 : 42$

(iii) $12 \div 3$ and $8 \div 2$

(iv) $25 : 40$ and $20 : 32$

(v) $\frac{15}{18}$ and $\frac{10}{12}$

(vi) $\frac{7}{8}$ and $14 : 16$

Solution:

(i) $8 : 16$ and $12 : 15$

$$\text{Since } 8 : 16 = \frac{8}{16} = \frac{1}{2}$$

$$\text{and } 12 : 15 = \frac{12}{15} = \frac{4}{5}$$

$\therefore$ Ratio $8 : 16 \neq$ ratio $12 : 15$, they are not in a proportion.

(ii) $16 : 28$ and $24 : 42$

$$\text{Since } 16 : 28 = \frac{16}{28} = \frac{4}{7}$$

$$\text{and } 24 : 42 = \frac{24}{42} = \frac{4}{7}$$

$\therefore$ Ratio $16 : 28$ and $24 : 42$ are equal, so they form a proportion.

(iii) $12 \div 3$ and $8 \div 2$

$$\text{Since } \frac{12}{3} = 4 \text{ and } \frac{8}{2} = 4$$

$\therefore$ Ratio $12 \div 3$ and $8 \div 2$ are equal, so they form a proportion.

(iv) $25 : 40$ and $20 : 32$

$$\text{Since } 25 : 40 = \frac{25}{40} = \frac{5}{8}$$

$$\text{and } 20 : 32 = \frac{20}{32} = \frac{5}{8}$$

$\therefore$ Ratio 25 : 20 and 20 : 32 are equal, so they form a proportion.

$$(v) \frac{15}{18} \text{ and } \frac{10}{12}$$

$$\text{Since } \frac{15}{18} = \frac{5}{6} \text{ and } \frac{10}{12} = \frac{5}{6}$$

$\therefore$ Ratio $\frac{15}{18}$ and $\frac{10}{12}$ are equal, so they form a proportion.

$$(vi) \frac{7}{8} \text{ and } 14 : 16$$

$$\text{Since } \frac{7}{8} = \frac{7}{8} \text{ and } 14 : 16 = \frac{14}{16} = \frac{7}{8}$$

$\therefore$ Ratio $\frac{7}{8}$ and 14 : 16 are equal, so they form a proportion.

Question 2.

Find the value of x in each of the following proportions :

$$(i) x : 4 = 6 : 8$$

$$(ii) 14 : x = 7 : 9$$

$$(iii) 4 : 6 = x : 18$$

$$(iv) 8 : 10 = x : 25$$

$$(v) 5 : 15 = 4 : x$$

$$(vi) 16 : 24 = 6 : x$$

Solution:

$$x : 4 = 6 : 8$$

$$\Rightarrow x \times 8 = 4 \times 6$$

$$\Rightarrow x = \frac{4 \times 6}{8} = 3$$

$$(ii) 14 : x = 7 : 9$$

$$\Rightarrow x \times 7 = 14 \times 9$$

$$\Rightarrow x = \frac{14 \times 9}{7} = 18$$

$$(iii) \quad 4 : 6 = x : 18$$

$$\Rightarrow 6 \times x = 4 \times 18$$

$$\Rightarrow x = \frac{4 \times 18}{6} = 12$$

$$(iv) \quad 8 : 10 = x : 25$$

$$\Rightarrow 10 \times x = 25 \times 8 \Rightarrow x = \frac{25 \times 8}{10} = 20$$

$$(v) \quad 5 : 15 = 4 : x$$

$$\Rightarrow 5 \times x = 15 \times 4 \Rightarrow x = \frac{15 \times 4}{5} = 12$$

$$(vi) \quad 16 : 24 = 6 : x$$

$$\Rightarrow 16 \times x = 24 \times 6 \Rightarrow x = \frac{24 \times 6}{16} = 9$$

Question 3.

Find the value of x so that the given four numbers are in proportion :

(i) x , 6, 10 and 15

(ii) x , 4, 15 and 30

(iii) 2, x , 10 and 25

(iv) 4, x , 6 and 18

(v) 9, 12, x and 8

(vi) 4, 10, 36 and x

(vii) 7, 21, x and 45

(viii) 6, 8, 12 and x .

Solution:

$$(i) x : 6 : 10 : 15$$

$$\Rightarrow x \times 15 = 6 \times 10 \Rightarrow x = \frac{6 \times 10}{15} = 4.$$

$$(ii) x : 4 : 15 : 30$$

$$\Rightarrow x \times 30 = 4 \times 15 \Rightarrow x = \frac{4 \times 15}{30} = 2.$$

$$(iii) 2 : x : 10 : 25$$

$$\Rightarrow x \times 10 = 2 \times 25 \Rightarrow x = \frac{2 \times 25}{10} = \frac{25}{5} = 5.$$

$$(iv) 4 : x : 6 : 18$$

$$\Rightarrow x \times 6 = 18 \times 4 \Rightarrow x = \frac{18 \times 4}{6} = 12.$$

$$(v) 9 : 12 : x : 8$$

$$\Rightarrow 12 \times x = 9 \times 8 \Rightarrow x = \frac{9 \times 8}{12} = 6.$$

$$(vi) 4 : 10 : 36 : x$$

$$\Rightarrow 4 \times x = 10 \times 36 \Rightarrow x = \frac{10 \times 36}{4} = 90.$$

$$(vii) 7 : 21 : x : 45$$

$$\Rightarrow 21 \times x = 7 \times 45$$

$$\Rightarrow x = \frac{7 \times 45}{21} = \frac{45}{3} = 15.$$

$$(viii) 6 : 8 : 12 : x$$

$$\Rightarrow 6 \times x = 12 \times 8 \Rightarrow x = \frac{12 \times 8}{6} = 16.$$

Question 4.

The first, second and the fourth terms of a proportion are 6, 18 and 75, respectively. Find its third term.

Solution:

Let the third term = x

$$6 : 18 :: x : 75$$

$$= 18 \times x = 6 \times 75$$

$$x = \frac{6 \times 75}{18} = \frac{75}{3} = 25$$

The third term of proportion is 25

Question 5.

Find the second term of the proportion whose first, third and fourth terms are 9, 8 and 24 respectively.

Solution:

Let the second term = x

$$9 : x :: 8 : 24$$

$$\Rightarrow x \times 8 = 24 \times 9$$

$$x = \frac{24 \times 9}{8} = 3 \times 9 = 27$$

The second term of proportion = 27

Question 6.

Find the fourth term of the proportion whose first, second and third terms are 18, 27, and 32 respectively.

Solution:

Let the fourth term = x

$$18 : 27 :: 32 : x$$

$$\Rightarrow 18 \times x = 27 \times 32$$

$$\Rightarrow x = \frac{27 \times 32}{18} = 3 \times 16 = 48$$

Fourth term = 48

Question 7.

The ratio of the length and the width of a school ground is 5 : 2. Find the length, if the width is 40 metres.

Solution:

Let the length = x m,

width = 40 m

The ratio of length to width = x : 40

as per given statement 5 : 2 = x : 40

$$\Rightarrow 2 \times x = 40 \times 5$$

$$x = \frac{40 \times 5}{2} = 20 \times 5 = 100 \text{ m}$$

Question 8.

The ratio of the sale of eggs on a Sunday and that of the whole week at a grocery shop was 2 : 9. If the total value of the sale of eggs in the same week was Rs 360, find the value of the sale of eggs that Sunday.

Solution:

Let, the sale of eggs on Sunday = x

Sale in week = Rs 360

According to question, 2 : 9 = x : 360

$$\Rightarrow 9 \times x = 360 \times 2$$

$$x = \frac{360 \times 2}{9} = \text{Rs } 80$$

Sale on Sunday = Rs 80

Question 9.

The ratio of copper and zinc in an alloy is 9 : 8. If the weight of zinc, in the alloy, is 9.6 kg ; find the weight of copper in the alloy.

Solution:

Let the weight of copper = x kg

Weight of zinc = 9.6 kg.

According to question,

$$9 : 8 = x : 9.6$$

$$\Rightarrow 8 \times x = 9 \times 9.6$$

$$\Rightarrow x = \frac{9 \times 9.6}{8} = 9 \times 1.2 = 10.8 \text{ kg.}$$

Weight of copper in alloy = 10.8

Question 10.

The ratio of the number of girls to the number of boys in a school is 2 : 5. If the number of boys is 225 ; find:

(i) the number of girls in the school.

(ii) the number of students in the school.

Solution:

Let, the number of girls in school = x

Number of boys in school = 225

According to question $2 : 5 = x : 225$

$$\Rightarrow 5 \times x = 2 \times 225$$

$$x = \frac{2 \times 225}{5} = 2 \times 45 = 90$$

Number of girls in school = 90

Total number of student in the school = (number of boys + number of girls) = (225 + 90) = 315

Question 11.

In a class, one out of every 5 students pass. If there are 225 students in all the sections of a class, find how many pass ?

Solution:

Total number of students in all sections = 225

Given, One of every five students pass

$$\text{Total students pass} = 225 \times \frac{1}{5} = 45 \text{ students}$$

Question 12.

Make set of all possible proportions from the numbers 15, 18, 35 and 42.

Solution:

The possible proportions that can be made from the numbers 15, 18, 35 and 42 are

(i) $15 : 35 :: 18 : 42$

(ii) $42 : 18 :: 35 : 15$

(iii) $42 : 35 :: 18 : 15$

(iv) $15 : 18 :: 35 : 42$

EXERCISE 12(B)

Question 1.

If x , y and z are in continued proportion, then which of the following is true :

- (i) $x : y = x : z$
- (ii) $x : x = z : y$
- (iii) $x : y = y : z$
- (iv) $y : x = y : z$

Solution:

- (iii) $x : y = y : z$

Question 2.

Which of the following numbers are in continued proportion :

- (i) 3, 6 and 15
- (ii) 15, 45 and 48
- (iii) 6, 12 and 24
- (iv) 12, 18 and 27

Solution:

- (iii) and (iv)

Question 3.

Find the mean proportion between

- (i) 3 and 27
- (ii) 0.06 and 0.96

Solution:

(i) Mean proportional between 3 and 27

$$= \sqrt{3 \times 27} = \sqrt{81} = 9$$

(ii) Mean proportional between 0.6 and 9.6

$$= \sqrt{0.6 \times 9.6} = \sqrt{\frac{6}{10} \times \frac{96}{10}}$$

$$= \sqrt{\frac{576}{100}} = \frac{24}{10} = 2.4$$

Question 4.

Find the third proportional to :

- (i) 36, 18
- (ii) 5.25, 7
- (iii) ₹ 1.60, ₹ 0.40

Solution:

(i) Let the required third proportional be x

$\therefore 36, 18, x$ are in continued proportion

$$\Rightarrow 36 : 18 = 18 : x$$

$$\Rightarrow 36x = 18 \times 18$$

$$\Rightarrow x = \frac{18 \times 18}{36}$$

$$\Rightarrow x = 9$$

$\therefore$ Required proportional = 9

(ii) Let the required third proportional be x

$\therefore 5.25, 7, x$ are in continued proportion

$$\Rightarrow 5.25 : 7 = 7 : x$$

$$\Rightarrow 5x = 7 \times 7$$

$$\Rightarrow x = \frac{7 \times 7}{5.25}$$

$$\Rightarrow x = \frac{49}{5.25} = \frac{28}{3}$$

$$\Rightarrow x = 9\frac{1}{3}$$

(iii) Let the required third proportional be x

$\therefore ₹ 1.60, ₹ 0.40, ₹ x$ are in continued proportion.

$$\Rightarrow 1.60 \times x = 0.40 \times 0.40$$

$$\Rightarrow x = \frac{0.40 \times 0.40}{1.60}$$

$$\Rightarrow x = 0.1$$

Question 5.

The ratio between 7 and 5 is same as the ratio between ₹ x and ₹ 20.50 ; find the value of x .

Solution:

Since, It is given that the ratio between 7 and 5 is same as the ratio between ₹ x and ₹

20.50

$$\therefore 7 : 5 = x = 20.50$$

$$\Rightarrow 5x = 7 \times 20.50$$

$$\Rightarrow x = \frac{7 \times 20.50}{5}$$

$$\Rightarrow x = 82.7$$

Question 6.

If $(4x + 3y) : (3x + 5y) = 6 : 7$, find :

(i) $x : y$

(ii) x , if $y = 10$

(iii) y , if $x = 27$

Solution:

$$(i) \quad 7x(4x + 3y) = 6x(3x + 5y)$$

$$28x + 21y = 18x + 30y$$

$$28x - 18x = 30y - 21y$$

$$10x = 9y$$

$$\frac{x}{y} = \frac{9}{10}$$

$$\therefore x : y = 9 : 10$$

$$(ii) \quad (4x + 3y) : (3x + 5y) = 6 : 7$$

$$\text{Given, } y = 10$$

$$\therefore (4x + 3 \times 10) : (3x + 5 \times 10) = 6 : 7$$

$$(4x + 30) : (3x + 50) = 6 : 7$$

$$7 \times (4x + 30) = 6 \times (3x + 50)$$

$$28x + 210 = 18x + 300$$

$$28x - 18x = 300 - 210$$

$$10x = 90$$

$$\Rightarrow x = \frac{90}{10} = 9$$

$$(iii) \quad (4x + 3y) : (3x + 5y) = 6 : 7$$

$$\text{Given, } x = 27$$

$$\therefore (4 \times 27 + 3y) : (3 \times 27 + 5y) = 6 : 7$$

$$(108 + 3y) : (81 + 5y) = 6 : 7$$

$$7 \times (108 + 3y) = 6 \times (81 + 5y)$$

$$756 + 21y = 486 + 30y$$

$$9y = 270$$

$$\Rightarrow y = \frac{270}{9} = 30$$

Question 7.

If $\frac{2y+5x}{3y-5x} = 2\frac{1}{2}$, find:

(i) $x : y$

(ii) x , if $y = 70$

(iii) y , if $x = 33$

Solution:

$$(i) \quad \frac{2y+5x}{3y-5x} = \frac{2 \times 2 + 1}{2}$$

$$\frac{2y+5x}{3y-5x} = \frac{5}{2}$$

$$\Rightarrow 2(2y + 5x) = 5 \times (3y - 5x)$$

$$\Rightarrow 4y + 10x = 15y - 25x$$

$$\Rightarrow 35x = 11y$$

$$\Rightarrow \frac{x}{y} = \frac{11}{35} \quad \text{i.e. } x : y = 11 : 35$$

$$(ii) \quad \frac{2y+5x}{3y-5x} = \frac{5}{2}$$

$$\text{Given } y = 70$$

$$\frac{2 \times 70 + 5x}{3 \times 70 - 5x} = \frac{5}{2} \Rightarrow \frac{140 + 5x}{210 - 5x} = \frac{5}{2}$$

$$\Rightarrow 2 \times (140 + 5x) = 5 \times (210 - 5x)$$

$$\Rightarrow 280 + 10x = 1050 - 25x$$

$$\Rightarrow 35x = 1050 - 280$$

$$\Rightarrow 35x = 770 \quad \Rightarrow x = \frac{770}{35} = 22$$

$$(iii) \quad \frac{2y + 5x}{3y - 5x} = \frac{5}{2}$$

Given $x = 33$

$$\frac{2y + 5 \times 33}{3y - 5 \times 33} = \frac{5}{2} \Rightarrow \frac{2y + 165}{3y - 165} = \frac{5}{2}$$

$$\Rightarrow 2 \times (2y + 165) = 5 \times (3y - 165)$$

$$\Rightarrow 4y + 330 = 15y - 825$$

$$\Rightarrow 11y = 1155$$

$$\Rightarrow y = \frac{1155}{11} = 105$$

EXERCISE 12(C)

Question 1.

Are the following numbers in proportion:

(i) 32, 40, 48 and 60 ?

(ii) 12, 15, 18 and 20 ?

Solution:

(i) 32, 40, 48 and 60 are in proportion

if $32 : 40 = 48 : 60$

if $32 \times 60 = 40 \times 48$

$$\left\{ \frac{a}{b} = \frac{c}{d} \implies ad = bc \right\}$$

if $1920 = 1920$

Which is true.

32, 40, 48 and 60 are in proportion

(ii) 12, 15, 18 and 20 are in proportion

if $12 : 15 = 18 : 20$

if $12 \times 20 = 15 \times 18$ { $ad = bc$ }

if $240 = 270$

which is not true.

12, 15, 18 and 20 are not in proportion.

Question 2.

Find the value of x in each of the following such that the given numbers are in proportion.

(i) 14, 42, x and 75

(ii) 45, 135, 90 and x

Solution:

14, 42, x and 75 are in proportion

$$\frac{14}{42} = \frac{x}{75}$$

$$\Rightarrow 14 \times 75 = x \times 42$$

$$\Rightarrow x = \frac{14 \times 75}{42} = 25$$

$$\therefore x = 25$$

(ii) $\because$ 45, 135, 90 and x are in proportion

$$\therefore \frac{45}{135} = \frac{90}{x} \Rightarrow 45 \times x = 90 \times 135$$

$$\Rightarrow x = \frac{90 \times 135}{45} = 270$$

$$\therefore x = 270$$

Question 3.

The costs of two articles are in the ratio 7 : 4. If the cost of the first article is Rs. 2,800 ; find the cost of the second article.

Solution:

Ratio in the cost of two articles = 7 : 4

Cost of first article = Rs. 2800

Let cost of the second article = x

$$7 : 4 = 2800 : x$$

$$\Rightarrow \frac{7}{4} = \frac{2800}{x} \Rightarrow 7 \times x = 2800 \times 4$$

$$\Rightarrow x = \frac{2800 \times 4}{7} = 1600$$

$$\therefore \text{Cost of second article} = \text{Rs. } 1600$$

Question 4.

The ratio of the length and the width of a rectangular sheet of paper is 8 : 5. If the width

of the sheet is 17.5 cm; find the length.

Solution:

Let length of sheet = x cm

Ratio in length and breadth = $8 : 5$

and width = 17.5 cm

$$8 : 5 = x : 17.5$$

$$\Rightarrow \frac{8}{5} = \frac{x}{17.5} \Rightarrow 8 \times 17.5 = x \times 5$$

$$\Rightarrow x = \frac{8 \times 17.5}{5} = 8 \times 3.5 = 28$$

Length of sheet = 28 cm

Question 5.

The ages of A and B are in the ratio $6 : 5$. If A's age is 18 years, find the age of B.

Solution:

Ratio in the ages of A and B = $6 : 5$

A's age = 18 years

Let B's age = x years

$$6 : 5 = 18 : x$$

$$\Rightarrow \frac{6}{5} = \frac{18}{x} \Rightarrow 6 \times x = 18 \times 5$$

$$\Rightarrow x = \frac{18 \times 5}{6} = 15$$

$\therefore$ B's age = 15 years.

Question 6.

A sum of Rs. 10, 500 is divided among A, B and C in the ratio $5 : 6 : 4$. Find the share of each.

Solution:

Total amount = Rs. 10, 500

Ratio in A, B, and C = $5 : 6 : 4$

Sum of ratio = $5 + 6 + 4 = 15$

$$\therefore \text{A's share} = \text{Rs. } \frac{10500}{15} \times 5$$

$$= \text{Rs. } 700 \times 5 = \text{Rs. } 3500$$

$$\text{B's share} = \text{Rs. } \frac{10500 \times 6}{15}$$

$$= \text{Rs. } 700 \times 6$$

$$= \text{Rs. } 4200$$

$$\text{and C's share} = \text{Rs. } \frac{10500 \times 4}{15}$$

$$= \text{Rs. } 700 \times 4 = \text{Rs. } 2800$$

Question 7.

Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution:

15 cm : 2 m :: 10 sec : 3 min

15 cm : 2×100 cm :: 10 sec : 30×60 sec

15 : 200 :: 10 : 1800

3 : 40 :: 1 : 180

No, they do not form a proportion

Question 8.

Do the ratios 2 kg : 80 kg and 25 g : 625 g form a proportion ?

Solution:

2 kg : 80 kg :: 25 g : 625 g

2 : 80 :: 25 : 625

1 : 40 :: 1 : 25

No, they do not form a proportion.

Question 9.

10 kg sugar cost ₹ 350. If x kg sugar of the same kind costs ₹ 175, find the value of x

Solution:

10 kg of sugar costs = ₹ 350

and x kg of sugar cost = ₹ 175

A.T.Q.

10 kg : x kg :: 350 : 175

$\Rightarrow 10 \times 175 = 350 \times x$

$\Rightarrow 350x = 1750$

$\Rightarrow x = \frac{1750}{350} = 5$

Hence, 5 kg of sugar costs ₹ 175

Question 10.

The length of two ropes are in the ratio 7 : 5. Find the length of:

(i) shorter rope, if the longer one is 22.5 m

(ii) longer rope, if the shorter is 9.8 m.

Solution:

Length of the ropes are in the ratio = 7 : 5

(i) Let the length of shorter rope = x

Length of longer rope = 22.5 m

A.T.Q.

$$7 : 5 = 22.5 : x$$

$$\Rightarrow 7x = 22.5 \times 5$$

$$\Rightarrow x = \frac{22.5 \times 5}{7}$$

$$\Rightarrow x = 16.07 \text{ m}$$

(ii) Let length of the longer side = x

length of shorter rope = 9.8 m

A.T.Q.

$$7 : 5 = x : 9.8$$

$$\Rightarrow 5 \times x = 9.8 \times 7$$

$$\Rightarrow x = \frac{9.8 \times 7}{5}$$

$$\Rightarrow x = 13.72 \text{ m}$$

Question 11.

If 4, x and 9 are in continued proportion, find the value of x.

Solution:

4, x and 9 are in continued proportion

$$\Rightarrow 4 : x = x : 9$$

$$\Rightarrow x^2 = 9 \times 4$$

$$\Rightarrow x = \sqrt{36}$$

$$x = 6$$

Question 12.

If 25, 35 and x are in continued proportion, find the value of x.

Solution:

25, 35 and x are in continued proportion

$$\Rightarrow 25 : 35 = 35 : x$$

$$\Rightarrow 25 \times x = 35 \times 35$$

$$\Rightarrow x = \frac{35 \times 35}{25}$$