

27. Hyperbola

Exercise 27.1

1. Question

The equation of the directrix of a hyperbola is $x - y + 3 = 0$. Its focus is $(-1, 1)$ and eccentricity 3. Find the equation of the hyperbola.

Answer

Given: Equation of directrix of a hyperbola is $x - y + 3 = 0$. Focus of hyperbola is $(-1, 1)$ and eccentricity (e) = 3

To find: equation of the hyperbola

Let M be the point on directrix and P(x, y) be any point of the hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is an eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\begin{aligned}\sqrt{(x+1)^2 + (y-1)^2} &= 3 \left| \frac{(x-y+3)}{\sqrt{1^2 + (-1)^2}} \right| \\ \Rightarrow \sqrt{(x+1)^2 + (y-1)^2} &= 3 \left| \frac{(x-y+3)}{\sqrt{1+1}} \right|\end{aligned}$$

Squaring both sides:

$$\begin{aligned}\Rightarrow \left(\sqrt{(x+1)^2 + (y-1)^2} \right)^2 &= \left(3 \left| \frac{(x-y+3)}{\sqrt{1+1}} \right| \right)^2 \\ \Rightarrow (x+1)^2 + (y-1)^2 &= \frac{3^2(x-y+3)^2}{2}\end{aligned}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow 2\{x^2 + 1 + 2x + y^2 + 1 - 2y\} = 9\{x^2 + y^2 + 9 + 6x - 6y - 2xy\}$$

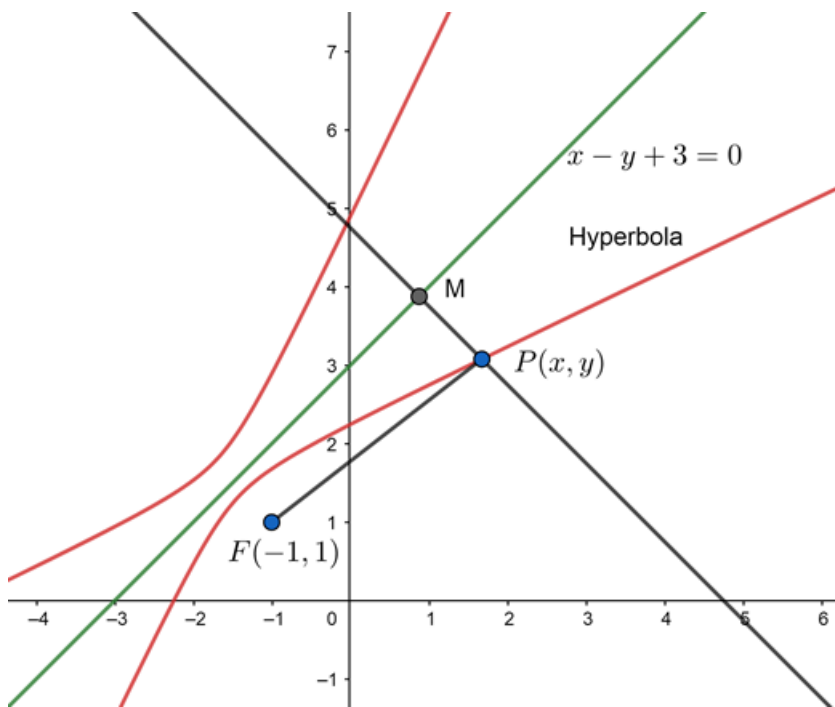
$$\Rightarrow 2x^2 + 2 + 4x + 2y^2 + 2 - 4y = 9x^2 + 9y^2 + 81 + 54x - 54y - 18xy$$

$$\Rightarrow 2x^2 + 4 + 4x + 2y^2 - 4y - 9x^2 - 9y^2 - 81 - 54x + 54y + 18xy = 0$$

$$\Rightarrow -7x^2 - 7y^2 - 50x + 50y + 18xy - 77 = 0$$

$$\Rightarrow \mathbf{7x^2 + 7y^2 + 50x - 50y - 18xy + 77 = 0}$$

This is the required equation of hyperbola



2 A. Question

Find the equation of the hyperbola whose

focus is $(0, 3)$, directrix is $x + y - 1 = 0$ and eccentricity $= 2$

Answer

Given: Equation of directrix of a hyperbola is $x + y - 1 = 0$. Focus of hyperbola is $(0, 3)$ and eccentricity $(e) = 2$

To find: equation of the hyperbola

Let M be the point on directrix and $P(x, y)$ be any point of the hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is an eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1^2+1^2}} \right|$$

$$\Rightarrow \sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-0)^2 + (y-3)^2} \right)^2 = \left(2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right| \right)^2$$

$$\Rightarrow (x-0)^2 + (y-3)^2 = \frac{2^2(x+y-1)^2}{2}$$

$$\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \& }$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$\Rightarrow 2\{x^2 + y^2 + 9 - 6y\} = 4\{x^2 + y^2 + 1 - 2x - 2y + 2xy\}$$

$$\Rightarrow 2x^2 + 2y^2 + 18 - 12y = 4x^2 + 4y^2 + 4 - 8x - 8y + 8xy$$

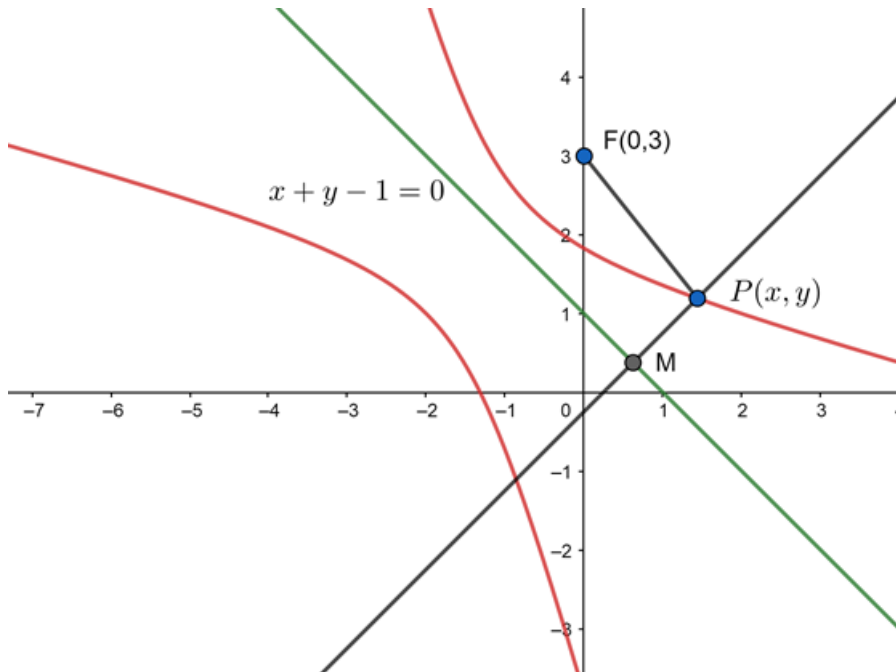
$$\Rightarrow 2x^2 + 2y^2 + 18 - 12y - 4x^2 - 4y^2 - 4 - 8x + 8y - 8xy = 0$$

$$\Rightarrow -2x^2 - 2y^2 - 8x - 4y - 8xy + 14 = 0$$

$$\Rightarrow -2(x^2 + y^2 - 4x + 2y + 4xy - 7) = 0$$

$$\Rightarrow \mathbf{x^2 + y^2 - 4x + 2y + 4xy - 7 = 0}$$

This is the required equation of hyperbola



2 B. Question

Find the equation of the hyperbola whose

focus is (1, 1), directrix is $3x + 4y + 8 = 0$ and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is $3x + 4y + 8 = 0$. Focus of hyperbola is (1, 1) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-1)^2 + (y-1)^2} = 2 \left| \frac{(3x+4y+8)}{\sqrt{3^2+4^2}} \right|$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = 2 \left| \frac{(3x+4y+8)}{\sqrt{9+16}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-1)^2 + (y-1)^2} \right)^2 = \left(2 \left| \frac{(3x+4y+8)}{\sqrt{25}} \right| \right)^2$$

$$\Rightarrow (x-1)^2 + (y-1)^2 = \frac{2^2(3x+4y+8)^2}{25}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow 25\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 4\{9x^2 + 16y^2 + 64 + 24xy + 64y + 48x\}$$

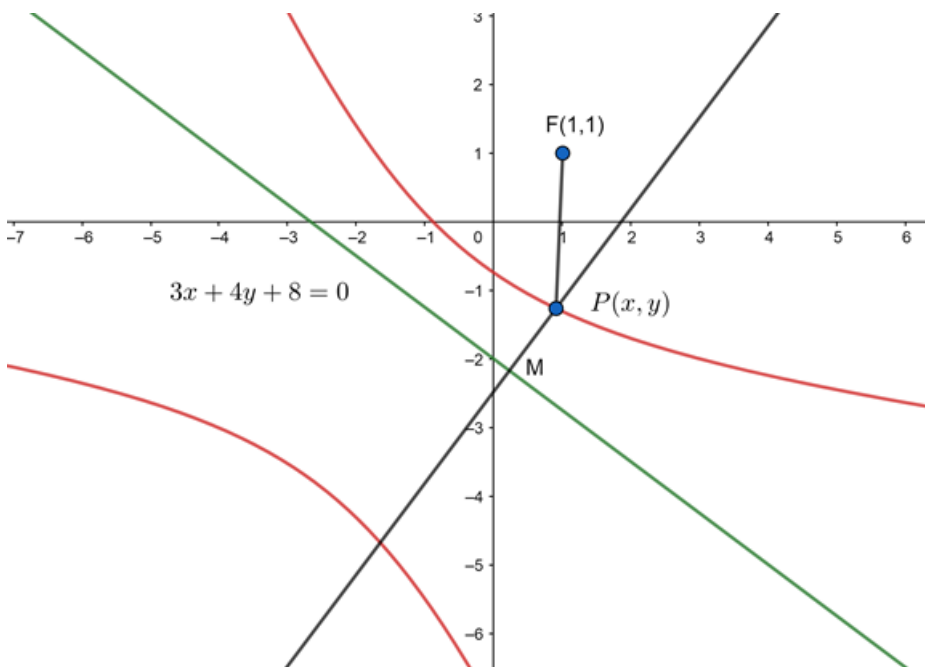
$$\Rightarrow 25x^2 + 25 - 50x + 25y^2 + 25 - 50y = 36x^2 + 64y^2 + 256 + 96xy + 256y + 192x$$

$$\Rightarrow 25x^2 + 25 - 50x + 25y^2 + 25 - 50y - 36x^2 - 64y^2 - 256 - 96xy - 256y - 192x = 0$$

$$\Rightarrow -11x^2 - 39y^2 - 242x - 306y - 96xy - 206 = 0$$

$$\Rightarrow \mathbf{11x^2 + 39y^2 + 242x + 306y + 96xy + 206 = 0}$$

This is the required equation of hyperbola



2 C. Question

Find the equation of the hyperbola whose

focus is (1, 1) directrix is $2x + y = 1$ and eccentricity $= \sqrt{3}$

Answer

Given: Equation of directrix of a hyperbola is $2x + y - 1 = 0$. Focus of hyperbola is (1, 1) and eccentricity (e) $= \sqrt{3}$

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-1)^2 + (y-1)^2} = \sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{2^2+1^2}} \right|$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = \sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{4+1}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-1)^2 + (y-1)^2} \right)^2 = \left(\sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{5}} \right| \right)^2$$

$$\Rightarrow (x-1)^2 + (y-1)^2 = \frac{3(2x+y-1)^2}{5}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow 5\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 3\{4x^2 + y^2 + 1 + 4xy - 2y - 4x\}$$

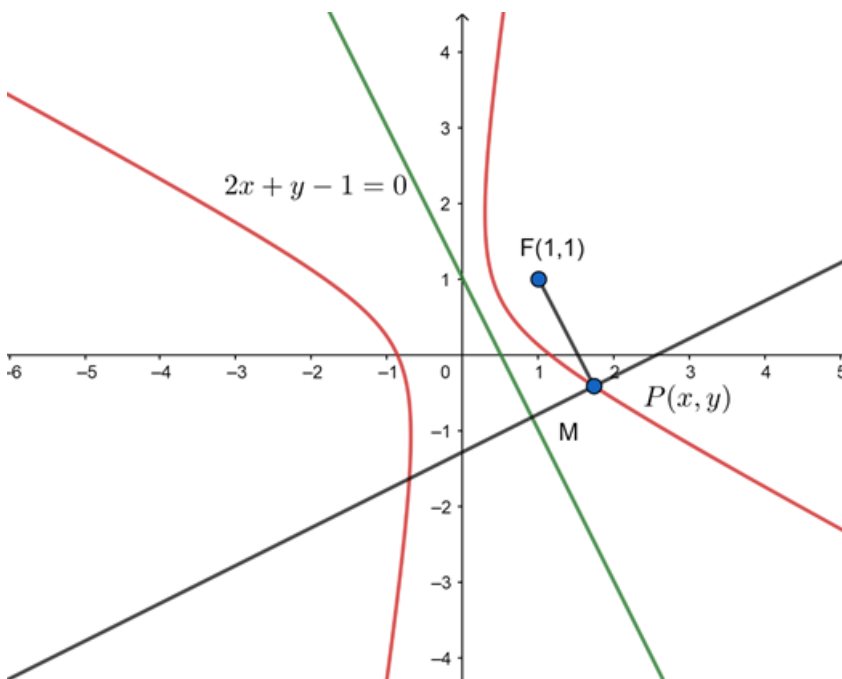
$$\Rightarrow 5x^2 + 5 - 10x + 5y^2 + 5 - 10y = 12x^2 + 3y^2 + 3 + 12xy - 6y - 12x$$

$$\Rightarrow 5x^2 + 5 - 10x + 5y^2 + 5 - 10y - 12x^2 - 3y^2 - 3 - 12xy + 6y + 12x = 0$$

$$\Rightarrow -7x^2 + 2y^2 + 2x - 4y - 12xy + 7 = 0$$

$$\Rightarrow 7x^2 - 2y^2 - 2x + 4y + 12xy - 7 = 0$$

This is the required equation of hyperbola.



2 D. Question

Find the equation of the hyperbola whose

focus is (2, -1), directrix is $2x + 3y = 1$ and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is $2x + 3y - 1 = 0$. Focus of hyperbola is (2, -1) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-2)^2 + (y+1)^2} = 2 \left| \frac{(2x+3y-1)}{\sqrt{2^2+3^2}} \right|$$

$$\Rightarrow \sqrt{(x-2)^2 + (y+1)^2} = 2 \left| \frac{(2x+3y-1)}{\sqrt{4+9}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-2)^2 + (y+1)^2} \right)^2 = \left(2 \left| \frac{(2x+3y-1)}{\sqrt{13}} \right| \right)^2$$

$$\Rightarrow (x-2)^2 + (y+1)^2 = \frac{4(2x+3y-1)^2}{13}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow 13\{x^2 + 4 - 4x + y^2 + 1 + 2y\} = 4\{4x^2 + 9y^2 + 1 + 12xy - 6y - 4x\}$$

$$\Rightarrow 13x^2 + 52 - 52x + 13y^2 + 13 + 26y = 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x$$

$$\Rightarrow 13x^2 + 52 - 52x + 13y^2 + 13 + 26y - 16x^2 - 36y^2 - 4 - 48xy + 24y + 16x = 0$$

$$\Rightarrow -3x^2 - 23y^2 - 36x + 50y - 48xy + 61 = 0$$

$$\Rightarrow \mathbf{3x^2 + 23y^2 + 36x - 50y + 48xy - 61 = 0}$$

This is the required equation of hyperbola.

2 E. Question

Find the equation of the hyperbola whose

focus is (a, 0), directrix is $2x + 3y = 1$ and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is $2x - y + a = 0$. Focus of hyperbola is (a, 0) and eccentricity (e) = $\frac{4}{3}$

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-a)^2 + (y-0)^2} = \frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{2^2+(-1)^2}} \right|$$

$$\Rightarrow \sqrt{(x-a)^2 + (y)^2} = \frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{4+1}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-a)^2 + (y)^2} \right)^2 = \left(\frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{5}} \right| \right)^2$$

$$\Rightarrow (x-a)^2 + (y)^2 = \frac{16(2x-y+a)^2}{9 \times 5}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow 117\{x^2 + a^2 - 2ax + y^2\} = 16\{4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax\}$$

$$\Rightarrow 117x^2 + 117a^2 - 234ax + 117y^2 = 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax$$

$$\Rightarrow 117x^2 + 117a^2 - 234ax + 117y^2 - 64x^2 - 16y^2 - 16a^2 + 64xy + 32ay - 64ax = 0$$

$$\Rightarrow \mathbf{53x^2 + 101y^2 - 298ax + 32ay + 64xy + 111a^2 = 0}$$

This is the required equation of hyperbola.

2 F. Question

Find the equation of the hyperbola whose

focus is (2, 2), directrix is $x + y = 9$ and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is $x + y - 9 = 0$. Focus of hyperbola is (2, 2) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-2)^2 + (y-2)^2} = 2 \left| \frac{(x+y-9)}{\sqrt{1^2+1^2}} \right|$$

$$\Rightarrow \sqrt{(x-2)^2 + (y-2)^2} = 2 \left| \frac{(x+y-9)}{\sqrt{1+1}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-2)^2 + (y-2)^2} \right)^2 = \left(2 \left| \frac{(x+y-9)}{\sqrt{2}} \right| \right)^2$$

$$\Rightarrow (x-2)^2 + (y-2)^2 = \frac{4(x+y-9)^2}{2}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\}$$

$$\Rightarrow x^2 + 4 - 4x + y^2 + 4 - 4y = 2\{x^2 + y^2 + 81 + 2xy - 18y - 18x\}$$

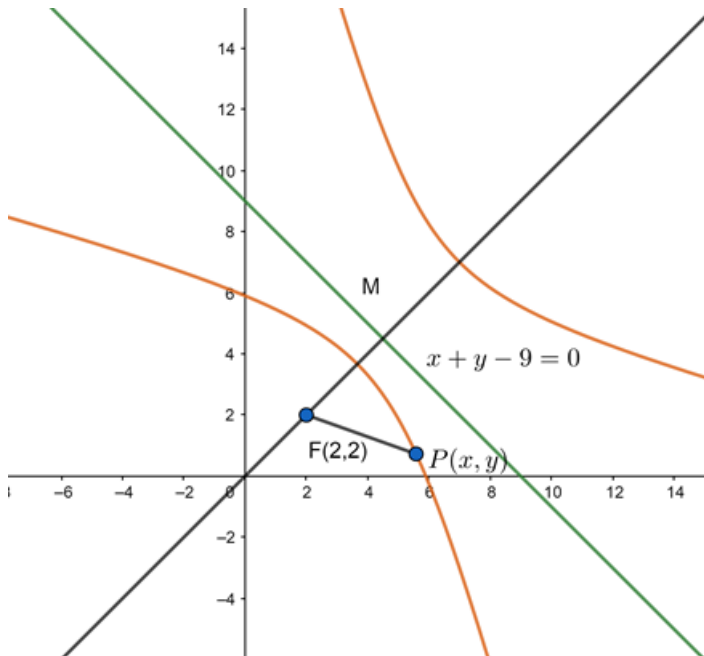
$$\Rightarrow x^2 - 4x + y^2 + 8 - 4y = 2x^2 + 2y^2 + 162 + 4xy - 36y - 36x$$

$$\Rightarrow x^2 - 4x + y^2 + 8 - 4y - 2x^2 - 2y^2 - 162 - 4xy + 36y + 36x = 0$$

$$\Rightarrow -x^2 - y^2 + 32x + 32y + 4xy - 154 = 0$$

$$\Rightarrow x^2 + y^2 - 32x - 32y + 4xy + 154 = 0$$

This is the required equation of hyperbola.



3 A. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$$9x^2 - 16y^2 = 144$$

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(\pm ae, 0)$

Equation of directrix are: $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 4$ and $b = 3$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

$$\Rightarrow ae = 4 \times \frac{5}{4} = 5$$

Foci: $(\pm 5, 0)$

Equation of directrix are:

$$x = \pm \frac{a}{e}$$

$$\Rightarrow x = \pm \frac{4}{\frac{5}{4}}$$

$$\Rightarrow x = \pm \frac{16}{5}$$

$$\Rightarrow 5x = \pm 16$$

$$\Rightarrow 5x \mp 16 = 0$$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2 \times (3)^2}{4}$$

$$= \frac{9}{2}$$

3 B. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$$16x^2 - 9y^2 = -144$$

Answer

Given: $16x^2 - 9y^2 = -144$

To find: eccentricity(e), coordinates of the foci $f(m,n)$, equation of directrix, length of latus-rectum of hyperbola.

$$\frac{9y^2}{144} - \frac{16x^2}{144} = 1$$

$$\Rightarrow \frac{y^2}{16} - \frac{x^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = -1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$:

Eccentricity(e) is given by,

$$e = \frac{c}{b}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(0, \pm be)$

The equation of directrix are: $y = \pm \frac{b}{e}$

Length of latus rectum is $\frac{2a^2}{b}$

Here, $a = 3$ and $b = 4$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

$$\Rightarrow be = 4 \times \frac{5}{4} = 5$$

Foci: $(0, \pm 5)$

The equation of directrix are:

$$y = \pm \frac{b}{e}$$

$$\Rightarrow y = \pm \frac{4}{\frac{5}{4}}$$

$$\Rightarrow y = \pm \frac{16}{5}$$

$$\Rightarrow 5y = \pm 16$$

$$\Rightarrow 5y \mp 16 = 0$$

Length of latus rectum,

$$= \frac{2a^2}{b}$$

$$= \frac{2 \times (3)^2}{4}$$

$$= \frac{9}{2}$$

3 C. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$$4x^2 - 3y^2 = 36$$

Answer

Given: $4x^2 - 3y^2 = 36$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{4x^2}{36} - \frac{3y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{(\sqrt{12})^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = \sqrt{12}$

$$c = \sqrt{3^2 + (\sqrt{12})^2}$$

$$\Rightarrow c = \sqrt{9 + 12}$$

$$\Rightarrow c = \sqrt{21}$$

Therefore,

$$e = \frac{\sqrt{21}}{3}$$

$$\Rightarrow ae = 3 \times \frac{\sqrt{21}}{3} = \sqrt{21}$$

Foci: $(\pm\sqrt{21}, 0)$

The equation of directrix are:

$$x = \pm \frac{a}{e}$$

$$\Rightarrow x = \pm \frac{3}{\frac{\sqrt{21}}{3}}$$

$$\Rightarrow x = \pm \frac{9}{\sqrt{21}}$$

$$\Rightarrow \sqrt{21}x = \pm 9$$

$$\Rightarrow \sqrt{21}x \mp 9 = 0$$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2 \times (\sqrt{12})^2}{3}$$

$$= \frac{2 \times 12}{3}$$

$$= 8$$

3 D. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$$3x^2 - y^2 = 4$$

Answer

Given: $3x^2 - y^2 = 4$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{(2)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = \frac{2}{\sqrt{3}}$ and $b = 2$

$$c = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + (2)^2}$$

$$\Rightarrow c = \sqrt{\frac{4}{3} + 4}$$

$$\Rightarrow c = \sqrt{\frac{4 + 12}{3}}$$

$$\Rightarrow c = \sqrt{\frac{16}{3}}$$

$$\Rightarrow c = \frac{4}{\sqrt{3}}$$

Therefore,

$$e = \frac{\frac{4}{\sqrt{3}}}{\frac{2}{\sqrt{3}}}$$

$$\Rightarrow e = 2$$

$$\Rightarrow ae = \frac{2}{\sqrt{3}} \times 2 = \frac{4}{\sqrt{3}}$$

$$\text{Foci: } \left(\pm \frac{4}{\sqrt{3}}, 0\right)$$

The equation of directrix are:

$$x = \pm \frac{a}{e}$$

$$\Rightarrow x = \pm \frac{\frac{2}{\sqrt{3}}}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x = \pm 1$$

$$\Rightarrow \sqrt{3}x \mp 1 = 0$$

Length of latus rectum,

$$= \frac{2(2)^2}{\frac{2}{\sqrt{3}}}$$

$$= 4\sqrt{3}$$

3 E. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$$2x^2 - 3y^2 = 5$$

Answer

Given: $2x^2 - 3y^2 = 5$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{2x^2}{5} - \frac{3y^2}{5} = 1$$

$$\Rightarrow \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2} - \frac{y^2}{\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = \frac{\sqrt{5}}{\sqrt{2}}$ and $b = \frac{\sqrt{5}}{\sqrt{3}}$

$$c = \sqrt{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow c = \sqrt{\frac{5}{2} + \frac{5}{3}}$$

$$\Rightarrow c = \sqrt{\frac{15 + 10}{6}}$$

$$\Rightarrow c = \sqrt{\frac{25}{6}}$$

$$\Rightarrow c = \frac{5}{\sqrt{6}}$$

Therefore,

$$e = \frac{\frac{5}{\sqrt{6}}}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$\Rightarrow e = \frac{\sqrt{5}}{\sqrt{3}}$$

$$\Rightarrow ae = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{3}} = \frac{5}{\sqrt{6}}$$

$$\text{Foci: } \left(\pm \frac{5}{\sqrt{6}}, 0 \right)$$

The equation of directrix are:

$$x = \pm \frac{a}{e}$$

$$\Rightarrow x = \pm \frac{\frac{\sqrt{5}}{\sqrt{2}}}{\frac{\sqrt{5}}{\sqrt{3}}}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{6}}$$

$$\Rightarrow \sqrt{6}x = \pm 1$$

$$\Rightarrow \sqrt{6}x \mp 1 = 0$$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2 \left(\frac{\sqrt{5}}{\sqrt{3}} \right)^2}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$= \frac{2 \times \frac{5}{3}}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$= \frac{2\sqrt{10}}{3}$$

4. Question

Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola $25x^2 - 36y^2 = 225$.

Answer

$$\text{Given: } 25x^2 - 36y^2 = 225$$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{25x^2}{225} - \frac{36y^2}{225} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{15}{5}\right)^2} - \frac{y^2}{\left(\frac{15}{6}\right)^2} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = \frac{5}{2}$

$$c = \sqrt{(3)^2 + \left(\frac{5}{2}\right)^2}$$

$$\Rightarrow c = \sqrt{9 + \frac{25}{4}}$$

$$\Rightarrow c = \sqrt{\frac{36 + 25}{4}}$$

$$\Rightarrow c = \sqrt{\frac{61}{4}}$$

$$\Rightarrow c = \frac{\sqrt{61}}{2}$$

Therefore,

$$e = \frac{\frac{\sqrt{61}}{2}}{3}$$

$$\Rightarrow e = \frac{\sqrt{61}}{6}$$

$$\Rightarrow ae = 3 \times \frac{\sqrt{61}}{6} = \frac{\sqrt{61}}{2}$$

$$\text{Foci: } \left(\pm \frac{\sqrt{61}}{2}, 0 \right)$$

The equation of directrix are:

$$x = \pm \frac{a}{e}$$

$$\Rightarrow x = \pm \frac{3}{\frac{\sqrt{61}}{6}}$$

$$\Rightarrow x = \pm \frac{18}{\sqrt{61}}$$

$$\Rightarrow \sqrt{61}x = \pm 18$$

$$\Rightarrow \sqrt{61}x \mp 18 = 0$$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2\left(\frac{5}{2}\right)^2}{3}$$

$$= \frac{2 \times \frac{25}{4}}{3}$$

$$= \frac{25}{6}$$

5 A. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

Answer

$$\textbf{Given: } 16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

$$\Rightarrow 16x^2 + 32x + 16 - 9y^2 + 36y - 36 - 16 + 36 - 164 = 0$$

$$\Rightarrow 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 16 + 36 - 164 = 0$$

$$\Rightarrow 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 144 = 0$$

$$\Rightarrow 16(x + 1)^2 - 9(y - 2)^2 = 144$$

$$\Rightarrow \frac{16(x+1)^2}{144} - \frac{9(y-2)^2}{144} = 1$$

$$\Rightarrow \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$$

$$\Rightarrow \frac{(x+1)^2}{3^2} - \frac{(y-2)^2}{4^2} = 1$$

Here, **center of the hyperbola is (-1, 2)**

Let $x + 1 = X$ and $y - 2 = Y$

$$\Rightarrow \frac{X^2}{3^2} - \frac{Y^2}{4^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = 4$

$$c = \sqrt{(3)^2 + (4)^2}$$

$$\Rightarrow c = \sqrt{9 + 16}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{3}$$

$$\Rightarrow ae = 3 \times \frac{5}{3} = 5$$

Foci: $(\pm ae, 0)$

$$\Rightarrow X = \pm 5 \text{ and } Y = 0$$

$$\Rightarrow x + 1 = \pm 5 \text{ and } y - 2 = 0$$

$$\Rightarrow x = \pm 5 - 1 \text{ and } y = 2$$

So, Foci: $(\pm 5 - 1, 2)$

Equation of directrix are:

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{3}{\frac{5}{3}}$$

$$\Rightarrow X = \pm \frac{9}{5}$$

$$\Rightarrow 5X = \pm 9$$

$$\Rightarrow 5X \mp 9 = 0$$

$$\Rightarrow 5(x + 1) \mp 9 = 0$$

$$\Rightarrow 5x + 5 \mp 9 = 0$$

$$\Rightarrow 5x + 5 - 9 = 0 \text{ and } 5x + 5 + 9 = 0$$

$$\Rightarrow 5x - 4 = 0 \text{ and } 5x + 14 = 0$$

5 B. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$$x^2 - y^2 + 4x = 0$$

Answer

Given: $x^2 - y^2 + 4x = 0$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$$x^2 - y^2 + 4x = 0$$

$$\Rightarrow x^2 + 4x + 4 - y^2 - 4 = 0$$

$$\Rightarrow (x + 2)^2 - y^2 = 4$$

$$\Rightarrow \frac{(x + 2)^2}{4} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{(x + 2)^2}{2^2} - \frac{y^2}{2^2} = 1$$

Here, **center of the hyperbola is (2, 0)**

Let $x - 2 = X$

$$\Rightarrow \frac{X^2}{2^2} - \frac{y^2}{2^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci are given by $(\pm ae, 0)$

The equation of directrix are $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 2$ and $b = 2$

$$c = \sqrt{(2)^2 + (2)^2}$$

$$\Rightarrow c = \sqrt{4 + 4}$$

$$\Rightarrow c = \sqrt{8}$$

$$\Rightarrow c = 2\sqrt{2}$$

Therefore,

$$e = \frac{2\sqrt{2}}{2}$$

$$\Rightarrow e = \sqrt{2}$$

$$\Rightarrow ae = 2 \times \sqrt{2} = 2\sqrt{2}$$

Foci: $(\pm ae, 0)$

$$\Rightarrow X = \pm 2\sqrt{2} \text{ and } y = 0$$

$$\Rightarrow x + 2 = \pm 2\sqrt{2} \text{ and } y = 0$$

$$\Rightarrow x = \pm 2\sqrt{2} - 2 \text{ and } y = 0$$

So, Foci: $(\pm 2\sqrt{2} - 2, 0)$

Equation of directrix are:

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \sqrt{2}$$

$$\Rightarrow X \mp \sqrt{2} = 0$$

$$\Rightarrow x + 2 \mp \sqrt{2} = 0$$

$$\Rightarrow x + 2 - \sqrt{2} = 0 \text{ and } x + 2 + \sqrt{2} = 0$$

5 C. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$$x^2 - 3y^2 - 2x = 8$$

Answer

Given: $x^2 - 3y^2 - 2x = 8$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$$x^2 - 3y^2 - 2x = 8$$

$$\Rightarrow x^2 - 2x + 1 - 3y^2 - 1 = 8$$

$$\Rightarrow (x - 1)^2 - 3y^2 = 9$$

$$\Rightarrow \frac{(x - 1)^2}{9} - \frac{3y^2}{9} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

Here, **center of the hyperbola is (1, 0)**

Let $x - 1 = X$

$$\Rightarrow \frac{X^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(\pm ae, 0)$

Equation of directrix are: $x = \pm \frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = \sqrt{3}$

$$c = \sqrt{(3)^2 + (\sqrt{3})^2}$$

$$\Rightarrow c = \sqrt{9 + 3}$$

$$\Rightarrow c = \sqrt{12} = 2\sqrt{3}$$

Therefore,

$$e = \frac{2\sqrt{3}}{3}$$

$$\Rightarrow ae = 3 \times \frac{2\sqrt{3}}{3} = 2\sqrt{3}$$

Foci: $(\pm ae, 0)$

$$\Rightarrow X = \pm 2\sqrt{3} \text{ and } y = 0$$

$$\Rightarrow x - 1 = \pm 2\sqrt{3} \text{ and } y = 0$$

$$\Rightarrow x = \pm 2\sqrt{3} + 1 \text{ and } y = 0$$

So, Foci: $(\pm 2\sqrt{3} + 1, 0)$

Equation of directrix are:

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{3}{\frac{2\sqrt{3}}{3}}$$

$$\Rightarrow X = \pm \frac{9}{2\sqrt{3}}$$

$$\Rightarrow 2\sqrt{3}X \mp 9 = 0$$

$$\Rightarrow 2\sqrt{3}(x - 1) \mp 9 = 0$$

$$\Rightarrow 2\sqrt{3}x - 2\sqrt{3} \mp 9 = 0$$

$$\Rightarrow 2\sqrt{3}x - 2\sqrt{3} - 9 = 0 \text{ and } 2\sqrt{3}x - 2\sqrt{3} + 9 = 0$$

6 A. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

the distance between the foci = 16 and eccentricity = $\sqrt{2}$

Answer

Given: the distance between the foci = 16 and eccentricity = $\sqrt{2}$

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

Therefore

$$2ae = 16$$

$$\Rightarrow ae = \frac{16}{2}$$

$$\Rightarrow a \times \sqrt{2} = 8$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a^2 = \frac{64}{2} = 32$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 32\{(\sqrt{2})^2 - 1\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$$\Rightarrow b^2 = 32$$

Equation of hyperbola is:

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow x^2 - y^2 = 32$$

Hence, required equation of hyperbola is $x^2 - y^2 = 32$

6 B. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

conjugate axis is 5 and the distance between foci = 13

Answer

Given: the distance between the foci = 13 and conjugate axis is 5

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

Length of conjugate axis is $2b$

Therefore

$$2b = 5 \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow b^2 = \frac{25}{4}$$

$$2ae = 13$$

$$\Rightarrow ae = \frac{13}{2}$$

$$\Rightarrow a^2e^2 = \frac{169}{4}$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = a^2e^2 - a^2$$

$$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$$

$$\Rightarrow a^2 = \frac{169}{4} - \frac{25}{4}$$

$$\Rightarrow a^2 = \frac{144}{4} = 36$$

Equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{4y^2}{25} = 1$$

$$\Rightarrow \frac{25x^2 - 144y^2}{900} = 1$$

$$\Rightarrow 25x^2 - 144y^2 = 900$$

Hence, required equation of hyperbola is **$25x^2 - 144y^2 = 900$**

6 C. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

conjugate axis is 7 and passes through the point (3, -2).

Answer

Given: conjugate axis is 5 and passes through the point (3, -2)

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Conjugate axis is $2b$

Therefore

$$2b = 5 \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow b^2 = \frac{25}{4}$$

The equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since it passes through (3, -2)

$$\Rightarrow \frac{(3)^2}{a^2} - \frac{(-2)^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4(4)}{25} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{16}{25} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 + \frac{16}{25}$$

$$\Rightarrow \frac{9}{a^2} = \frac{25 + 16}{25}$$

$$\Rightarrow \frac{9}{a^2} = \frac{41}{25}$$

$$\Rightarrow a^2 = \frac{25}{41} \times 9$$

$$\Rightarrow a^2 = \frac{225}{41}$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Since, } a^2 = \frac{225}{41} \text{ and } b^2 = \frac{25}{4}$$

$$\Rightarrow \frac{x^2}{\frac{225}{41}} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{41x^2}{225} - \frac{4y^2}{25} = 1$$

$$\Rightarrow \frac{65x^2 - 36y^2}{441} = 1$$

$$\Rightarrow 65x^2 - 36y^2 = 441$$

Hence, required equation of hyperbola is **$65x^2 - 36y^2 = 441$**

7 A. Question

Find the equation of the hyperbola whose

foci are (6, 4) and (-4, 4) and eccentricity is 2.

Answer

Given: Foci are (6, 4) and (-4, 4) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m - a)^2 + (n - b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m + a}{2}, \frac{n + b}{2} \right)$$

Center of hyperbola having foci (6, 4) and (-4, 4) is given by

$$= \left(\frac{6 - 4}{2}, \frac{4 + 4}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right)$$

$$= (1, 4)$$

The distance between the foci is $2ae$, and Foci are (6, 4) and (-4, 4)

$$\Rightarrow \sqrt{(6 + 4)^2 + (4 - 4)^2} = 2ae$$

$$\Rightarrow \sqrt{(10)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{100} = 2ae$$

$$\Rightarrow 10 = 2ae$$

$$\Rightarrow \frac{10}{2} = ae$$

$$\Rightarrow ae = 5$$

$$\{\because e = 2\}$$

$$\Rightarrow a \times 2 = 5$$

$$\Rightarrow a = \frac{5}{2}$$

$$\Rightarrow a^2 = \frac{25}{4}$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = \frac{25}{4}\{(2)^2 - 1\}$$

$$\Rightarrow b^2 = \frac{25}{4}(4 - 1)$$

$$\Rightarrow b^2 = \frac{25}{4}(3)$$

$$\Rightarrow b^2 = \frac{75}{4}$$

The equation of hyperbola:

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{\frac{25}{4}} - \frac{(y - 4)^2}{\frac{75}{4}} = 1$$

$$\Rightarrow \frac{4(x - 1)^2}{25} - \frac{4(y - 4)^2}{75} = 1$$

$$\Rightarrow \frac{12(x - 1)^2 - 4(y - 4)^2}{75} = 1$$

$$\Rightarrow 12(x - 1)^2 - 4(y - 4)^2 = 75$$

$$\Rightarrow 12(x^2 + 1 - 2x) - 4(y^2 + 16 - 8y) = 75$$

$$\Rightarrow 12x^2 + 12 - 24x - 4y^2 - 64 + 32y - 75 = 0$$

$$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 127 = 0$$

Hence, required equation of hyperbola is **$12x^2 - 4y^2 - 24x + 32y - 127 = 0$**

7 B. Question

Find the equation of the hyperbola whose

vertices are (-8, -1) and (16, -1) and focus is (17, -1)

Answer

Given: Vertices are (-8, -1) and (16, -1) and focus is (17, -1)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is 2a

The distance between the foci and vertex is $ae - a$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Center of hyperbola having vertices (-8, -1) and (16, -1) is given by

$$= \left(\frac{-8+16}{2}, \frac{-1-1}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{-2}{2}\right)$$

$$= (4, -1)$$

The distance between two vertices is $2a$ and vertices are (-8, -1) and (16, -1)

$$\Rightarrow \sqrt{(16+8)^2 + (-1+1)^2} = 2a$$

$$\Rightarrow \sqrt{(24)^2 + (0)^2} = 2a$$

$$\Rightarrow \sqrt{576} = 2a$$

$$\Rightarrow 24 = 2a$$

$$\Rightarrow a = 12$$

The distance between the foci and vertex is $ae - a$, Foci is (17, -1) and the vertex is (16, -1)

$$\Rightarrow \sqrt{(17-16)^2 + (-1+1)^2} = ae - a$$

$$\Rightarrow \sqrt{(1)^2 + (0)^2} = a(e - 1)$$

$$\Rightarrow \sqrt{1} = 12(e - 1)$$

$$\Rightarrow \frac{1}{12} = e - 1$$

$$\Rightarrow e = 1 + \frac{1}{12}$$

$$\Rightarrow e = \frac{13}{12}$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 12 \text{ and } e = \frac{13}{12} \right\}$$

$$\Rightarrow b^2 = (12)^2 \left\{ \left(\frac{13}{12}\right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 144 \left(\frac{169}{144} - 1 \right)$$

$$\Rightarrow b^2 = 144 \left(\frac{169 - 144}{144} \right)$$

$$\Rightarrow b^2 = 25$$

The equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-4)^2}{144} - \frac{(y+1)^2}{25} = 1$$

$$\Rightarrow \frac{25(x-4)^2 - 144(y+1)^2}{3600} = 1$$

$$\Rightarrow 25(x-4)^2 - 144(y+1)^2 = 3600$$

$$\Rightarrow 25(x^2 + 16 - 8x) - 144(y^2 + 1 + 2y) = 3600$$

$$\Rightarrow 25x^2 + 400 - 200x - 144y^2 - 144 - 288y - 3600 = 0$$

$$\Rightarrow 25x^2 - 144y^2 - 200x - 288y - 3344 = 0$$

Hence, required equation of hyperbola is **$25x^2 - 144y^2 - 200x - 288y - 3344 = 0$**

7 C. Question

Find the equation of the hyperbola whose

foci are (4, 2) and (8, 2) and eccentricity is 2.

Answer

Given: Foci are (4, 2) and (8, 2) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2} \right)$$

Center of hyperbola having foci (4, 2) and (8, 2) is given by

$$= \left(\frac{4+8}{2}, \frac{2+2}{2} \right)$$

$$= \left(\frac{12}{2}, \frac{4}{2} \right)$$

$$= (6, 2)$$

The distance between the foci is $2ae$ and Foci are (4, 2) and (8, 2)

$$\Rightarrow \sqrt{(4-8)^2 + (2-2)^2} = 2ae$$

$$\Rightarrow \sqrt{(-4)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{16} = 2ae$$

$$\Rightarrow 4 = 2ae$$

$$\Rightarrow \frac{4}{2} = ae$$

$$\Rightarrow ae = 2$$

$$\{\because e = 2\}$$

$$\Rightarrow a \times 2 = 2$$

$$\Rightarrow a = 1$$

$$\Rightarrow a^2 = 1$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1\{(2)^2 - 1\}$$

$$\Rightarrow b^2 = 1(4 - 1)$$

$$\Rightarrow b^2 = 3$$

The equation of hyperbola:

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x - 6)^2}{1} - \frac{(y - 2)^2}{3} = 1$$

$$\Rightarrow \frac{(x - 6)^2}{1} - \frac{(y - 2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x - 6)^2 - (y - 2)^2}{3} = 1$$

$$\Rightarrow 3(x - 6)^2 - (y - 2)^2 = 3$$

$$\Rightarrow 3(x^2 + 36 - 12x) - (y^2 + 4 - 4y) = 3$$

$$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y - 3 = 0$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$$

Hence, required equation of hyperbola is **$3x^2 - y^2 - 36x + 4y + 101 = 0$**

7 D. Question

Find the equation of the hyperbola whose

vertices are at $(0, \pm 7)$ and foci at $\left(0, \pm \frac{28}{3}\right)$

Answer

Given: Vertices are $(0, \pm 7)$ and foci are $\left(0, \pm \frac{28}{3}\right)$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Vertices of the hyperbola are given by $(0, \pm b)$

Foci of the hyperbola are given by $(0, \pm be)$

Vertices are $(0, \pm 7)$ and foci are $(0, \pm \frac{28}{3})$

Therefore,

$$b = 7 \text{ and } be = \frac{28}{3}$$

$$\Rightarrow 7 \times e = \frac{28}{3}$$

$$\Rightarrow e = \frac{4}{3}$$

$$a^2 = b^2(e^2 - 1)$$

$$\left\{ \because b = 7 \text{ and } e = \frac{4}{3} \right\}$$

$$\Rightarrow a^2 = 7^2 \left\{ \left(\frac{4}{3} \right)^2 - 1 \right\}$$

$$\Rightarrow a^2 = 49 \left(\frac{16}{9} - 1 \right)$$

$$\Rightarrow a^2 = 49 \left(\frac{16 - 9}{9} \right)$$

$$\Rightarrow a^2 = 49 \times \frac{7}{9}$$

$$\Rightarrow a^2 = \frac{343}{9}$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{\frac{343}{9}} - \frac{y^2}{49} = -1$$

$$\Rightarrow \frac{9x^2}{343} - \frac{y^2}{49} = -1$$

$$\Rightarrow \frac{9x^2 - 7y^2}{343} = -1$$

$$\Rightarrow 9x^2 - 7y^2 = -343$$

$$\Rightarrow 9x^2 - 7y^2 + 343 = 0$$

Hence, required equation of hyperbola is **$9x^2 - 7y^2 + 343 = 0$**

7 E. Question

Find the equation of the hyperbola whose

vertices are at $(\pm 6, 0)$ and one of the directrices is $x = 4$.

Answer

Given: Vertices are $(\pm 6, 0)$ and one of the directrices is $x = 4$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of the hyperbola are given by $(\pm a, 0)$

The equation of the directrices: $x = \pm \frac{a}{e}$

Vertices are $(\pm 6, 0)$ and one of the directrices is $x = 4$

Therefore,

$$a = 6 \text{ and } \frac{a}{e} = 4$$

$$\Rightarrow \frac{6}{e} = 4$$

$$\Rightarrow e = \frac{6}{4}$$

$$\Rightarrow e = \frac{3}{2}$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 6 \text{ and } e = \frac{3}{2} \right\}$$

$$\Rightarrow b^2 = (6)^2 \left\{ \left(\frac{3}{2} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 36 \left(\frac{9}{4} - 1 \right)$$

$$\Rightarrow b^2 = 36 \left(\frac{9 - 4}{4} \right)$$

$$\Rightarrow b^2 = 36 \times \frac{5}{4}$$

$$\Rightarrow b^2 = 45$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{45} = 1$$

$$\Rightarrow \frac{5x^2 - 4y^2}{180} = 1$$

$$\Rightarrow 5x^2 - 4y^2 = 180$$

$$\Rightarrow 5x^2 - 4y^2 - 180 = 0$$

Hence, required equation of hyperbola is **$5x^2 - 4y^2 - 180 = 0$**

7 F. Question

Find the equation of the hyperbola whose

foci at $(\pm 2, 0)$ and eccentricity is $3/2$.

Answer

Given: Foci are $(2, 0)$ and $(-2, 0)$ and eccentricity is $\frac{3}{2}$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m - a)^2 + (n - b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m + a}{2}, \frac{n + b}{2} \right)$$

Center of hyperbola having Foci $(2, 0)$ and $(-2, 0)$ is given by

$$= \left(\frac{2 - 2}{2}, \frac{0 - 0}{2} \right)$$

$$= \left(\frac{0}{2}, \frac{0}{2} \right)$$

$$= (0, 0)$$

The distance between the foci is $2ae$, and Foci are $(2, 0)$ and $(-2, 0)$

$$\Rightarrow \sqrt{(2 + 2)^2 + (0 - 0)^2} = 2ae$$

$$\Rightarrow \sqrt{(4)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{16} = 2ae$$

$$\Rightarrow 4 = 2ae$$

$$\Rightarrow \frac{4}{2} = ae$$

$$\Rightarrow ae = 2$$

$$\left\{ \because e = \frac{3}{2} \right\}$$

$$\Rightarrow a \times \frac{3}{2} = 2$$

$$\Rightarrow a = \frac{4}{3}$$

$$\Rightarrow a^2 = \frac{16}{9}$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = \frac{16}{9} \left\{ \left(\frac{3}{2} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = \frac{16}{9} \left(\frac{9}{4} - 1 \right)$$

$$\Rightarrow b^2 = \frac{16}{9} \left(\frac{9-4}{4} \right)$$

$$\Rightarrow b^2 = \frac{16}{9} \left(\frac{5}{4} \right)$$

$$\Rightarrow b^2 = \frac{20}{9}$$

The equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-0)^2}{\frac{16}{9}} - \frac{(y-0)^2}{\frac{20}{9}} = 1$$

$$\Rightarrow \frac{9x^2}{16} - \frac{9y^2}{20} = 1$$

$$\Rightarrow \frac{45x^2 - 36y^2}{80} = 1$$

$$\Rightarrow 45x^2 - 36y^2 = 80$$

$$\Rightarrow 45x^2 - 36y^2 - 80 = 0$$

Hence, required equation of hyperbola is **$45x^2 - 36y^2 - 80 = 0$**

8. Question

Find the eccentricity of the hyperbola, the length of whose conjugate axis is $\frac{3}{4}$ of the length of the transverse axis.

Answer

Given: the length of whose conjugate axis is $\frac{3}{4}$ of the length of the transverse axis

To find: eccentricity of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of the conjugate axis is $2b$ and length of transverse axis is $2a$

According to question:

$$2b = \frac{3}{4} \times 2a$$

$$\Rightarrow \frac{b}{a} = \frac{3}{4}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{9}{16}$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{9}{16}}$$

$$\Rightarrow e = \sqrt{\frac{16+9}{16}}$$

$$\Rightarrow e = \sqrt{\frac{25}{16}}$$

$$\Rightarrow e = \frac{5}{4}$$

Hence, the eccentricity of the hyperbola is $\frac{5}{4}$

9 A. Question

Find the equation of the hyperbola whose

the focus is at (5, 2), vertices at (4, 2) and (2, 2) and centre at (3, 2)

Answer

Given: Vertices are (4, 2) and (2, 2), the focus is (5, 2) and centre (3, 2)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is $2a$

The distance between the foci and vertex is $ae - a$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2} \right)$$

The distance between two vertices is $2a$ and vertices are $(4, 2)$ and $(2, 2)$

$$\Rightarrow \sqrt{(4-2)^2 + (2-2)^2} = 2a$$

$$\Rightarrow \sqrt{(2)^2 + (0)^2} = 2a$$

$$\Rightarrow \sqrt{4} = 2a$$

$$\Rightarrow 2 = 2a$$

$$\Rightarrow a = 1$$

The distance between the foci and vertex is $ae - a$, Foci is $(5, 2)$ and the vertex is $(4, 2)$

$$\Rightarrow \sqrt{(5-4)^2 + (2-2)^2} = ae - a$$

$$\Rightarrow \sqrt{(1)^2 + (0)^2} = a(e - 1)$$

$$\Rightarrow \sqrt{1} = 1(e - 1)$$

$$\Rightarrow 1 = e - 1$$

$$\Rightarrow e = 1 + 1$$

$$\Rightarrow e = 2$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 1 \text{ and } e = 2 \right\}$$

$$\Rightarrow b^2 = (1)^2\{(2)^2 - 1\}$$

$$\Rightarrow b^2 = 1(4 - 1)$$

$$\Rightarrow b^2 = 1(3)$$

$$\Rightarrow b^2 = 3$$

The equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$\{\because \text{Centre } (3, 2)\}$

$$\Rightarrow \frac{(x-3)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x-3)^2 - (y-2)^2}{3} = 1$$

$$\Rightarrow 3(x-3)^2 - (y-2)^2 = 3$$

$$\Rightarrow 3(x^2 + 9 - 6x) - (y^2 + 4 - 4y) = 3$$

$$\Rightarrow 3x^2 + 27 - 18x - y^2 - 4 + 4y - 3 = 0$$

$$\Rightarrow 3x^2 - y^2 - 18x + 4y + 20 = 0$$

Hence, required equation of hyperbola is **$3x^2 - y^2 - 18x + 4y + 20 = 0$**

9 B. Question

Find the equation of the hyperbola whose

focus is at (4, 2), centre at (6, 2) and $e = 2$.

Answer

Given: Foci is (4, 2), $e = 2$ and center at (6, 2)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is $2a$

The distance between the foci and vertex is $ae - a$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Therefore

Let one of the two foci is (m, n) and the other one is (4, 2)

Since, Centre(6, 2)

$$\left(\frac{m+4}{2} = 6, \frac{n+2}{2} = 2\right)$$

$$\Rightarrow (m+4 = 12, n+2 = 4)$$

$$\Rightarrow (m = 8, n = 2)$$

Foci are (4, 2) and (8, 2)

The distance between the foci is $2ae$ and Foci are (4, 2) and (8, 2)

$$\Rightarrow \sqrt{(4-8)^2 + (2-2)^2} = 2ae$$

$$\Rightarrow \sqrt{(-4)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{16} = 2ae$$

$$\Rightarrow 4 = 2ae$$

$$\Rightarrow \frac{4}{2} = ae$$

$$\Rightarrow ae = 2$$

$$\{\because e = 2\}$$

$$\Rightarrow a \times 2 = 2$$

$$\Rightarrow a = \frac{2}{2} = 1$$

$$\Rightarrow a^2 = 1$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1\{(2)^2 - 1\}$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

The equation of hyperbola:

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x - 6)^2}{1} - \frac{(y - 2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x - 6)^2 - (y - 2)^2}{3} = 1$$

$$\Rightarrow 3(x^2 + 36 - 12x) - (y^2 + 4 - 4y) = 3$$

$$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y - 3 = 0$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$$

Hence, required equation of hyperbola is **$3x^2 - y^2 - 36x + 4y + 101 = 0$**

10. Question

If P is any point on the hyperbola whose axis are equal, prove that $SP.S'P = CP^2$

Answer

Given: Axis of the hyperbola are equal, i.e. $a = b$

To prove: $SP.S'P = CP^2$

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{a^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + 1}$$

$$\Rightarrow e = \sqrt{2}$$

Foci of the hyperbola are given by $(\pm ae, 0)$

$\Rightarrow$ Foci of hyperbola are given by $(\pm\sqrt{2}a, 0)$

So, $S(\sqrt{2}a, 0)$ and $S'(-\sqrt{2}a, 0)$

Let P (m, n) be any point on the hyperbola

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m - a)^2 + (n - b)^2}$

$$SP = \sqrt{(m - \sqrt{2}a)^2 + (n - 0)^2}$$

$$\Rightarrow SP^2 = m^2 + 2a^2 - 2\sqrt{2}am + n^2$$

$$S'P = \sqrt{(m + \sqrt{2}a)^2 + (n - 0)^2}$$

$$\Rightarrow S'P^2 = m^2 + 2a^2 + 2\sqrt{2}am + n^2$$

C is Centre with coordinates (0, 0)

$$CP = \sqrt{(m - 0)^2 + (n - 0)^2}$$

$$\Rightarrow CP^4 = (m^2 + n^2)^2$$

$$\Rightarrow CP^4 = m^4 + n^4 + 2m^2n^2 \dots \dots \dots (i)$$

Now,

$$SP^2 \cdot S'P^2 = (m^2 + 2a^2 + n^2 - 2\sqrt{2}am)(m^2 + 2a^2 + n^2 + 2\sqrt{2}am)$$

$$\Rightarrow SP^2 \cdot S'P^2 = (m^2 + 2a^2 + n^2)^2 - (2\sqrt{2}am)^2$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + 4a^4 + n^4 + 4a^2m^2 + 4a^2n^2 + 2m^2n^2 - 8a^2m^2$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + 4a^4 + n^4 + 4a^2n^2 + 2m^2n^2 - 4a^2m^2$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(a^2 + n^2 - m^2)$$

$$\{\because a^2 = m^2 - n^2\}$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(m^2 - n^2 + n^2 - m^2)$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(0)$$

$$\Rightarrow SP^2 \cdot S'P^2 = m^4 + n^4 + 2m^2n^2$$

From (i):

$$\Rightarrow SP^2 \cdot S'P^2 = CP^4$$

Taking square root both sides:

$$\Rightarrow \sqrt{SP^2 \cdot S'P^2} = \sqrt{CP^4}$$

$$\Rightarrow SP \cdot S'P = CP^2$$

Hence Proved

11 A. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices ($\pm 2, 0$), foci ($\pm 3, 0$)

Answer

Given: Vertices are ($\pm 2, 0$) and foci are ($\pm 3, 0$)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of the hyperbola are given by $(\pm a, 0)$

Foci of the hyperbola are given by $(\pm ae, 0)$

Vertices are $(\pm 2, 0)$ and foci are $(\pm 3, 0)$

Therefore,

$$a = 2 \text{ and } ae = 3$$

$$\Rightarrow 2 \times e = 3$$

$$\Rightarrow e = \frac{3}{2}$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 2 \text{ and } e = \frac{3}{2} \right\}$$

$$\Rightarrow b^2 = 2^2 \left\{ \left(\frac{3}{2} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 4 \left(\frac{9}{4} - 1 \right)$$

$$\Rightarrow b^2 = 4 \left(\frac{9 - 4}{4} \right)$$

$$\Rightarrow b^2 = 4 \times \frac{5}{4}$$

$$\Rightarrow b^2 = 5$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$$

$$\Rightarrow \frac{5x^2 - 4y^2}{20} = 1$$

$$\Rightarrow 5x^2 - 4y^2 = 20$$

$$\Rightarrow 5x^2 - 4y^2 - 20 = 0$$

Hence, required equation of hyperbola is $5x^2 - 4y^2 - 20 = 0$

11 B. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices $(0, \pm 5)$, foci $(0, \pm 8)$

Answer

Given: Vertices are $(0, \pm 5)$ and foci are $(0, \pm 8)$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Vertices of the hyperbola are given by $(0, \pm b)$

Foci of the hyperbola are given by $(0, \pm be)$

Vertices are $(0, \pm 5)$ and foci are $(0, \pm 8)$

Therefore,

$$b = 5 \text{ and } be = 8$$

$$\Rightarrow 5 \times e = 8$$

$$\Rightarrow e = \frac{8}{5}$$

$$a^2 = b^2(e^2 - 1)$$

$$\left\{ \because b = 5 \text{ and } e = \frac{8}{5} \right\}$$

$$\Rightarrow a^2 = 5^2 \left\{ \left(\frac{8}{5} \right)^2 - 1 \right\}$$

$$\Rightarrow a^2 = 25 \left(\frac{64}{25} - 1 \right)$$

$$\Rightarrow a^2 = 25 \left(\frac{64 - 25}{25} \right)$$

$$\Rightarrow a^2 = 25 \times \frac{39}{25}$$

$$\Rightarrow a^2 = 39$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{39} - \frac{y^2}{25} = -1$$

Hence, the required equation of the hyperbola is $\frac{x^2}{39} - \frac{y^2}{25} = -1$

11 C. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices $(0, \pm 3)$, foci $(0, \pm 5)$

Answer

Given: Vertices are $(0, \pm 3)$ and foci are $(0, \pm 5)$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Vertices of the hyperbola are given by $(0, \pm b)$

Foci of the hyperbola are given by $(0, \pm be)$

Vertices are $(0, \pm 3)$ and foci are $(0, \pm 5)$

Therefore,

$$b = 3 \text{ and } be = 5$$

$$\Rightarrow 3 \times e = 5$$

$$\Rightarrow e = \frac{5}{3}$$

$$a^2 = b^2(e^2 - 1)$$

$$\left\{ \because b = 3 \text{ and } e = \frac{5}{3} \right\}$$

$$\Rightarrow a^2 = 3^2 \left\{ \left(\frac{5}{3} \right)^2 - 1 \right\}$$

$$\Rightarrow a^2 = 9 \left(\frac{25}{9} - 1 \right)$$

$$\Rightarrow a^2 = 9 \left(\frac{25 - 9}{9} \right)$$

$$\Rightarrow a^2 = 9 \times \frac{16}{9}$$

$$\Rightarrow a^2 = 16$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = -1$$

Hence, the required equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = -1$

11 D. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(\pm 5, 0)$, transverse axis = 8

Answer

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis is $2a$

Coordinates of the foci for a standard hyperbola is given by $(\pm ae, 0)$

According to question:

$$2a = 8 \text{ and } ae = 5$$

$$2a = 8$$

$$\Rightarrow a = \frac{8}{2}$$

$$\Rightarrow a = 4$$

$$\Rightarrow a^2 = 16$$

$$ae = 5$$

$$\Rightarrow 4 \times e = 5$$

$$\Rightarrow e = \frac{5}{4}$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 16 \left\{ \left(\frac{5}{4} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 16 \left(\frac{25}{4} - 1 \right)$$

$$\Rightarrow b^2 = 16 \left(\frac{25 - 4}{4} \right)$$

$$\Rightarrow b^2 = 16 \left(\frac{21}{4} \right)$$

$$\Rightarrow b^2 = 4(21)$$

$$\Rightarrow b^2 = 84$$

Hence, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{84} = 1$$

11 E. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(0, \pm 13)$, conjugate axis = 24

Answer

Given: foci $(0, \pm 13)$ and the conjugate axis is 24

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Length of the conjugate axis is $2b$

Coordinates of the foci for a standard hyperbola is given by $(0, \pm be)$

According to question:

$$2b = 24 \text{ and } be = 13$$

$$2b = 24$$

$$\Rightarrow b = \frac{24}{2}$$

$$\Rightarrow b = 12$$

$$\Rightarrow b^2 = 144$$

$$be = 12$$

$$\Rightarrow 12 \times e = 13$$

$$\Rightarrow e = \frac{13}{12}$$

We know,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = 144 \left\{ \left(\frac{13}{12} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 144 \left(\frac{169}{144} - 1 \right)$$

$$\Rightarrow b^2 = 144 \left(\frac{169 - 144}{144} \right)$$

$$\Rightarrow b^2 = 144 \left(\frac{25}{144} \right)$$

$$\Rightarrow b^2 = 25$$

Hence, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{144} - \frac{y^2}{25} = -1$$

11 F. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(\pm 3\sqrt{5}, 0)$, the latus-rectum = 8

Answer

Given: Foci $(\pm 3\sqrt{5}, 0)$ and the latus-rectum = 8

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Coordinates of the foci for a standard hyperbola is given by $(\pm ae, 0)$

Length of latus rectum is $\frac{2b^2}{a}$

According to the question:

$$ae = 3\sqrt{5} \text{ and } \frac{2b^2}{a} = 8$$

$$ae = 3\sqrt{5}$$

$$\Rightarrow e = \frac{3\sqrt{5}}{a}$$

$$\Rightarrow e^2 = \left(\frac{3\sqrt{5}}{a}\right)^2$$

$$\Rightarrow e^2 = \frac{45}{a^2}$$

$$\frac{2b^2}{a} = 8$$

$$b^2 = \frac{8a}{2}$$

$$b^2 = 4a$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow 4a = a^2 \left\{ \frac{45}{a^2} - 1 \right\}$$

$$\Rightarrow 4a = a^2 \left(\frac{45 - a^2}{a^2} \right)$$

$$\Rightarrow 4a = 45 - a^2$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow a^2 + 9a - 5a - 45 = 0$$

$$\Rightarrow a(a + 9) - 5(a + 9) = 0$$

$$\Rightarrow (a + 9)(a - 5) = 0$$

$$\Rightarrow a = -9 \text{ or } a = 5$$

Since a is a distance, and it can't be negative

$$\Rightarrow a = 5$$

$$\Rightarrow a^2 = 25$$

$$b^2 = 4a$$

$$\Rightarrow b^2 = 4(5)$$

$$\Rightarrow b^2 = 20$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{25} - \frac{y^2}{20} = 1$$

11 G. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(\pm 4, 0)$, the latus-rectum = 12

Answer

Given: Foci $(\pm 4, 0)$, the latus-rectum = 12

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Coordinates of the foci for a standard hyperbola is given by $(\pm ae, 0)$

Length of latus rectum is $\frac{2b^2}{a}$

According to the question:

$$ae = 4 \text{ and } \frac{2b^2}{a} = 12$$

$$ae = 4$$

$$\Rightarrow e = \frac{4}{a}$$

$$\Rightarrow e^2 = \left(\frac{4}{a}\right)^2$$

$$\Rightarrow e^2 = \frac{16}{a^2}$$

$$\frac{2b^2}{a} = 12$$

$$b^2 = \frac{12a}{2}$$

$$b^2 = 6a$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow 6a = a^2 \left\{ \frac{16}{a^2} - 1 \right\}$$

$$\Rightarrow 6a = a^2 \left(\frac{16 - a^2}{a^2} \right)$$

$$\Rightarrow 6a = 16 - a^2$$

$$\Rightarrow a^2 + 6a - 16 = 0$$

$$\Rightarrow a^2 + 8a - 2a - 16 = 0$$

$$\Rightarrow a(a + 8) - 2(a + 8) = 0$$

$$\Rightarrow (a + 8)(a - 2) = 0$$

$$\Rightarrow a = -8 \text{ or } a = 2$$

Since a is a distance, and it can't be negative,

$$\Rightarrow a = 2$$

$$\Rightarrow a^2 = 4$$

$$b^2 = 6a$$

$$\Rightarrow b^2 = 6(2)$$

$$\Rightarrow b^2 = 12$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 1$$

1 H. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

$$\text{vertices } (0, \pm 6), e = \frac{5}{3}$$

Answer

$$\textbf{Given:} \text{ Vertices } (0, \pm 6), e = \frac{5}{3}$$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Coordinates of the vertices for a standard hyperbola is given by $(0, \pm b)$

According to question:

$$b = 6 \Rightarrow b^2 = 36$$

We know,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = 6^2 \left\{ \left(\frac{5}{3} \right)^2 - 1 \right\}$$

$$\left\{ \because e = \frac{5}{3} \right\}$$

$$\Rightarrow a^2 = 36 \left(\frac{25}{9} - 1 \right)$$

$$\Rightarrow a^2 = 36 \left(\frac{25 - 9}{9} \right)$$

$$\Rightarrow a^2 = 36\left(\frac{16}{9}\right)$$

$$\Rightarrow a^2 = 64$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{64} - \frac{y^2}{36} = -1$$

11 I. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Answer

Given: Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Coordinates of the foci for a standard hyperbola is given by $(0, \pm be)$

According to the question:

$$be = \sqrt{10}$$

$$\Rightarrow b^2e^2 = 10$$

Since $(2, 3)$ passing through hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Therefore,

$$\frac{2^2}{a^2} - \frac{3^2}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$$

$$\{\because a^2 = b^2(e^2 - 1)\}$$

$$\Rightarrow \frac{4}{b^2(e^2 - 1)} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4}{b^2e^2 - b^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4}{10 - b^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4b^2 - 9(10 - b^2)}{(10 - b^2)b^2} = -1$$

$$\Rightarrow \frac{4b^2 - 90 + 9b^2}{(10 - b^2)b^2} = -1$$

$$\Rightarrow \frac{-(90 - 13b^2)}{(10 - b^2)b^2} = -1$$

$$\Rightarrow 90 - 13b^2 = (10 - b^2)b^2$$

$$\Rightarrow 90 - 13b^2 = 10b^2 - b^4$$

$$\Rightarrow 90 - 13b^2 - 10b^2 + b^4 = 0$$

$$\Rightarrow b^4 - 23b^2 + 90 = 0$$

$$\Rightarrow b^4 - 18b^2 - 5b^2 + 90 = 0$$

$$\Rightarrow b^2(b^2 - 18) - 5(b^2 - 18) = 0$$

$$\Rightarrow (b^2 - 18)(b^2 - 5) = 0$$

$$\Rightarrow b^2 = 18 \text{ or } 5$$

Case 1: $b^2 = 18$ and $b^2e^2 = 10$

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = b^2e^2 - b^2$$

$$\Rightarrow a^2 = 10 - 18$$

$$\Rightarrow a^2 = -8$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{-8} - \frac{y^2}{10} = -1$$

$$\Rightarrow -\left(\frac{x^2}{8} + \frac{y^2}{10}\right) = -1$$

$$\Rightarrow \frac{x^2}{8} + \frac{y^2}{10} = 1$$

Case 2: $b^2 = 5$ and $b^2e^2 = 10$

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = b^2e^2 - b^2$$

$$\Rightarrow a^2 = 10 - 5$$

$$\Rightarrow a^2 = 5$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2}{10} = -1$$

11 J. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(0, \pm 12)$, latus-rectum = 36.

Answer

Given: Foci $(0, \pm 12)$, the latus-rectum = 36

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Coordinates of the foci for a standard hyperbola is given by $(0, \pm be)$

Length of latus rectum is $\frac{2a^2}{b}$

According to the question:

$$be = 12 \text{ and } \frac{2a^2}{b} = 36$$

$$be = 12$$

$$\Rightarrow e = \frac{12}{b}$$

$$\Rightarrow e^2 = \left(\frac{12}{b}\right)^2$$

$$\Rightarrow e^2 = \frac{144}{b^2}$$

$$\frac{2a^2}{b} = 36$$

$$\Rightarrow a^2 = \frac{36b}{2}$$

$$\Rightarrow a^2 = 18b$$

We know,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow 18b = b^2 \left\{ \frac{144}{b^2} - 1 \right\}$$

$$\Rightarrow 18b = b^2 \left(\frac{144 - b^2}{b^2} \right)$$

$$\Rightarrow 18b = 144 - b^2$$

$$\Rightarrow b^2 + 18b - 144 = 0$$

$$\Rightarrow b^2 + 24b - 6b - 144 = 0$$

$$\Rightarrow b(b + 24) - 6(b + 24) = 0$$

$$\Rightarrow (b + 24)(b - 6) = 0$$

$$\Rightarrow b = -24 \text{ or } b = 6$$

Since b is a distance, and it can't be negative

$$\Rightarrow b = 6$$

$$\Rightarrow b^2 = 36$$

$$a^2 = 18b$$

$$\Rightarrow a^2 = 18(6)$$

$$\Rightarrow b^2 = 108$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{108} = -1$$

12. Question

If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain its equation.

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The distance between foci is given by $2ae$

According to question:

$$2ae = 16$$

$$\Rightarrow a = \frac{16}{2e}$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a = 4\sqrt{2}$$

$$\Rightarrow a^2 = 32$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 32\{(\sqrt{2})^2 - 1\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$$\Rightarrow b^2 = 32$$

Hence, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow x^2 - y^2 = 32$$

13. Question

Show that the set of all points such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2 represents a hyperbola.

Answer

To prove: the set of all points under given conditions represents a hyperbola

Let a point P be (x, y) such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2.

Formula used:

The distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2 + (n-b)^2}$$

The distance of P(x, y) from (4, 0) is $\sqrt{(x-4)^2 + (y-0)^2}$

The distance of P(x, y) from (-4, 0) is $\sqrt{(x+4)^2 + (y-0)^2}$

Since, the difference of their distances from (4, 0) and (-4, 0) is always equal to 2

Therefore,

$$\sqrt{(x+4)^2 + (y-0)^2} - \sqrt{(x-4)^2 + (y-0)^2} = 2$$

$$\Rightarrow \sqrt{(x+4)^2 + (y-0)^2} = 2 + \sqrt{(x-4)^2 + (y-0)^2}$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x+4)^2 + (y-0)^2} \right)^2 = \left(2 + \sqrt{(x-4)^2 + (y-0)^2} \right)^2$$

$$\begin{aligned} \Rightarrow \left(\sqrt{(x+4)^2 + (y-0)^2} \right)^2 \\ = 2^2 + \left(\sqrt{(x-4)^2 + (y-0)^2} \right)^2 + 4\sqrt{(x-4)^2 + (y-0)^2} \end{aligned}$$

$$\Rightarrow (x+4)^2 + (y)^2 = 4 + (x-4)^2 + (y)^2 + 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow x^2 + 16 + 8x + y^2 = 4 + x^2 + 16 - 8x + y^2 + 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow x^2 + 16 + 8x + y^2 - 4 - x^2 - 16 + 8x - y^2 = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 16x - 4 = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 4(4x - 1) = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 4x - 1 = \sqrt{(x-4)^2 + (y)^2}$$

Squaring both sides:

$$\Rightarrow (4x - 1)^2 = \left(\sqrt{(x - 4)^2 + (y)^2} \right)^2$$

$$\Rightarrow 16x^2 + 1 - 8x = (x - 4)^2 + y^2$$

$$\Rightarrow 16x^2 + 1 - 8x = x^2 + 16 - 8x + y^2$$

$$\Rightarrow 16x^2 + 1 - 8x - x^2 - 16 + 8x - y^2 = 0$$

$$\Rightarrow 15x^2 - y^2 - 15 = 0$$

Hence, required equation of hyperbola is $15x^2 - y^2 - 15 = 0$

Very Short Answer

1. Question

Write the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e)

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Here, $a = 4$ and $b = 3$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

Hence, eccentricity is $\frac{5}{4}$

2. Question

Write the eccentricity of the hyperbola whose latus-rectum is half of its transverse axis.

Answer

Given: Latus-rectum is half of its transverse axis

To find: eccentricity of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis is $2a$

Latus-rectum of the hyperbola is $\frac{2b^2}{a}$

According to question:

Latus-rectum is half of its transverse axis

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow 2b^2 = a^2$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{2b^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{2+1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

Hence, eccentricity is $\sqrt{\frac{3}{2}}$

3. Question

Write the coordinates of the foci of the hyperbola $9x^2 - 16y^2 = 144$.

Answer

Given: $9x^2 - 16y^2 = 144$

To find: coordinates of the foci $f(m,n)$

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(\pm ae, 0)$

Here, $a = 4$ and $b = 3$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

$$\Rightarrow ae = 4 \times \frac{5}{4} = 5$$

Foci: $(\pm 5, 0)$

4. Question

Write the equation of the hyperbola of eccentricity $\sqrt{2}$, if it is known that the distance between its foci is 16.

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between foci is given by $2ae$

According to question:

$$2ae = 16$$

$$\Rightarrow a = \frac{16}{2e}$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a = 4\sqrt{2}$$

$$\Rightarrow a^2 = 32$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 32\{(\sqrt{2})^2 - 1\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$$\Rightarrow b^2 = 32$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow x^2 - y^2 = 32$$

5. Question

If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, write the value of b^2

Answer

To find: value of b^2

Given: foci of given ellipse and hyperbola coincide

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{25x^2}{441} - \frac{25y^2}{81} = 1$$

$$\Rightarrow \frac{x^2}{\frac{441}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Formula used:

Coordinates of the foci for standard ellipse is given by $(\pm c_1, 0)$ where $c_1^2 = a_1^2 - b_1^2$

Coordinates of the foci for standard hyperbola is given by $(\pm c_2, 0)$ where $c_2^2 = a_2^2 + b_2^2$

Since, their foci coincide

$$\Rightarrow c_1^2 = c_2^2$$

$$\Rightarrow a_1^2 - b_1^2 = a_2^2 + b_2^2$$

Here $a_1 = 4$, $b_1 = b$, $a_2 = \frac{12}{5}$ and $b_2 = \frac{9}{5}$

$$\Rightarrow 4^2 - b^2 = \frac{12^2}{5^2} + \frac{9^2}{5^2}$$

$$\Rightarrow 16 - b^2 = \frac{144}{25} + \frac{81}{25}$$

$$\Rightarrow 16 - b^2 = \frac{225}{25}$$

$$\Rightarrow 16 - b^2 = 9$$

$$\Rightarrow b^2 = 16 - 9$$

$$\Rightarrow \mathbf{b^2 = 7}$$

6. Question

Write the length of the latus-rectum of the hyperbola $16x^2 - 9y^2 = 144$.

Answer

Given: $16x^2 - 9y^2 = 144$

To find: length of latus-rectum of hyperbola.

$$16x^2 - 9y^2 = 144$$

$$\Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = 4$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2 \times (4)^2}{3}$$

$$= \frac{32}{3}$$

7. Question

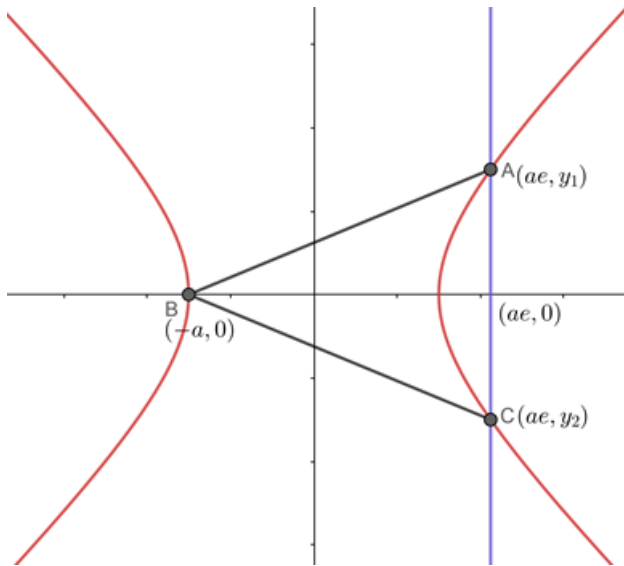
If the latus-rectum through one focus of a hyperbola subtends a right angle at the farther vertex, then write the eccentricity of the hyperbola.

Answer

Given: latus-rectum through one focus of a hyperbola subtends a right angle at the farther vertex

To find: eccentricity of hyperbola

Let B is vertex of hyperbola and A and C are point of intersection of latus-rectum and hyperbola



Standard equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since, A and C lie on hyperbola

Therefore

$$\frac{(ae)^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{a^2 e^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow e^2 - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{y_1^2}{b^2} = e^2 - 1$$

$$\Rightarrow y_1^2 = b^2(e^2 - 1) \dots \dots \dots (i)$$

As angle between AB and AC is 90°

$$\Rightarrow \text{Slope}_{AB} \times \text{Slope}_{BC} = -1$$

$$\Rightarrow \frac{y_1}{ae + a} \times -\frac{y_1}{ae + a} = -1$$

$$\Rightarrow \left(\frac{y_1}{ae + a} \right)^2 = 1$$

$$\Rightarrow \frac{y_1^2}{\{a(e + 1)\}^2} = 1$$

From (i):

$$\Rightarrow \frac{b^2(e^2 - 1)}{a^2(e + 1)^2} = 1$$

$$\Rightarrow \frac{a^2(e^2 - 1)(e^2 - 1)}{a^2(e + 1)^2} = 1$$

$$\{\because b^2 = a^2(e^2 - 1)\}$$

$$\Rightarrow \frac{(e^2 - 1)^2}{(e + 1)^2} = 1$$

$$\Rightarrow (e^2 - 1)^2 = (e + 1)^2$$

$$\Rightarrow e^4 + 1 - 2e^2 = e^2 + 1 + 2e$$

$$\Rightarrow e^4 + 1 - 2e^2 - e^2 - 1 - 2e = 0$$

$$\Rightarrow e^4 - 3e^2 - 2e = 0$$

$$\Rightarrow e(e^3 - 3e - 2) = 0$$

$$\Rightarrow e(e - 2)(e + 1)^2 = 0$$

$$\Rightarrow e = 0 \text{ or } 2 \text{ or } -1$$

But e should be greater than or equal to 1 for hyperbola

$$\Rightarrow e = 2$$

Hence, eccentricity of hyperbola is 2

8. Question

Write the distance between the directrices of the hyperbola

$$x = 8 \sec \theta, y = 8 \tan \theta.$$

Answer

Given: Hyperbola is $x = 8 \sec \theta$ and $y = 8 \tan \theta$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between directrix is given by $\frac{2a}{e}$

$$x = 8 \sec \theta \text{ and } y = 8 \tan \theta$$

$$\Rightarrow \sec \theta = \frac{x}{8} \text{ and } \tan \theta = \frac{y}{8}$$

We know,

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \left(\frac{x}{8}\right)^2 - \left(\frac{y}{8}\right)^2 = 1$$

$$\Rightarrow \frac{x^2}{8^2} - \frac{y^2}{8^2} = 1$$

Here $a = 8$ and $b = 8$

Now,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{8^2}{8^2}}$$

$$\Rightarrow e = \sqrt{1 + 1}$$

$$\Rightarrow e = \sqrt{2}$$

Hence, distance between directrix,

$$= \frac{2a}{e}$$

$$= \frac{2(8)}{\sqrt{2}}$$

$$= 8\sqrt{2}$$

9. Question

Write the equation of the hyperbola whose vertices are $(\pm 3, 0)$ and foci at $(\pm 5, 0)$.

Answer

Given: Vertices are $(\pm 3, 0)$ and foci are $(\pm 5, 0)$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of hyperbola are given by $(\pm a, 0)$

Foci of hyperbola are given by $(\pm ae, 0)$

Vertices are $(\pm 3, 0)$ and foci are $(\pm 5, 0)$

Therefore,

$$a = 3 \text{ and } ae = 5$$

$$\Rightarrow 3 \times e = 5$$

$$\Rightarrow e = \frac{5}{3}$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 3 \text{ and } e = \frac{5}{3} \right\}$$

$$\Rightarrow b^2 = 3^2 \left\{ \left(\frac{5}{3} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 9 \left(\frac{25}{9} - 1 \right)$$

$$\Rightarrow b^2 = 9 \left(\frac{25 - 9}{9} \right)$$

$$\Rightarrow b^2 = 9 \times \frac{16}{9}$$

$$\Rightarrow b^2 = 16$$

Equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

Hence, required equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

10. Question

If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, then write the value of $2e_1^2 + e_2^2$.

Answer

Given: e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

To find: value of $2e_1^2 + e_2^2$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{ where } c = \sqrt{a^2 + b^2}$$

Here $a = 3$ and $b = 2$

$$c = \sqrt{3^2 + 2^2}$$

$$\Rightarrow c = \sqrt{9 + 4}$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

$$e_2 = \frac{\sqrt{13}}{3} \dots \dots \dots (1)$$

For ellipse:

$$\frac{x^2}{18} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{(\sqrt{18})^2} + \frac{y^2}{2^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{ where } c = \sqrt{b^2 - a^2}$$

Here $a = \sqrt{18}$ and $b = 2$

$$c = \sqrt{(\sqrt{18})^2 - 2^2}$$

$$\Rightarrow c = \sqrt{18 - 4}$$

$$\Rightarrow c = \sqrt{14}$$

Therefore,

$$e_1 = \frac{\sqrt{14}}{\sqrt{18}}$$

$$\Rightarrow e_1 = \sqrt{\frac{14}{18}}$$

$$\Rightarrow e_1 = \sqrt{\frac{7}{9}} \dots \dots \dots (2)$$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$$2e_1^2 + e_2^2$$

$$= 2\left(\sqrt{\frac{7}{9}}\right)^2 + \left(\frac{\sqrt{13}}{3}\right)^2$$

$$= 2\left(\frac{7}{9}\right) + \frac{13}{9}$$

$$= \frac{14}{9} + \frac{13}{9}$$

$$= \frac{14 + 13}{9}$$

$$= \frac{27}{9}$$

$$= 3$$

Hence, value of $2e_1^2 + e_2^2$ is **3**

MCQ

1. Question

Equation of the hyperbola whose vertices are $(\pm 3, 0)$ and foci at $(\pm 5, 0)$, is

A. $16x^2 - 9y^2 = 144$

B. $9x^2 - 16y^2 = 144$

C. $25x^2 - 9y^2 = 225$

D. $9x^2 - 25y^2 = 81$

Answer

Given: Vertices are $(\pm 3, 0)$ and foci are $(\pm 5, 0)$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of hyperbola are given by $(\pm a, 0)$

Foci of hyperbola are given by $(\pm ae, 0)$

Vertices are $(\pm 3, 0)$ and foci are $(\pm 5, 0)$

Therefore,

$$a = 3 \text{ and } ae = 5$$

$$\Rightarrow 3 \times e = 5$$

$$\Rightarrow e = \frac{5}{3}$$

$$b^2 = a^2(e^2 - 1)$$

$$\left\{ \because a = 3 \text{ and } e = \frac{5}{3} \right\}$$

$$\Rightarrow b^2 = 3^2 \left\{ \left(\frac{5}{3} \right)^2 - 1 \right\}$$

$$\Rightarrow b^2 = 9 \left(\frac{25}{9} - 1 \right)$$

$$\Rightarrow b^2 = 9 \left(\frac{25 - 9}{9} \right)$$

$$\Rightarrow b^2 = 9 \times \frac{16}{9}$$

$$\Rightarrow b^2 = 16$$

Equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\Rightarrow \frac{16x^2 - 9y^2}{144} = 1$$

$$\Rightarrow 16x^2 - 9y^2 = 144$$

Hence, required equation of hyperbola is **$16x^2 - 9y^2 = 144$**

2. Question

If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, then the relation between e_1 and e_2 is

A. $3e_1^2 + e_2^2 = 2$

B. $e_1^2 + 2e_2^2 = 3$

C. $2e_1^2 + e_2^2 = 3$

$$D. e_1^2 + 3e_2^2 = 2$$

Answer

Given: e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

To find: value of $2e_1^2 + e_2^2$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{ where } c = \sqrt{a^2 + b^2}$$

Here $a = 3$ and $b = 2$

$$c = \sqrt{3^2 + 2^2}$$

$$\Rightarrow c = \sqrt{9 + 4}$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

$$e_2 = \frac{\sqrt{13}}{3} \dots \dots \dots (1)$$

For ellipse:

$$\frac{x^2}{18} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{(\sqrt{18})^2} + \frac{y^2}{2^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{a} \text{ where } c = \sqrt{a^2 - b^2}$$

Here $a = \sqrt{18}$ and $b = 2$

$$c = \sqrt{(\sqrt{18})^2 - 2^2}$$

$$\Rightarrow c = \sqrt{18 - 4}$$

$$\Rightarrow c = \sqrt{14}$$

Therefore,

$$e_1 = \frac{\sqrt{14}}{\sqrt{18}}$$

$$\Rightarrow e_1 = \sqrt{\frac{14}{18}}$$

$$\Rightarrow e_1 = \sqrt{\frac{7}{9}} \dots \dots \dots (2)$$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$$2e_1^2 + e_2^2$$

$$= 2 \left(\sqrt{\frac{7}{9}} \right)^2 + \left(\frac{\sqrt{13}}{3} \right)^2$$

$$= 2 \left(\frac{7}{9} \right) + \frac{13}{9}$$

$$= \frac{14}{9} + \frac{13}{9}$$

$$= \frac{14 + 13}{9}$$

$$= \frac{27}{9}$$

$$= 3$$

Hence, value of $2e_1^2 + e_2^2$ is **3**

3. Question

The distance between the directrices of the hyperbola $x = 8 \sec \theta$, $y = 8 \tan \theta$, is

A. $8\sqrt{2}$

B. $16\sqrt{2}$

C. $4\sqrt{2}$

D. $6\sqrt{2}$

Answer

Given: Hyperbola is $x = 8 \sec \theta$ and $y = 8 \tan \theta$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between directrix is given by $\frac{2a}{e}$

$$x = 8 \sec \theta \text{ and } y = 8 \tan \theta$$

$$\Rightarrow \sec \theta = \frac{x}{8} \text{ and } \tan \theta = \frac{y}{8}$$

We know,

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \left(\frac{x}{8}\right)^2 - \left(\frac{y}{8}\right)^2 = 1$$

$$\Rightarrow \frac{x^2}{8^2} - \frac{y^2}{8^2} = 1$$

Here $a = 8$ and $b = 8$

Now,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{8^2}{8^2}}$$

$$\Rightarrow e = \sqrt{1 + 1}$$

$$\Rightarrow e = \sqrt{2}$$

Hence, distance between directrix,

$$= \frac{2a}{e}$$

$$= \frac{2(8)}{\sqrt{2}}$$

$$= 8\sqrt{2}$$

4. Question

The equation of the conic with focus at $(1, -1)$ directrix along $x - y + 1 = 0$ and eccentricity $\sqrt{2}$ is

A. $xy = 1$

B. $2xy + 4x - 4y - 1 = 0$

C. $x^2 - y^2 = 1$

D. $2xy - 4x + 4y + 1 = 0$

Answer

Given: Equation of directrix of a hyperbola is $x - y + 1 = 0$. Focus of hyperbola is $(1, -1)$ and eccentricity (e) is $\sqrt{2}$

To find: equation of conic

Let M be the point on directrix and $P(x, y)$ be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2} \left| \frac{(x-y+1)}{\sqrt{1^2 + (-1)^2}} \right|$$

$$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2} \left| \frac{(x-y+1)}{\sqrt{1+1}} \right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-1)^2 + (y+1)^2} \right)^2 = \left(\sqrt{2} \left| \frac{(x-y+1)}{\sqrt{1+1}} \right| \right)^2$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{(\sqrt{2})^2 (x-y+1)^2}{2}$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{2(x-y+1)^2}{2}$$

$$\{\because (a-b)^2 = a^2 + b^2 + 2ab\}$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + y^2 + 1 - 2xy + 2x - 2y$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y - x^2 - y^2 + 2xy - 1 - 2x + 2y = 0$$

$$\Rightarrow \mathbf{2xy - 4x + 4y + 1 = 0}$$

This is the required equation of hyperbola

5. Question

The eccentricity of the conic $9x^2 - 16y^2 = 144$ is

A. $\frac{5}{4}$

B. $\frac{4}{3}$

C. $\frac{4}{5}$

D. $\sqrt{7}$

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e)

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Here, $a = 4$ and $b = 3$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

Hence, eccentricity is $\frac{5}{4}$

6. Question

A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation $PA - PB = k (k \neq 0)$, then the locus of P is

- A. a hyperbola
- B. a branch of the hyperbola
- C. a parabola
- D. an ellipse

Answer

We know it by the fact that when difference in distances is constant, it forms as hyperbola

7. Question

The eccentricity of the hyperbola whose latus-rectum is half of its transverse axis, is

- A. $\frac{1}{\sqrt{2}}$
- B. $\sqrt{\frac{2}{3}}$
- C. $\sqrt{\frac{3}{2}}$
- D. none of these

Answer

Given: Latus-rectum is half of its transverse axis

To find: eccentricity of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis is $2a$

Latus-rectum of the hyperbola is $\frac{2b^2}{a}$

According to question:

Latus-rectum is half of its transverse axis

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow 2b^2 = a^2$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{2b^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{2+1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

Hence, eccentricity is $\sqrt{\frac{3}{2}}$

8. Question

The eccentricity of the hyperbola $x^2 - 4y^2 = 1$ is

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{5}}{2}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{2}{\sqrt{5}}$

Answer

Given: $x^2 - 4y^2 = 1$

To find: eccentricity(e)

$$x^2 - 4y^2 = 1$$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{\frac{1}{4}} = 1$$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Here, $a = 1$ and $b = \frac{1}{2}$

$$c = \sqrt{1^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow c = \sqrt{1 + \frac{1}{4}}$$

$$\Rightarrow c = \sqrt{\frac{5}{4}}$$

$$\Rightarrow c = \frac{\sqrt{5}}{2}$$

Therefore,

$$e = \frac{\frac{\sqrt{5}}{2}}{1}$$

$$\Rightarrow e = \frac{\sqrt{5}}{2}$$

Hence, eccentricity is $\frac{\sqrt{5}}{2}$

9. Question

The difference of the focal distances of any point on the hyperbola is equal to

- A. length of the conjugate axis
- B. eccentricity
- C. length of the transverse axis
- D. Latus-rectum

Answer

This is definition of eccentricity.

Eccentricity is difference of the focal distances of any point on the hyperbola

10. Question

the foci of the hyperbola $9x^2 - 16y^2 = 144$ are

- A. $(\pm 4, 0)$
- B. $(0, \pm 4)$
- C. $(\pm 5, 0)$
- D. $(0, \pm 5)$

Answer

Given: $9x^2 - 16y^2 = 144$

To find: coordinates of the foci $f(m,n)$

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(\pm ae, 0)$

Here, $a = 4$ and $b = 3$

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

$$\Rightarrow c = 5$$

Therefore,

$$e = \frac{5}{4}$$

$$\Rightarrow ae = 4 \times \frac{5}{4} = 5$$

Foci: $(\pm 5, 0)$

11. Question

The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then equation of the hyperbola is

- A. $x^2 + y^2 = 32$
- B. $x^2 - y^2 = 16$

C. $x^2 + y^2 = 16$

D. $x^2 - y^2 = 32$

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between foci is given by $2ae$

According to question:

$$2ae = 16$$

$$\Rightarrow a = \frac{16}{2e}$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a = 4\sqrt{2}$$

$$\Rightarrow a^2 = 32$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 32\{(\sqrt{2})^2 - 1\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$$\Rightarrow b^2 = 32$$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow x^2 - y^2 = 32$$

12. Question

If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic $9x^2 - 4y^2 = 36$, then

A. $e_1^2 - e_2^2 = 2$

B. $2 < e_2^2 - e_1^2 < 3$

C. $e_2^2 - e_1^2 = 2$

D. $e_2^2 - e_1^2 > 3$

Answer

Given: e_1 and e_2 are respectively the eccentricities of $9x^2 + 4y^2 = 36$ and $9x^2 - 4y^2 = 36$ respectively

To find: $e_1^2 - e_2^2$

$$9x^2 - 4y^2 = 36$$

$$\Rightarrow \frac{9x^2}{36} - \frac{4y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{ where } c = \sqrt{a^2 + b^2}$$

Here $a = 2$ and $b = 3$

$$c = \sqrt{2^2 + 3^2}$$

$$\Rightarrow c = \sqrt{4 + 9}$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

$$e_2 = \frac{\sqrt{13}}{2} \dots \dots \dots (1)$$

For ellipse:

$$9x^2 + 4y^2 = 36$$

$$\Rightarrow \frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{ where } c = \sqrt{b^2 - a^2}$$

Here $a = 2$ and $b = 3$

$$c = \sqrt{3^2 - 2^2}$$

$$\Rightarrow c = \sqrt{9 - 4}$$

$$\Rightarrow c = \sqrt{5}$$

Therefore,

$$e_1 = \frac{\sqrt{5}}{3} \dots \dots \dots (2)$$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$$e_1^2 - e_2^2$$

$$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2$$

$$= \frac{5}{9} - \frac{13}{4}$$

$$= \frac{20 - 117}{36}$$

$$= \frac{-97}{36}$$

$$\Rightarrow e_2^2 - e_1^2$$

$$= \frac{97}{36}$$

Hence, value of $2 < e_2^2 - e_1^2 < 3$

13. Question

If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2 \alpha + y^2 = 25$, then $\alpha =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer

Given: e_1 and e_2 are respectively the eccentricities of $x^2 - y^2 \sec^2 \alpha = 5$ and $x^2 \sec^2 \alpha + y^2 = 25$ respectively

To find: value of α

$$x^2 - y^2 \sec^2 \alpha = 5$$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2 \sec^2 \alpha}{5} = 1$$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2}{\frac{5}{\sec^2 \alpha}} = 1$$

$$\Rightarrow \frac{x^2}{(\sqrt{5})^2} - \frac{y^2}{\left(\frac{\sqrt{5}}{\sec \alpha}\right)^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{ where } c = \sqrt{a^2 + b^2}$$

$$\text{Here } a = \sqrt{5} \text{ and } b = \frac{\sqrt{5}}{\sec \alpha}$$

$$c = \sqrt{(\sqrt{5})^2 + \left(\frac{\sqrt{5}}{\sec \alpha}\right)^2}$$

$$\Rightarrow c = \sqrt{5 + \frac{5}{\sec^2 \alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5 \sec^2 \alpha + 5}{\sec^2 \alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5(\sec^2 \alpha + 1)}{\sec^2 \alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5\left(\frac{1}{\cos^2 \alpha} + 1\right)}{\frac{1}{\cos^2 \alpha}}}$$

$$\Rightarrow c = \sqrt{\frac{5\left(\frac{1 + \cos^2 \alpha}{\cos^2 \alpha}\right)}{\frac{1}{\cos^2 \alpha}}}$$

$$\Rightarrow c = \sqrt{5(1 + \cos^2 \alpha)}$$

Therefore,

$$e_2 = \frac{\sqrt{5(1 + \cos^2 \alpha)}}{\sqrt{5}}$$

$$\Rightarrow e_2 = \sqrt{1 + \cos^2 \alpha} \dots \dots \dots (1)$$

For ellipse:

$$x^2 \sec^2 \alpha + y^2 = 25$$

$$\Rightarrow \frac{x^2 \sec^2 \alpha}{25} + \frac{y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{\frac{25}{\sec^2 \alpha}} + \frac{y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{5}{\sec \alpha}\right)^2} + \frac{y^2}{5^2} = 1$$

$$\Rightarrow \frac{x^2}{(5 \cos \alpha)^2} + \frac{y^2}{5^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{ where } c = \sqrt{b^2 - a^2}$$

Here $a = 5 \cos \alpha$ and $b = 5$

$$c = \sqrt{5^2 - (5 \cos \alpha)^2}$$

$$\Rightarrow c = \sqrt{25 - 25 \cos^2 \alpha}$$

$$\Rightarrow c = \sqrt{25(1 - \cos^2 \alpha)}$$

$$\Rightarrow c = 5\sqrt{1 - \cos^2 \alpha}$$

Therefore,

$$e_1 = \frac{5\sqrt{1 - \cos^2 \alpha}}{5}$$

$$\Rightarrow e_1 = \sqrt{1 - \cos^2 \alpha} \dots \dots \dots (2)$$

According to question:

Eccentricity of given hyperbola is $\sqrt{3}$ time eccentricity of given ellipse

$$\Rightarrow e_2 = \sqrt{3}e_1$$

From (1) and (2):

$$\Rightarrow \sqrt{1 + \cos^2 \alpha} = \sqrt{3}\sqrt{1 - \cos^2 \alpha}$$

Squaring both sides:

$$\Rightarrow 1 + \cos^2 \alpha = 3(1 - \cos^2 \alpha)$$

$$\Rightarrow 1 + \cos^2 \alpha = 3 - 3 \cos^2 \alpha$$

$$\Rightarrow 3 \cos^2 \alpha + \cos^2 \alpha = 3 - 1$$

$$\Rightarrow 4 \cos^2 \alpha = 2$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \alpha = \frac{\pi}{4}$$

14. Question

The equation of the hyperbola whose foci are (6, 4) and (-4, 4) and eccentricity 2, is

A. $\frac{(x-1)^2}{25/4} - \frac{(y-4)^2}{75/4} = 1$

B. $\frac{(x+1)^2}{25/4} - \frac{(y+4)^2}{75/4} = 1$

C. $\frac{(x-1)^2}{75/4} - \frac{(y-4)^2}{25/4} = 1$

D. none of these

Answer

Given: Foci are (6, 4) and (-4, 4) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

Distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2 + (n-b)^2}$$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2} \right)$$

Center of hyperbola having foci (6, 4) and (-4, 4) is given by

$$= \left(\frac{6-4}{2}, \frac{4+4}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right)$$

$$= (1, 4)$$

Distance between the foci is $2ae$ and Foci are (6, 4) and (-4, 4)

$$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2} = 2ae$$

$$\Rightarrow \sqrt{(10)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{100} = 2ae$$

$$\Rightarrow 10 = 2ae$$

$$\Rightarrow \frac{10}{2} = ae$$

$$\Rightarrow ae = 5$$

$$\{\because e = 2\}$$

$$\Rightarrow a \times 2 = 5$$

$$\Rightarrow a = \frac{5}{2}$$

$$\Rightarrow a^2 = \frac{25}{4}$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = \frac{25}{4} \{(2)^2 - 1\}$$

$$\Rightarrow b^2 = \frac{25}{4}(4-1)$$

$$\Rightarrow b^2 = \frac{25}{4}(3)$$

$$\Rightarrow b^2 = \frac{75}{4}$$

Equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$$

Hence, required equation of hyperbola is $\frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$

15. Question

The length of the straight line $x - 3y = 1$ intercepted by the hyperbola $x^2 - 4y^2 = 1$ is

A. $\frac{6}{\sqrt{5}}$

B. $3\sqrt{\frac{2}{5}}$

C. $6\sqrt{\frac{2}{5}}$

D. none of these

Answer

Given: A straight line $x - 3y = 1$ intercepts hyperbola $x^2 - 4y^2 = 1$

To find: Length of the intercepted line

Formula used:

Distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2 + (n-b)^2}$$

Firstly we will find point of intersections of given line and hyperbola

$$x - 3y = 1 \Rightarrow x = 1 + 3y$$

$$x^2 - 4y^2 = 1$$

$$\Rightarrow (1 + 3y)^2 - 4y^2 = 1$$

$$\Rightarrow 1 + 9y^2 + 6y - 4y^2 = 1$$

$$\Rightarrow 5y^2 + 6y = 0$$

$$\Rightarrow y(5y + 6) = 0$$

$$\Rightarrow y = 0 \text{ or } 5y + 6 = 0$$

$$\Rightarrow y = 0 \text{ or } -\frac{6}{5}$$

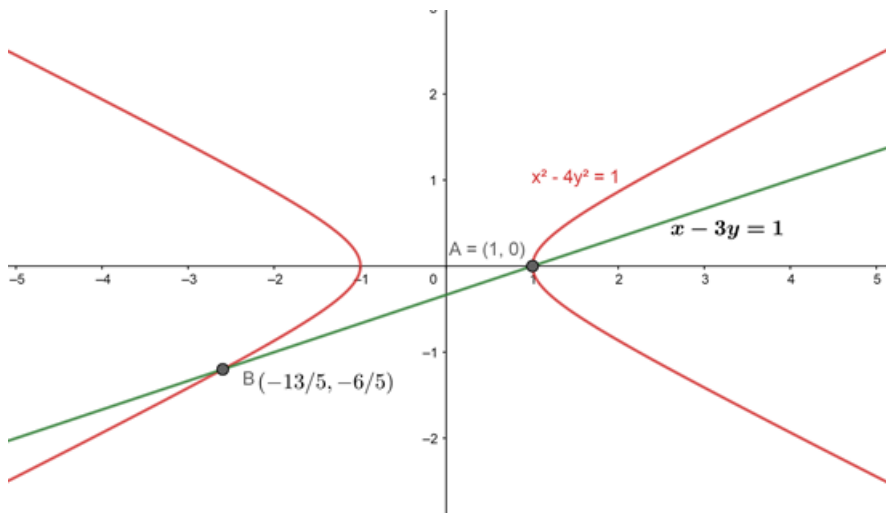
$$\text{Now, } x = 1 + 3y$$

$$\Rightarrow x = 1 + 3(0) \text{ or } 1 + 3\left(-\frac{6}{5}\right)$$

$$\Rightarrow x = 1 \text{ or } 1 - \frac{18}{5}$$

$$\Rightarrow x = 1 \text{ or } -\frac{13}{5}$$

So, Point of intersections are $A(1, 0)$ and $B\left(-\frac{13}{5}, -\frac{6}{5}\right)$



Distance between point of intersections is

$$= \sqrt{\left(1 - \left(-\frac{13}{5}\right)\right)^2 + \left(0 - \left(-\frac{6}{5}\right)\right)^2}$$

$$= \sqrt{\left(1 + \frac{13}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

$$= \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

$$= \sqrt{\frac{324}{25} + \frac{36}{25}}$$

$$= \sqrt{\frac{360}{25}}$$

$$= \sqrt{\frac{72}{5}}$$

$$= 6\sqrt{\frac{2}{5}}$$

6. Question

The latus-rectum of the hyperbola $16x^2 - 9y^2 = 144$ is

- A. $16/3$
- B. $32/3$
- C. $8/3$
- D. $4/3$

Answer

Given: $16x^2 - 9y^2 = 144$

To find: length of latus-rectum of hyperbola.

$$16x^2 - 9y^2 = 144$$

$$\Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Length of latus rectum is $\frac{2b^2}{a}$

Here, $a = 3$ and $b = 4$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2 \times (4)^2}{3}$$

$$= \frac{32}{3}$$

17. Question

The foci of the hyperbola $2x^2 - 3y^2 = 5$ are

- A. $(\pm 5\sqrt{6}, 0)$
- B. $(\pm 5/6, 0)$
- C. $(\pm \sqrt{5}/6, 0)$
- D. none of these

Answer

Given: $2x^2 - 3y^2 = 5$

To find: coordinates of the foci $f(m,n)$

$$2x^2 - 3y^2 = 5$$

$$\Rightarrow \frac{2x^2}{5} - \frac{3y^2}{5} = 1$$

$$\Rightarrow \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\sqrt{\frac{5}{2}}\right)^2} - \frac{y^2}{\left(\sqrt{\frac{5}{3}}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Foci is given by $(\pm ae, 0)$

$$\text{Here, } a = \sqrt{\frac{5}{2}} \text{ and } b = \sqrt{\frac{5}{3}}$$

$$c = \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 + \left(\sqrt{\frac{5}{3}}\right)^2}$$

$$\Rightarrow c = \sqrt{\frac{5}{2} + \frac{5}{3}}$$

$$\Rightarrow c = \sqrt{\frac{15 + 10}{6}}$$

$$\Rightarrow c = \sqrt{\frac{25}{6}}$$

$$\Rightarrow c = \frac{5}{\sqrt{6}}$$

Therefore,

$$e = \frac{\frac{5}{\sqrt{6}}}{\sqrt{\frac{5}{2}}}$$

$$\Rightarrow e = \sqrt{\frac{5}{3}}$$

$$\Rightarrow ae = \sqrt{\frac{5}{2}} \times \sqrt{\frac{5}{3}} = \frac{5}{\sqrt{6}}$$

Foci: $\left(\pm \frac{5}{\sqrt{6}}, 0\right)$

18. Question

The eccentricity the hyperbola $x = \frac{a}{2}\left(t + \frac{1}{t}\right), y = \frac{a}{2}\left(t - \frac{1}{t}\right)$ is

- A. $\sqrt{2}$
- B. $\sqrt{3}$
- C. $2\sqrt{3}$
- D. $3\sqrt{2}$

Answer

Given: Equation of hyperbola $x = \frac{a}{2}\left(t + \frac{1}{t}\right), y = \frac{a}{2}\left(t - \frac{1}{t}\right)$

To find: Eccentricity of the hyperbola

$$x = \frac{a}{2}\left(t + \frac{1}{t}\right)$$

$$\Rightarrow \frac{2x}{a} = t + \frac{1}{t}$$

Squaring both sides:

$$\Rightarrow \left(\frac{2x}{a}\right)^2 = \left(t + \frac{1}{t}\right)^2$$

$$\Rightarrow \frac{4x^2}{a^2} = t^2 + \frac{1}{t^2} + 2(t)\left(\frac{1}{t}\right)$$

$$\Rightarrow \frac{4x^2}{a^2} = t^2 + \frac{1}{t^2} + 2$$

$$\Rightarrow t^2 + \frac{1}{t^2} = \frac{4x^2}{a^2} - 2 \dots \dots \dots (1)$$

$$y = \frac{a}{2}\left(t - \frac{1}{t}\right)$$

$$\Rightarrow \frac{2y}{a} = t - \frac{1}{t}$$

Squaring both sides:

$$\Rightarrow \left(\frac{2y}{a}\right)^2 = \left(t - \frac{1}{t}\right)^2$$

$$\Rightarrow \frac{4y^2}{a^2} = t^2 + \frac{1}{t^2} - 2(t)\left(\frac{1}{t}\right)$$

$$\Rightarrow \frac{4y^2}{a^2} = t^2 + \frac{1}{t^2} - 2$$

$$\Rightarrow t^2 + \frac{1}{t^2} = \frac{4y^2}{a^2} + 2 \dots \dots \dots (2)$$

From (1) and (2):

$$\frac{4x^2}{a^2} - 2 = \frac{4y^2}{a^2} + 2$$

$$\Rightarrow \frac{4x^2}{a^2} - \frac{4y^2}{a^2} = 4$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

Here a = a, b = a

$$c = \sqrt{(a)^2 + (a)^2}$$

$$\Rightarrow c = \sqrt{2a^2}$$

$$\Rightarrow c = \sqrt{2}a$$

Therefore,

$$e = \frac{\sqrt{2}a}{a}$$

$$\Rightarrow e = \sqrt{2}$$

Hence, the **eccentricity of the hyperbola is $\sqrt{2}$**

19. Question

The equation of the hyperbola whose centre is (6, 2) one focus is (4, 2) and of eccentricity 2 is

A. $3(x - 6)^2 - (y - 2)^2 = 3$

B. $(x - 6)^2 - 3(y - 2)^2 = 1$

C. $(x - 6)^2 - 2(y - 2)^2 = 1$

D. $2(x - 6)^2 - (y - 2)^2 = 1$

Answer

Given: Foci is (4, 2), $e = 2$ and center at (6, 2)

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is $2a$

The distance between the foci and vertex is $ae - a$ and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2 + (n-b)^2}$$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Therefore

Let one of the two foci is (m, n) and the other one is (4, 2)

Since, Centre(6, 2)

$$\left(\frac{m+4}{2} = 6, \frac{n+2}{2} = 2\right)$$

$$\Rightarrow (m+4 = 12, n+2 = 4)$$

$$\Rightarrow (m = 8, n = 2)$$

Foci are (4, 2) and (8, 2)

The distance between the foci is $2ae$ and Foci are (4, 2) and (8, 2)

$$\Rightarrow \sqrt{(4-8)^2 + (2-2)^2} = 2ae$$

$$\Rightarrow \sqrt{(-4)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{16} = 2ae$$

$$\Rightarrow 4 = 2ae$$

$$\Rightarrow \frac{4}{2} = ae$$

$$\Rightarrow ae = 2$$

$$\{\because e = 2\}$$

$$\Rightarrow a \times 2 = 2$$

$$\Rightarrow a = \frac{2}{2} = 1$$

$$\Rightarrow a^2 = 1$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1\{(2)^2 - 1\}$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

Equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$$

$$\Rightarrow 3(x-6)^2 - (y-2)^2 = 3$$

Hence, required equation of hyperbola is $3(x-6)^2 - (y-2)^2 = 3$

20. Question

The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}\lambda = 0$ and $\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0$ is a hyperbola of eccentricity

- A. 1
- B. 2
- C. 3
- D. 4

Answer

Given: A hyperbola is formed by the locus of the point of intersection of lines

$$\sqrt{3}x - y - 4\sqrt{3}\lambda = 0 \text{ and } \sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0$$

To find: Eccentricity of the hyperbola

$$\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0 \dots \dots \dots (1)$$

$$\sqrt{3}x - y - 4\sqrt{3}\lambda = 0$$

Multiply by λ :

$$\Rightarrow \sqrt{3}\lambda x - \lambda y - 4\sqrt{3}\lambda^2 = 0 \dots \dots \dots (2)$$

Adding (1) and (2):

$$\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} + \sqrt{3}\lambda x - \lambda y - 4\sqrt{3}\lambda^2 = 0 + 0$$

$$\Rightarrow 2\sqrt{3}\lambda x - 4\sqrt{3} - 4\sqrt{3}\lambda^2 = 0$$

$$\Rightarrow 2\sqrt{3}\lambda x = 4\sqrt{3} + 4\sqrt{3}\lambda^2$$

$$\Rightarrow 2\sqrt{3}\lambda x = 4\sqrt{3}(1 + \lambda^2)$$

$$\Rightarrow x = \frac{4\sqrt{3}(1 + \lambda^2)}{2\sqrt{3}\lambda}$$

$$\Rightarrow x = \frac{2(1 + \lambda^2)}{\lambda}$$

$$\Rightarrow x = 2\left(\lambda + \frac{1}{\lambda}\right)$$

Now, From (1):

$$\sqrt{3}\lambda\left(2\left(\lambda + \frac{1}{\lambda}\right)\right) + \lambda y - 4\sqrt{3} = 0$$

$$\Rightarrow 2\sqrt{3}\lambda\left(\lambda + \frac{1}{\lambda}\right) + \lambda y = 4\sqrt{3}$$

$$\Rightarrow 2\sqrt{3}\left(\lambda + \frac{1}{\lambda}\right) + y = \frac{4\sqrt{3}}{\lambda}$$

$$\Rightarrow y = \frac{4\sqrt{3}}{\lambda} - 2\sqrt{3}\left(\lambda + \frac{1}{\lambda}\right)$$

$$\Rightarrow y = \frac{4\sqrt{3}}{\lambda} - 2\sqrt{3}\lambda - \frac{2\sqrt{3}}{\lambda}$$

$$\Rightarrow y = \frac{2\sqrt{3}}{\lambda} - 2\sqrt{3}\lambda$$

$$\Rightarrow y = 2\sqrt{3}\left(\frac{1}{\lambda} - \lambda\right)$$

$$x = 2\left(\lambda + \frac{1}{\lambda}\right) \text{ and } y = 2\sqrt{3}\left(\frac{1}{\lambda} - \lambda\right)$$

$$\Rightarrow \frac{x}{2} = \left(\lambda + \frac{1}{\lambda}\right) \text{ and } \frac{y}{2\sqrt{3}} = \left(\frac{1}{\lambda} - \lambda\right)$$

Squaring both sides:

$$\Rightarrow \left(\frac{x}{2}\right)^2 = \left(\lambda + \frac{1}{\lambda}\right)^2 \text{ and } \left(\frac{y}{2\sqrt{3}}\right)^2 = \left(\frac{1}{\lambda} - \lambda\right)^2$$

$$\Rightarrow \left(\frac{x}{2}\right)^2 = \lambda^2 + \frac{1}{\lambda^2} + 2(\lambda)\left(\frac{1}{\lambda}\right)$$

$$\Rightarrow \frac{x^2}{4} = \lambda^2 + \frac{1}{\lambda^2} + 2$$

$$\Rightarrow \lambda^2 + \frac{1}{\lambda^2} = \frac{x^2}{4} - 2 \dots \dots \dots (3)$$

$$\Rightarrow \frac{y^2}{12} = \lambda^2 + \frac{1}{\lambda^2} - 2(\lambda)\left(\frac{1}{\lambda}\right)$$

$$\Rightarrow \frac{y^2}{12} = \lambda^2 + \frac{1}{\lambda^2} - 2$$

$$\Rightarrow \lambda^2 + \frac{1}{\lambda^2} = \frac{y^2}{12} + 2 \dots \dots \dots (4)$$

From (3) and (4):

$$\frac{x^2}{4} - 2 = \frac{y^2}{12} + 2$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 4$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{(4\sqrt{3})^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{ where } c = \sqrt{a^2 + b^2}$$

$$\text{Here } a = 4 \text{ and } b = 4\sqrt{3}$$

$$c = \sqrt{(4)^2 + (4\sqrt{3})^2}$$

$$\Rightarrow c = \sqrt{16 + 48}$$

$$\Rightarrow c = \sqrt{64}$$

$$\Rightarrow c = 8$$

Therefore,

$$e = \frac{8}{4}$$

$$\Rightarrow e = 2$$

Hence, **eccentricity of hyperbola is 2**