Substitution

IMPORTANT POINTS

1. **Substitution :** The value of an expression depends on the value of its variable (s).
2. **Use of Brackets :**
   The Symbols —, ( ), { }, [ ] are called brackets.
   If an expression is enclosed within a bracket, it is considered a single quantity, even if it is made up of many terms.
   Keep in Mind :
   - While simplifying an expression containing a bracket, first of all, the terms inside the bracket are operated (combined).
   - ( ) is called a small bracket or Parenthesis.
   - { } is called a middle bracket or Curly bracket.
   - [ ] is called big or square bracket.
   - If one more bracket is needed, then we use the bar bracket.
     i.e. a line ———— is drawn over a group of terms.
     Thus, in $3x + 4y - 5z$, the line over 4y – 5z serves as the bar bracket and is called Vinculum.

EXERCISE 20(A)

Question 1.
Fill in the following blanks, when :
x = 3,y = 6, z = 18, a = 2, b = 8, c = 32 and d = 0.

(i) $x + y =$

(ii) $y - x =$ .....

(iii) $\frac{y}{x} =$ .....

(iv) $c \div b =$ .....

(v) $z \div x =$ .....

(vi) $y \times d =$ .....

(vii) $d \div x =$ .....

(viii) $ab + y =$ .....

(ix) $a + b + x =$ .....

(x) $b + z - d =$ .....

(xi) $a - b + y =$ .....

(xii) $z - a - b =$ .....

(xiii) $d - a + x =$ .....

(xiv) $xy - bd =$ .....

(xv) $xz + cd =$ .....

Solution:

(*i*) $x + y = 3 + 6 = \mathbf{9}$

(*ii*) $y - x = 6 - 3 = \mathbf{3}$

(*iii*) $\frac{y}{x} = \frac{6}{3} = \mathbf{2}$

(*iv*) $c \div b = \frac{c}{b} = \frac{32}{8} = \mathbf{4}$

(*v*) $z \div x = \frac{z}{x} = \frac{18}{3} = \mathbf{6}$

(*vi*) $y \times d = 6 \times 0 = \mathbf{0}$

(*vii*) $d \div x = \frac{d}{x} = \frac{0}{3} = \mathbf{0}$

(*viii*) $ab + y = 2 \times 8 + 6 = 16 + 6 = \mathbf{22}$

(*ix*) $a + b + x = 2 + 8 + 3 = \mathbf{13}$

(*x*) $b + z - d = 8 + 18 - 0 = \mathbf{26}$

(*xi*) $a - b + y = 2 - 8 + 6 = 8 - 8 = \mathbf{0}$

(*xii*) $z - a - b = 18 - 2 - 8 = 18 - 10 = \mathbf{8}$

(*xiii*) $d - a + x = 0 - 2 + 3 = \mathbf{1}$

(*xiv*) $xy - bd = 3 \times 6 - 8 \times 0 = 18 - 0 = \mathbf{18}$

(*xv*) $xz + cd = 3 \times 18 + 32 \times 0 = 54 + 0 = \mathbf{54}$

Question 2.

Find the value of :

(*i*) $p + 2q + 3r$, when $p = 1$, $q = 5$ and $r = 2$

(*ii*) $2a + 4b + 5c$, when $a = 5$, $b = 10$ and $c = 20$

(*iii*) $3a - 2b$, when $a = 8$ and $b = 10$

(*iv*) $5x + 3y - 6z$, when $x = 3$, $y = 5$ and $z = 4$

(*v*) $2p - 3q + 4r - 8s$, when $p = 10$, $q = 8$, $r = 6$, and $s = 2$

(*vi*) $6m - 2n - 5p - 3q$, when $m = 20$, $n = 10$, $p = 2$ and $q = 9$

Solution:

(*i*) $p + 2q + 3r$

$= 1 + 2 \times 5 + 3 \times 2$

$= 1 + 10 + 6 = \mathbf{17}$

(*ii*) $2a + 4b + 5c$

$= 2 \times 5 + 4 \times 10 + 5 \times 20$

$= 10 + 40 + 100 = \mathbf{150}$

(*iii*) $3a - 2b = 3 \times 8 - 2 \times 10$

$= 24 - 20 = \mathbf{4}$

(*iv*) $5x + 3y - 6z$

$= 5 \times 3 + 3 \times 5 - 6 \times 4$

$= 15 + 15 - 24 = 30 - 24 = \mathbf{6}$

(*v*) $2p - 3q + 4r - 8s$

$= 2 \times 10 - 3 \times 8 + 4 \times 6 - 8 \times 2$

$= 20 - 24 + 24 - 16$

$= 20 - 16 = \mathbf{4}$

(*vi*) $6m - 2n - 5p - 3q$

$= 6 \times 20 - 2 \times 10 - 5 \times 2 - 3 \times 9$

$= 120 - 20 - 10 - 27$

$= 120 - 57 = \mathbf{63}$

Question 3.

Find the value of :

(*i*) $4pq \times 2r$, when $p = 5$, $q = 3$ and $r = 1/2$

(*ii*) $\frac{yx}{z}$, when $x = 8$, $y = 4$ and $z = 16$

(*iii*) $\frac{a+b-c}{2a}$, when $a = 5$, $b = 7$ and $c = 2$

Solution:

(*i*) $4pq \times 2r = 4 \times 5 \times 3 \times 2 \times \frac{1}{2} = \mathbf{60}$

(*ii*) $\frac{yx}{z} = \frac{4 \times 8}{16} = \frac{32}{16} = \mathbf{2}$

(*iii*) $\frac{a+b-c}{2a} = \frac{5+7-2}{2 \times 5} = \frac{12-2}{10} = \frac{10}{10} = \mathbf{1}$

Question 4.

If a = 3, b = 0, c = 2 and d = 1, find the value of :

(*i*) $3a + 2b - 6c + 4d$

(*ii*) $6a - 3b - 4c - 2d$

(*iii*) $ab - bc + cd - da$

(*iv*) $abc - bcd + cda$

(*v*) $a^2 + 2b^2 - 3c^2$

(*vi*) $a^2 + b^2 - c^2 + d^2$

(*vii*) $2a^2 - 3b^2 + 4c^2 - 5d^2$

Solution:

(*i*) $3a + 2b - 6c + 4d$

$= 3 \times 3 + 2 \times 0 - 6 \times 2 + 4 \times 1$

$= 9 + 0 - 12 + 4 = 13 - 12 = \mathbf{1}$

(*ii*) $6a - 3b - 4c - 2d$

$= 6 \times 3 - 3 \times 0 - 4 \times 2 - 2 \times 1$

$= 18 - 0 - 8 - 2 = 18 - 10 = \mathbf{8}$

(*iii*) $ab - bc + cd - da$

$= 3 \times 0 - 0 \times 2 + 2 \times 1 - 1 \times 3$

$= 0 + 0 + 2 - 3 = \mathbf{-1}$

(*iv*) $abc - bcd + cda$

$= 3 \times 0 \times 2 - 0 \times 2 \times 1 + 2 \times 1 \times 3$

$= 0 - 0 + 6 = \mathbf{6}$

(*v*) $a^2 + 2b^2 - 3c^2$

$= 3^2 + 2 \times 0^2 - 3 \times 2^2$

$= 9 + 0 - 3 \times 4$

$= 9 - 12 = \mathbf{-3}$

(*vi*) $a^2 + b^2 - c^2 + d^2$

$= (3)^2 + (0)^2 - (2^2) + (1)^2$

$= 9 + 0 - 4 + 1 = \mathbf{6}$

(*vii*) $2a^2 - 3b^2 + 4c^2 - 5d^2$

$= 2\,(3)^2 - 3(0)^2 + 4\,(2)^2 - 5\,(1)^2$

$= 2 \times 9 - 0 + 4 \times 4 - 5 \times 1$

$= 18 - 0 + 16 - 5 = 34 - 5 = \mathbf{29}$

Question 5.

Find the value of $5x^2 - 3x + 2$, when $x = 2$.

Solution:

$5x^2 - 3x + 2 = 5\,(2)^2 - 3(2) + 2$

$= 5\,(4) - 6 + 2$

$= 20 + 2 - 6 = \mathbf{16}$

Question 6.

Find the value of $3x^3 - 4x^2 + 5x - 6$, when x = -1.

Solution:

$$3x^3 - 4x^2 + 5x - 6$$
$$= 3(-1)^3 - 4(-1)^2 + 5(-1) - 6$$
$$= 3 \times -1 - 4 \times 1 - 5 - 6$$
$$= -3 - 4 - 5 - 6 = \mathbf{-18}$$

Question 7.

Show that the value of $x^3 - 8x^2 + 12x - 5$ is zero, when x = 1.

Solution:

$$x^3 - 8x^2 + 12x - 5$$
$$= (1)^3 - 8(1)^2 + 12\,(1) - 5$$
$$= 1 - 8 + 12 - 5$$
$$= 13 - 13 = \mathbf{0}\ \textbf{Hence proved.}$$

Question 8.

State true and false :

(i) The value of x + 5 = 6, when x = 1

(ii) The value of 2x – 3 = 1, when x = 0

(iii) $\frac{2x-4}{x+1}$ = -1,when x = 1

Solution:

(*i*) True, verification $x + 5 = 6$,

When $x = 1$, $1 + 5 = 6$

$6 = \mathbf{6}$ **Hence proved.**

(*ii*) $2x - 3 = 1$, when $x = 0$

$2 \times 0 - 3 = 1$, $0 - 3 = 1$,

$-3 = \mathbf{1}$ **False.**

(*iii*) $\frac{2x-4}{x+1} = -1$, when $x = 1 = \frac{2\times1-4}{1+1} = -1$

$$= \frac{2-4}{2} = -1 \quad = \frac{-2}{2} = -1$$

$= -1 = \mathbf{-1}$ **True.**

Question 9.

If x = 2, y = 5 and z = 4, find the value of each of the following :

(i) $\frac{x}{2x^2}$ (ii) $\frac{xz}{yz}$

(iii) z^x (iv) y^x

(v) $\frac{x^2y^2z^2}{xz}$ (vi) $\frac{5x^4y^2z^2}{2x^2}$

(vii) $xy \div y^2z$ (viii) $\frac{x^2y^x}{x}$

Solution:

(i) $\frac{x}{2x^2} = \frac{2}{2(2)^2} = \frac{2}{2\times4} = \frac{1}{4}$

(ii) $\frac{xz}{yz} = \frac{2\times4}{5\times4} = \frac{2}{5}$

(iii) $z^x = 4^2 = 4 \times 4 = 16$

(iv) $y^x = 5^2 = 5 \times 5 = 25$

(v) $\frac{x^2y^2z^2}{xz} = \frac{(2)^2\times(5)^2\times(4)^2}{2\times4}$

$= (2)^{2-1} \times (5)^2 \times (4)^{2-1}$

$= 2 \times 5 \times 5 \times 4 = 200$

(vi) $\frac{5x^4y^2z^2}{2x^2} = \frac{5x^{4-2}y^2z^2}{2} = \frac{5x^2y^2z^2}{2}$

$= \frac{5(2)^2(5)^2(4)^2}{2} = \frac{5\times4\times25\times16}{2}$

$= 5 \times 2 \times 25 \times 16 = 4{,}000$

(vii) $\frac{xy}{y^2z} = \frac{x}{y^{2-1}z} = \frac{x}{yz} = \frac{2}{5\times4} = \frac{1}{10}$

(viii) $\frac{x^2y^x}{x} = x^{2-1}y^x = xy^x = (2)\ (5)^2$

$= 2 \times 25 = 50$

Question 10.

If a = 3, find the values of a^2 and 2^a.

Solution:

$a^2 = (3)^2 = 3 \times 3 = 9$

$2^a = (2)^3 = 2 \times 2 \times 2 = 8$

Question 11.

If m = 2, find the difference between the values of $4m^3$ and $3m^4$.

Solution:

$4m^3 = 4\ (2)^3 = 4 \times 2 \times 2 \times 2 = 32$

$3m^4 = 3\ (2)^4 = 3 \times 2 \times 2 \times 2 \times 2 = 48$

Now, a difference $3m^4 - 4m^3 = 48 - 32 = 16$

EXERCISE 20(B)

Question 1.

Evaluate :

(i) (23 – 15) + 4

(ii) 5x + (3x + 7x)

(iii) 6m – (4m – m)

(iv) (9a – 3a) + 4a

(v) 35b – (16b + 9b)

(vi) (3y + 8y) – 5y

Solution:

(i) (23 – 15) + 4 = 8 + 4 = 12

(ii) 5x + (3x + 7x) = 5x + 10x = 15x

(iii) 6m – (4m – m) = 6m – 3m = 3m

(iv) (9a – 3a) + 4a = 6a + 4a = 10a

(v) 35b – (16b + 9b)= 35b – 25b = 10b

(vi) (3y + 8y) – 5y = 11y – 5y = 6y

Question 2.

Simplify :

(*i*) $12x - (5x + 2x)$

(*ii*) $10m + (4n - 3n) - 5n$

(*iii*) $(15b - 6b) - (8b + 4b)$

(*iv*) $-(-4a - 8a)$

(*v*) $x - (x - y) - (-x + y)$

(*vi*) $p + (-q - r - s) - (p - q - r)$

(*vii*) $(a + b) - (c + d) - (e - f)$

(*viii*) $3x + (8x - 5x) - (7x - x)$

(*ix*) $a - (a - b - c)$

(*x*) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$

(*xi*) $2m - (3m + 2n - 6n)$

(*xii*) $-m - n - (-m) - m$

(*xiii*) $x + y - (x + \overline{y - x})$

(*xiv*) $25y - (5x - 10y + 6x - 3y)$

(*xv*) $3x + (2x - \overline{x + 2})$

(*xvi*) $a - (2a - \overline{4a + 3a})$

(*xvii*) $5x^2 - (3x - \overline{x^2 - 4})$

(*xviii*) $-(y - x) - (x + y - \overline{2x + y})$

Solution:

(*i*) $12x - (5x + 2x) = 12x - 7x = \mathbf{5x}$

(*ii*) $10m + (4n - 3n) - 5n$

$= 10\ m + n - 5n = \mathbf{10m - 4n}$

(*iii*) $(15b - 6b) - (8b + 4b)$

$= 9b - 12b = \mathbf{-3b}$

(*iv*) $-(-4a - 8a) = -(-12a) = \mathbf{12a}$

(*v*) $x - (x - y) - (-x + y)$

$= x - x + x + y - y = \mathbf{x}$

(*vi*) $p + (-q - r - s) - (p - q - r)$

$= p - q - r - s - p + q + r$

$= p - p - q + q - r + r - s = \mathbf{-s}$

(*vii*) $(a + b) - (c + d) - (e - f)$

$= \mathbf{a + b - c - d - e + f}$

(*viii*) $3x + (8x - 5x) - (7x - x)$

$= 3x + 3x - 6x = 6x - 6x = \mathbf{0}$

(*ix*) $a - (a - b - c) = a - a + b + c$

$= \mathbf{b + c}$

(*x*) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$

$= 6a^2 + a^2 - a^2 + b^2 = \mathbf{6a^2 + b^2}$

(*xi*) $2m - (3m + 2n - 6n)$

$= 2m - 3m - 2n + 6n$

$= -m + 4n = \mathbf{4n - m}$

(*xii*) $-m - n - (-m) - m$

$= -m - n + m - m = \mathbf{-m - n}$

(*xiii*) $x + y - (x + \overline{y - x})$

$= x + y - (x + y - x)$

$= x + y - x - y + x$

$= x - x + x + y - y = \mathbf{x}.$

(*xiv*) $25y - (5x - 10y + 6x - 3y)$

$= 25y - 5x + 10y - 6x + 3y$

$= 25y + 10y + 3y - 5x - 6x$

$= \mathbf{38y - 11x}$

(*xv*) $3x + (2x - \overline{x + 2})$

$= 3x + (2x - x - 2)$

$= 3x + 2x - x - 2 = \mathbf{4x - 2}$

(xvi) $a - (2a - \overline{4a + 3a})$

$= a - (2a - 4a - 3a)$

$= a - 2a + 4a + 3a = 8a - 2a = \mathbf{6a}.$

(xvii) $5x^2 - (3x - \overline{x^2 - 4})$

$= 5x^2 - (3x - x^2 + 4) = 5x^2 - 3x + x^2 - 4$

$= 5x^2 + x^2 - 3x - 4 = \mathbf{6x^2 - 3x - 4}$

(xviii) $-(y - x) - (x + y - \overline{2x + y})$

$= -(y - x) - (x + y - 2x - y)$

$= -y + x - x - y + 2x + y$

$= x - x + 2x - y - y + y = \mathbf{2x - y}$

Question 3.

Simplify :

(i) $x - (y - z) + x + (y - z) + y - (z + x)$

(ii) $x - [y + \{x - (y + x)\}]$

(iii) $4x + 3\,(2x - 5y)$

(iv) $2\,(3a - b) - 5\,(a - 3b)$

(v) $p + 2\,(q - \overline{r + p})$

(vi) $a - [-\{-(a - \overline{b - c})\}]$

(vii) $3x - [5y - \{6y + 2\,(10y - x)\}]$

(viii) $5\{a^2 - a(a - \overline{a - 2})\}$

Solution:

(*i*) $x-(y-z)+x+(y-z)+y-(z+x)$

$=x-y+z+x+y-z+y-z-x$

$=x+x-x-y+y+y+z-z-z$

$=\mathbf{x+y-z}$

(*ii*) $x-[y+\{x-(y+x)\}]$

$=x-[y+\{x-y-x\}]$

$=x-[y+x-y-x]$

$=x-y-x+y+x$

$=x-x+x-y+y=\mathbf{x}$

(*iii*) $4x+3(2x-5y)$

$=4x+6x-15y$

$=\mathbf{10x-15y}$

(*iv*) $2(3a-b)-5(a-3b)$

$=6a-2b-5a+15b$

$=6a-5a+15b-2b=\mathbf{a+13b}$

(*v*) $p+2(q-\overline{r+p})$

$=p+2(q-r-p)$

$=p+2q-2r-2p=\mathbf{2q-2r-p}$

(*vi*) $a-[-\{-(a-\overline{b-c})\}]$

$=a-[-\{-(a-b+c)\}]$

$=a-[-\{-a+b-c\}]$

$=a-[+a-b+c]$

$=a-a+b-c=\mathbf{b-c}$

(*vii*) $3x-[5y-\{6y+2(10y-x)\}]$

$=3x-[5y-\{6y+20y-2x\}]$

$=3x-[5y-6y-20y+2x]$

$=3x-5y+6y+20y-2x$

$=3x-2x+6y+20y-5y$

$=\mathbf{x+21y}$

(*viii*) $5\{a^2-a(a-\overline{a-2})\}$

$=5\{a^2-a(a-a+2)\}$

$=5\{a^2-a^2+a^2-2a)$

$=5a^2-5a^2+5a^2-10a$

$=\mathbf{5a^2-10a}$

EXERCISE 20(C)

Question 1.

Fill in the blanks :

(*i*) $2a + b - c = 2a + (.....)$

(*ii*) $3x - z + y = 3x - (.....)$

(*iii*) $6p - 5x + q = 6p - (.....)$

(*iv*) $a + b - c + d = a + (.....)$

(*v*) $5a + 4b + 4x - 2c = 4x - (.....)$

(*vi*) $7x + 2z + 4y - 3 = -3 + 4y + (.....)$

(*vii*) $3m - 2n + 6 = 6 - (.....)$

(*viii*) $2t + r - p - q + s = 2t + r - (.....)$

Solution:

(*i*) $2a + b - c = 2a + \mathbf{(b - c)}$

(*ii*) $3x - z + y = 3x - \mathbf{(z - y)}$

(*iii*) $6p - 5x + q = 6p - \mathbf{(5x - q)}$

(*iv*) $a + b - c + d = a + \mathbf{(b - c + d)}$

(*v*) $5a + 4b + 4x - 2c = 4x - \mathbf{(2c - 5a - 4b)}$

(*vi*) $7x + 2z + 4y - 3 = -3 + 4y + \mathbf{(7x + 2z)}$

(*vii*) $3m - 2n + 6 = 6 - \mathbf{(2n - 3m)}$

(viii) 2t + r – p – q + s = 2t + r – (p + q – s)

Question 2.

Insert the bracket as indicated :

(*i*) $x - 2y = -(.....)$

(*ii*) $m + n - p = -(.....)$

(*iii*) $a + 4b - 4c = a + (.....)$

(*iv*) $a - 3b + 5c = a - (.....)$

(*v*) $x^2 - y^2 + z^2 = x^2 - (.....)$

(*vi*) $m^2 + x^2 - p^2 = -(.....)$

(*vii*) $2x - y + 2z = 2z - (.....)$

(*viii*) $ab + 2bc - 3ac = 2bc - (.....)$

Solution:

(i) $x - 2y = -(2y - x)$

(ii) $m + n - p = -(p - m - n)$

(iii) $a + 4b - 4c = a + (4b - 4c)$

(iv) $a - 3b + 5c = a - (3b - 5c)$

(v) $x^2 - y^2 + z^2 = x^2 - (y^2 - z^2)$

(vi) $m^2 + x^2 - p^2 = -(p^2 - m^2 - x^2)$

(vii) $2x - y + 2z = 2z - (y - 2x)$

(viii) $ab + 2bc - 3ac = 2bc - (3ac - ab)$

REVISION EXERCISE

Question 1.

Find the value of 3ab + 10bc – 2abc when a = 2, b = 5 and c = 8.

Solution:

$a = 2, b = 5, c = 8$

$\therefore\ 3ab + 10bc - 2abc$

$= 3 \times 2 \times 5 + 10 \times 5 \times 8 - 2 \times 2 \times 5 \times 8$

$= 30 + 400 - 160 = 430 - 160$

$= 270$

Question 2.

If x = 2, = 3 and z = 4, find the value of $3x^2 - 4y^2 + 2z^2$.

Solution:

$x = 2, y = 3, z = 4$

$\therefore\ 3x^2 - 4y^2 + 2z^2 = 3\,(2)^2 - 4\,(3)^2 + 2\,(4)^2$

$= 3 \times 4 - 4 \times 9 + 2 \times 16$

$= 12 - 36 + 32$

$= 12 + 32 - 36 = 44 - 36 = 8$

Question 3.

If x = 3, y = 2 and z = 1; find the value of:

(i) x^y

(ii) y^x

(iii) $3x^2 - 5y^2$

(iv) $2x - 3y + 4z + 5$

(v) $y^2 - x^2 + 6z^2$

(vi) $xy + y^2z - 4zx$

Solution:

$x = 3, y = 2, z = 1$

(*i*) $x^y = 3^2 = 3 \times 3 = \mathbf{9}$

(*ii*) $y^x = 2^3 = 2 \times 2 \times 2 = \mathbf{8}$

(*iii*) $3x^2 - 5y^2 = 3\,(3)^2 - 5\,(2)^2$

$= 3 \times 9 - 5 \times 4 = 27 - 20 = 7$

(*iv*) $2x - 3y + 4z + 5 = 2 \times 3 - 3 \times 2 + 4 \times 1 + 5$

$= 6 - 6 + 4 + 5 = 15 - 6 = \mathbf{9}$

(*v*) $y^2 - x^2 + 6z^2$

$= (2)^2 - (3)^2 + 6\,(1)^2$

$= 4 - 9 + 6 \times 1 = 4 - 9 + 6$

$= 10 - 9 = \mathbf{1}$

(*vi*) $xy + y^2z - 4zx$

$= 3 \times 2 + (2)^2 \times 1 - 4 \times 1 \times 3$

$= 6 + 4 - 12 = 10 - 12 = -2$

Question 4.

If P = $-12x^2 - 10xy + 5y^2$, Q = $7x^2 + 6xy + 2y^2$, and R = $5x + 2xy + 4y$; find :

(i) P – Q

(ii) Q + P

(iii) P – Q + R

(iv) P + Q + R

Solution:

$P = -12x^2 - 10xy + 5y^2$

$Q = 7x^2 + 6xy + 2y^2$

$R = 5x^2 + 2xy + 4y^2$

(*i*) $P - Q = (-12x^2 - 10xy + 5y^2) - (7x^2 + 6xy + 2y^2)$

$= -12x^2 - 10xy + 5y^2 - 7x^2 - 6xy - 2y^2$

$= -12x^2 - 7x^2 - 10xy - 6xy + 5y^2 - 2y^2$

$= -19x^2 - 16xy + 3y^2$

(*ii*) $Q + P = (7x^2 + 6xy + 2y^2) + (-12x^2 - 10xy + 5y^2)$

$= 7x^2 + 6xy + 2y^2 - 12x^2 - 10xy + 5y^2$

$= 7x^2 - 12x^2 + 6xy - 10xy + 2y^2 + 5y^2$

$= -5x^2 - 4xy + 7y^2$

(*iii*) $P - Q + R = (-12x^2 - 10xy + 5y^2) - (7x^2 + 6xy + 2y^2) + (5x^2 + 2xy + 4y^2)$

$= -12x^2 - 10xy + 5y^2 - 7x^2 - 6xy - 2y^2 + 5x^2 + 2xy + 4y^2$

$= -12x^2 - 7x^2 + 5x^2 - 10xy - 6xy + 2xy + 5y^2 - 2y^2 + 4y^2$

$= -14x^2 - 14xy + 7y^2$

(*iv*) $P + Q + R = -12x^2 - 10xy + 5y^2 + 7x^2 + 6xy + 2y^2 + 5x^2 + 2xy + 4y^2$

$= -12x^2 + 7x^2 + 5x^2 - 10xy + 6xy + 2xy + 5y^2 + 2y^2 + 4y^2$

$= 0 - 2xy + 11y^2$

$= -2xy + 11y^2$

Question 5.

If $x = a^2 - bc$, $y = b^2 - ca$ and $z = c^2 - ab$; find the value of :

(i) $ax + by + cz$

(ii) $ay - bx + cz$

Solution:

$x = a^2 - bc, y = b^2 - ca, z = c^2 - ab$

(i) $ax + by + cz = a(a^2 - bc) + b(b^2 - ca) + c(c^2 - ab)$

$= a^3 - abc + b^3 - abc + c^3 - abc$

$= a^3 + b^3 + c^3 - 3abc$

(ii) $ay - bx + cz = a(b^2 - ca) - b(a^2 - bc) + c(c^2 - ab)$

$= ab^2 - ca^2 - a^2b + b^2c + c^3 - abc$

Question 6.

Multiply and then evaluate :

(i) (4x + y) and (x – 2y); when x = 2 and y = 1.

(ii) (x^2 – y) and (xy – y^2); when x = 1 and y = 2.

(iii) (x – 2y + z) and (x – 3z); when x = -2, y = -1 and z = 1.

Solution:

(i) $(4x + y) \times (x - 2y)$

$= 4x(x - 2y) + y(x - 2y)$

$= 4x^2 - 8xy + xy - 2y^2$

$= 4x^2 - 7xy - 2y^2$

Verification :

When $x = 2, y = 1$

L.H.S. $= (4x + y)(x - 2y)$

$= (4 \times 2 + 1)(2 - 2 \times 1)$

$= (8 + 1)(2 - 2) = 9 \times 0 = 0$

R.H.S. $= 4x^2 - 7xy - 2y^2$

$= 4\ (2)^2 - 7 \times 2 \times 1 - 2\ (1)^2$

$= 4 \times 4 - 14 - 2 = 16 - 16 = 0$

$\therefore$ L.H.S. = R.H.S.

(*ii*) $(x^2 - y) \times (xy - y^2)$

$= x^2\ (xy - y^2) - y\ (xy - y^2)$

$= x^3y - x^2y^2 - xy^2 + y^3$

Verification :

When $x = 1, y = 2$

$\therefore$ L.H.S. $= (x^2 - y)\ (xy - y^2)$

$= [(1)^2 - 2]\ [1 \times 2 - (2)^2]$

$= (1 - 2)\ (2 - 4) = -1 \times -2 = 2$

R.H.S. $= x^3y - x^2y^2 - xy^2 + y^3$

$= (1)^3 \times 2 - (1)^2\ (2)^2 - 1(2)^2 + (2)^3$

$= 1 \times 2 - 1 \times 4 - 1 \times 4 + 8$

$= 2 - 4 - 4 + 8 = 10 - 8 = 2$

$\therefore$ L.H.S. = R.H.S.

(*iii*) $(x - 2y + z) \times (x - 3z)$

$= x\ (x - 3z) - 2y\ (x - 3z) + z\ (x - 3z)$

$= x^2 - 3zx - 2xy + 6yz + zx - 3z^2$

$= x^2 - 2zx - 2xy + 6yz - 3z^2$

Verification :

When $x = -2, y = -1, z = 1$

L.H.S. $= (x - 2y + z) \times (x - 3z)$

$= [-2 - 2 \times (-1) + 1] \times [-2 - 3 \times 1]$

$= (-2 + 2 + 1) \times (-2 - 3) = 1 \times (-5) = -5$

R.H.S. $= x^2 - 2zx - 2xy + 6yz - 3z^2$

$= (-2)^2 - 2\ (1)\ (-2) - 2\ (-2)\ (-1) + 6\ (-1)\ (1)$
$- 3(1)^2$

$= 4 + 4 - 4 - 6 - 3$

$= 8 - 13 = -5$

$\therefore$ L.H.S. = R.H.S.

Question 7.

Simplify :

(i) 5 (x + 3y) – 2 (3x – 4y)

(ii) 3x – 8 (5x – 10)

(iii) 6 {3x – 8 (5x – 10)}

(iv) 3x – 6 {3x – 8 (5x – 10)}

(v) 2 (3x – 4x – 8) – (3 – 5x – 2x)

(vi) $8x - (3x - \overline{2x - 3})$

(vii) $12x^2 - (7x - \overline{3x^2 + 15})$

Solution:

$(i)\ 5(x + 3y) - 2(3x - 4y)$

$= 5x + 15y - 6x + 8y$

$= 5x - 6x + 15y + 8y = -x + 23y$

$(ii)\ 3x - 8(5x - 10)$

$= 3x - 40x + 80$

$= -37x + 80$

$(iii)\ 6\{3x - 8(5x - 10)\}$

$= 6\{3x - 40x + 80\}$

$= 18x - 240x + 480$

$= -222x + 480$

$(iv)\ 3x - 6\{3x - 8(5x - 10)\}$

$= 3x - 6\{3x - 40x + 80\}$

$= 3x - 18x + 240x - 480$

$= 243x - 18x - 480 = 225x - 480$

$(v)\ 2(3x^2 - 4x - 8) - (3 - 5x - 2x^2)$

$= (6x^2 - 8x - 16) - (3 - 5x - 2x^2)$

$= 6x^2 - 8x - 16 - 3 + 5x + 2x^2$

$= 6x^2 + 2x^2 - 8x + 5x - 16 - 3$

$= 8x^2 - 3x - 19$

$(vi)\ 8x - (3x - \overline{2x - 3})$

$= 8x - (3x - 2x + 3) = 8x - 3x + 2x - 3$

$= 10x - 3x - 3 = 7x - 3$

$(vii)\ 12x^2 - (7x - \overline{3x^2 + 15})$

$= 12x^2 - (7x - 3x^2 - 15)$

$= 12x^2 - 7x + 3x^2 + 15$

$= 12x^2 + 3x^2 - 7x + 15 = 15x^2 - 7x + 15$

Question 8.

If x = -3, find the value of : $2x^3 + 8x^2 - 15$.

Solution:

$x = -3$

$\therefore\ 2x^3 + 8x^2 - 15$

$= 2(-3)^3 + 8(-3)^2 - 15$

$= 2(-27) + 8(9) - 15$

$= -54 + 72 - 15 = -54 - 15 + 72$

$= -69 + 72 = \mathbf{3}$