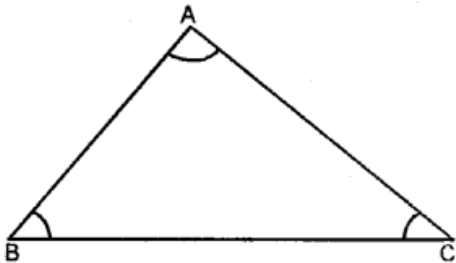


Triangle

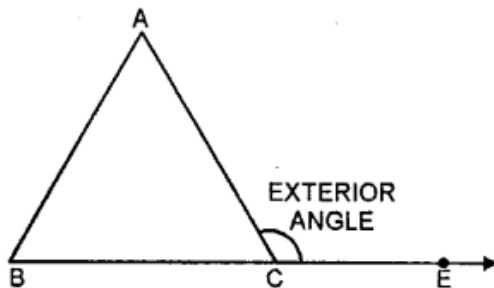
IMPORTANT POINTS

- 1. Collinear Points:** Three or more points which lie on the same straight line, are called collinear points.
- 2. Non-Collinear Points:** Three or more points which do not lie on the same line, are called non- collinear points.
- 3. Triangle:** By joining the three non-collinear points, a triangle is formed or A triangle is a figure which is enclosed by three line segments. In the figure, ABC is a triangle.
- 4. Parts of triangle:** A triangle has six parts, three sides and three angles which are on the vertices of the triangle.
- 5. Sum of angles of a triangle:** The sum of the three angles of a triangle is 180° .



- 6. Exterior angle of a triangle:** If one side of a triangle is produced then the exterior angle is formed. Exterior angle of a triangle is equal to sum of its interior opposite angles. In other words, we can say that exterior angle of a triangle is greater than each of its interior opposite angles. In the figure.

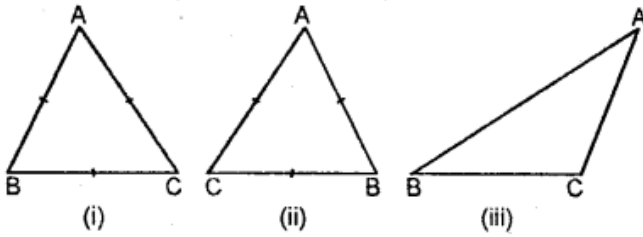
$\angle ACE$ is exterior angle and $\angle ACE = \angle A + \angle B$ and also $\angle ACE > \angle A$ and $\angle ACE > \angle B$.



7. Classification of triangles :

(A) According to their sides.

- (i) Equilateral Triangle:** If three sides of a triangle are equal, it is called an equilateral triangle.
- (ii) Isosceles Triangle:** If any two sides of a triangle are equal, then it is called an isosceles triangle.
- (iii) Scalene Triangle:** If no two sides of the triangle are equal. Then it is called a scalene triangle.

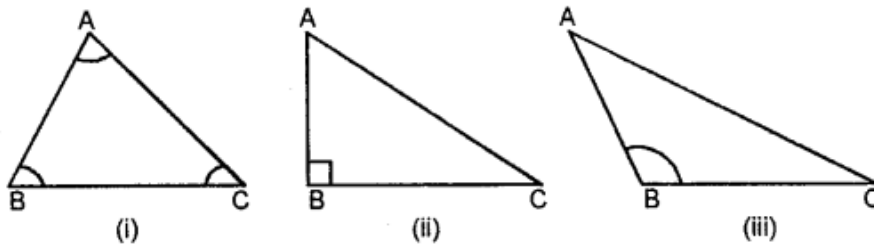


(B) According to Angles :

(i) Acute-angled Triangle: A triangle whose each angle is acute, is called an acute angled triangle.

(ii) Right-angled Triangle: A triangle whose one angle is a right angle i.e. 90° , is called a right angled triangle.

(iii) Obtused-angled Triangle: A triangle whose one angle is an obtused angle, is called an obtused-angled triangle.



8. Some special properties of a triangle:

(i) If one angle of a triangle is equal to the sum of other two angles, the triangle is a right angled triangle.

(ii) If the acute angles of a right angled triangle are equal, then the triangle is a right angled isosceles triangle and its each acute angle will be of 45° .

(iii) Sum of two sides of a triangle is greater than the third side.

(iv) There can be only one right angle in a triangle.

(v) There can be only one obtuse angle in a triangle.

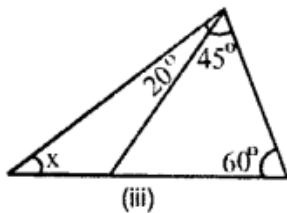
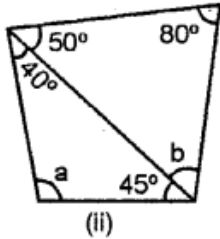
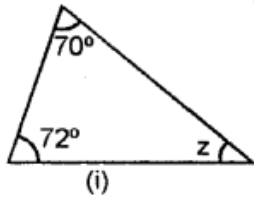
(vi) Side opposite to greater angle is greater.

(vii) Angle opposite to shorter side is shorter.

EXERCISE 26 (A)

Question 1.

In each of the following, find the marked unknown angles :



Solution:

(i) Since, sum of all angles of triangle = 180°

Hence, $70^\circ + 72^\circ + z = 180^\circ$

$\Rightarrow 142^\circ + z = 180^\circ$

$\Rightarrow z = 180^\circ - 142^\circ$

$z = 38^\circ$

(ii) Since, sum of all angles of a triangle = 180°

1st Triangle $50^\circ + 80^\circ + b = 180^\circ$

$\Rightarrow 130^\circ + b = 180^\circ$

$\Rightarrow b = 180^\circ - 130^\circ$

$b = 50^\circ$

2nd Triangle $40^\circ + 45^\circ + a = 180^\circ$

$\Rightarrow 85^\circ + a = 180^\circ$

$\Rightarrow a = 180^\circ - 85^\circ$

$a = 95^\circ$

(iii) $60^\circ + 45^\circ + 20^\circ + x = 180^\circ$

$\Rightarrow 125^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 125^\circ \Rightarrow x = 55^\circ$

Question 2.

Can a triangle together have the following angles ?

(i) 55° , 55° and 80°

(ii) 33° , 74° and 73°

(iii) 85° , 95° and 22° .

Solution:

(i) Sum of all angles of a triangle = 180° Here, $55^\circ + 55^\circ + 80^\circ = 180^\circ$

$$190^\circ \neq 180^\circ$$

No.

$$(ii) 33^\circ + 74^\circ + 73^\circ = 180^\circ$$

$$180^\circ = 180^\circ$$

Yes.

$$(iii) 85^\circ + 95^\circ + 22^\circ = 180^\circ$$

$$202^\circ \neq 180^\circ$$

No.

Question 3.

Find x , if the angles of a triangle are:

$$(i) x^\circ, x^\circ, x^\circ$$

$$(ii) x^\circ, 2x^\circ, 2x^\circ$$

$$(iii) 2x^\circ, 4x^\circ, 6x^\circ$$

Solution:

(i) Since, sum of all the angles of a triangle $= 180$

$$x^\circ + x^\circ + x^\circ = 180$$

$$\Rightarrow 3x^\circ = 180$$

$$\Rightarrow x^\circ = \frac{180}{3}$$

$$x = 60$$

$$(ii) x^\circ + 2x^\circ + 2x^\circ = 180$$

$$5x^\circ = 180$$

$$x^\circ = \frac{180}{5}$$

$$x^\circ = 36$$

$$(iii) 2x^\circ + 4x^\circ + 6x^\circ = 180$$

$$12x^\circ = 180$$

$$x^\circ = \frac{180}{12}$$

$$x^\circ = 15$$

Question 4.

One angle of a right-angled triangle is 70° . Find the other acute angle.

Solution:

We know that, sum of angles of a triangle $= 180^\circ$.

Let, the acute angle be ' x '

$$\therefore x + 90^\circ + 70^\circ = 180^\circ$$

$$\Rightarrow x + 160^\circ = 180^\circ$$

$$\Rightarrow x = 180^\circ - 160^\circ$$

$$\Rightarrow x = 20^\circ$$

$\therefore$ The acute angle is 20° .

Question 5.

In $\triangle ABC$, $\angle A = \angle B = 62^\circ$; find $\angle C$.

Solution:

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\begin{aligned}\Rightarrow 62^\circ + 62^\circ + \angle C &= 180^\circ \\ \Rightarrow 124^\circ + \angle C &= 180^\circ \\ \Rightarrow \angle C &= 180^\circ - 124^\circ \\ \Rightarrow \angle C &= 56^\circ\end{aligned}$$

Question 6.

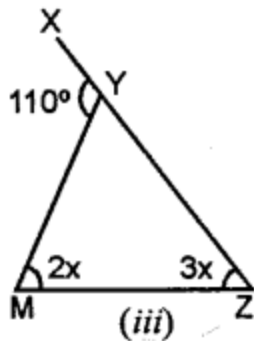
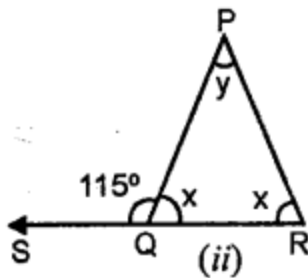
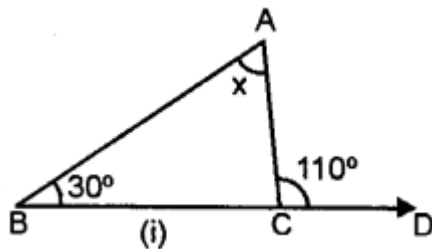
In $\triangle ABC$, $\angle C = 56^\circ$, $\angle B = \angle C$ and $\angle A = 100^\circ$; find $\angle B$.

Solution:

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ \Rightarrow 100^\circ + \angle B + \angle B &= 180^\circ \\ \Rightarrow 2\angle B &= 180^\circ - 100^\circ \\ \angle B &= \frac{80}{2}^\circ \\ \angle B &= 40^\circ \\ \angle C &= \angle B = 40^\circ\end{aligned}$$

Question 7.

Find, giving reasons, the unknown marked angles, in each triangle drawn below:



Solution:

We know that,

Exterior angle of a triangle is always equal to the sum of its two interior opposite angles

(property)

(i) $\therefore 110^\circ = x + 30^\circ$ (by property)

$$\Rightarrow x = 110^\circ - 30^\circ \Rightarrow x = 80^\circ$$

(ii) $x + 115^\circ = 180^\circ$

(linear property of angles)

$$\Rightarrow x = 180^\circ - 115^\circ \Rightarrow x = 65^\circ$$

$$\therefore 115^\circ = x + y$$

$$\Rightarrow 115^\circ = 65^\circ + y \Rightarrow y = 115^\circ - 65^\circ = 50^\circ$$

$$y = 50^\circ$$

(iii) $110^\circ = 2x + 3x$

$$5x = 110^\circ$$

$$x = \frac{110}{5}^\circ$$

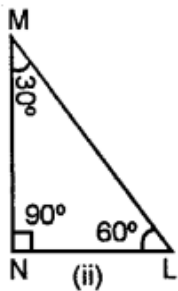
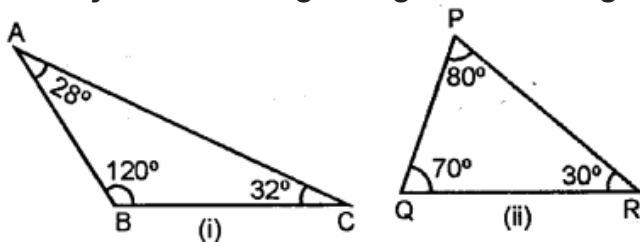
$$x = 22^\circ$$

$$\therefore 2x = 2 \times 22 = 44^\circ$$

$$3x = 3 \times 22 = 66^\circ$$

Question 8.

Classify the following triangles according to angle :



Solution:

(i) Since, it has an obtuse angle of 120° Hence, it is obtuse angled triangle.

(ii) Since, all the angle of triangle is less than 90° .

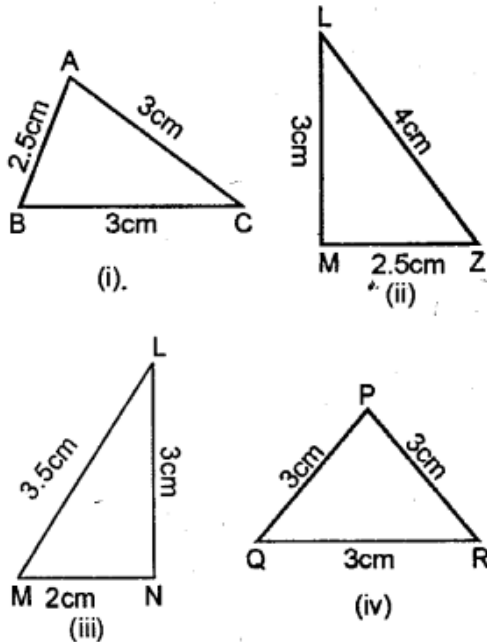
Hence, it is an acute angled triangle.

(iii) Since $\angle MNL = 90^\circ$, and sum of two acute angle $\angle M + \angle N = 30^\circ + 60^\circ = 90^\circ$.

Hence, it is a right angled triangle.

Question 9.

Classify the following triangles according to sides :



Solution:

(i) Since, two sides are equal.

Hence, Isosceles triangle.

(ii) Since, all the three sides are unequal.

Hence, Scalene, triangle.

(iii) Since, all the three sides are unequal Hence, Scalene triangle.

(iv) All the three sides are equal.

Hence, equilateral triangle.

EXERCISE 26 (B)

Construct triangle ABC, when :

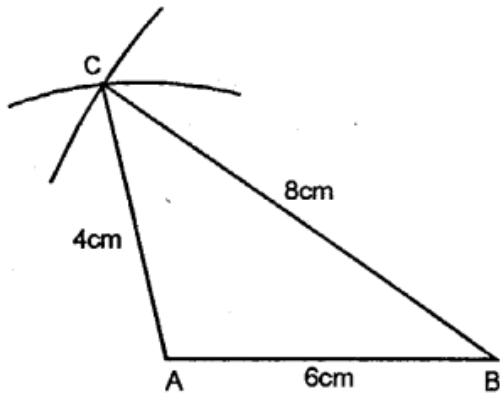
Question 1.

AB = 6 cm, BC = 8 cm and AC = 4 cm.

Solution:

Steps of Construction:

(1) Draw a line AB = 6 cm.



- (2) compasses and taking B as centre, draw an arc of 8 cm radius.
 (3) With A as centre, draw an arc of 4 cm radius, which cuts the previous arc at C.
 (4) Join AC and BC.
 Triangle ABC, obtained, is the required triangle.

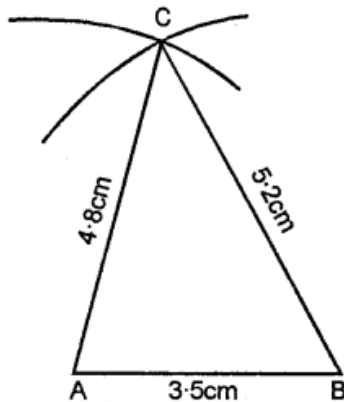
Question 2.

AB = 3.5 cm, AC = 4.8 cm and BC = 5.2 cm.

Solution:

Steps of Construction :

- (1) Draw a line AB = 3.5 cm.
 (2) Using compasses and taking B as centre, draw an arc of 5.2 cm radius.
 (3) With A as centre, draw an arc of 4.8



- (4) Join AC and BC

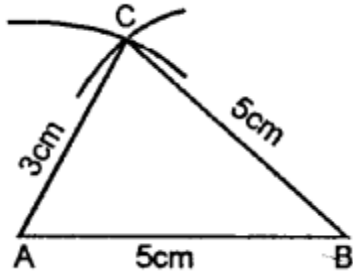
Question 3.

AB = BC = 5 cm and AC = 3 cm. Measure angles A and C. Is $\angle A = \angle C$?

Solution:

Steps of Construction :

- (1) Draw a line AB = 5 cm.



- (2) Using compasses and taking B as centre, draw an arc of 5 cm radius.
- (3) With A as centre, draw an arc of 3 cm radius, which cuts the previous arc at C.
- (4) Join AC and BC.

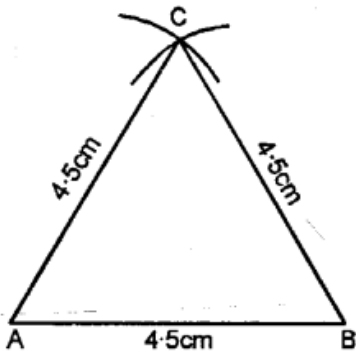
Question 4.

$AB = BC = CA = 4.5$ cm. Measure all the angles of the triangle. Are they equal ?

Solution:

Steps of Construction :

- (1) Draw a line $AB = 4.5$



- (2) Using compasses and taking B as centre, draw an arc of 4.5 cm radius.
- (3) With A as centre, draw an arc of 4.5 cm radius, which cuts the previous arc at C.
- (4) Join AC and BC.
- (5) Measurement, $\angle A = \angle B = \angle C = 60^\circ$.

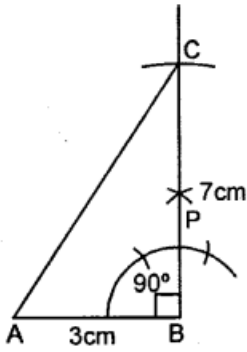
Question 5.

$AB = 3$ cm, $BC = 7$ cm and $\angle B = 90^\circ$.

Solution:

Steps of Construction :

- (1) Draw a line segment $AB = 3$ cm.



- (2) With the help of compasses, construct $\angle ABC = 90^\circ$.
- (3) With B as centre, draw an arc of 7 cm length which cuts BP at C.
- (4) Join A and C.
- (5) Triangle ABC, so obtained, is the required triangle.

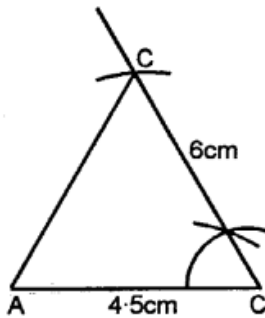
Question 6.

AC = 4.5 cm, BC = 6 cm and $\angle C = 60^\circ$.

Solution:

Steps of Construction :

- (1) Draw a line AC = 4.5 cm.



- (2) With the help of compasses, construct $\angle ACB = 60^\circ$.
- (3) With C as centre, draw an arc of 6 cm radius, which cuts CB at C.
- (4) Join B and A.

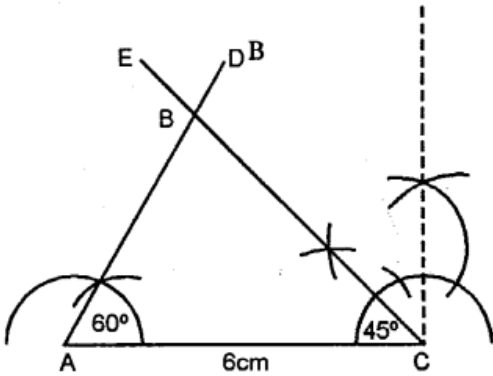
Question 7.

AC = 6 cm, $\angle A = 60^\circ$ and $\angle C = 45^\circ$. Measure AB and BC.

Solution:

Steps of Construction :

- (1) Draw a line segment AC = 6 cm.



- (2) At A construct an angle $\angle A = 60^\circ$.
- (3) At C construct an angle $\angle C = 45^\circ$.
- (4) AD and CE intersect each other at B.
- (5) $\therefore \triangle ABC$ is the required triangle.
- (6) On measuring $AB = 4.4$ cm, $BC = 5.4$ cm.

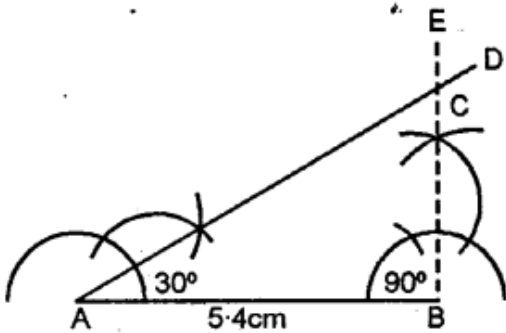
Question 8.

AB = 5.4 cm, $\angle A = 30^\circ$ and $\angle B = 90^\circ$. Measure $\angle C$ and side BC.

Solution:

Steps of Construction :

- (1) Draw a line segment $AB = 5.4$ cm.



- (2) At A construct an angle $\angle A = 30^\circ$.
- (3) At B construct an angle $\angle B = 90^\circ$.
- (4) AD and BE intersect each other at C.
- (5) $\therefore \triangle ABC$ is the required triangle.
- (6) On measuring $\angle C = 60^\circ$ side $BC = 3.1$ cm.

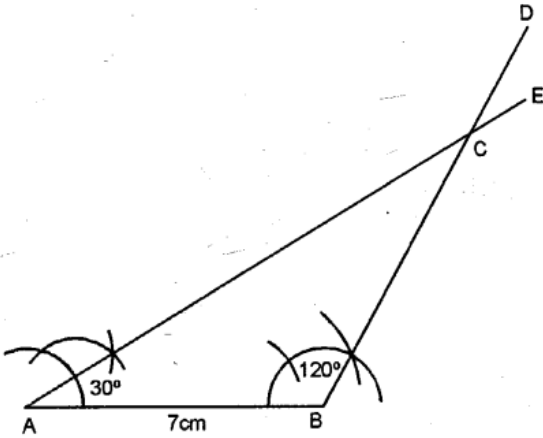
Question 9.

AB = 7 cm, $\angle B = 120^\circ$ and $\angle A = 30^\circ$. Measure AC and BC.

Solution:

Steps of Construction :

- (1) Draw a line segment $AB = 7$ cm



- (2) At A construct an angle $\angle A = 30^\circ$.
- (3) At B construct an angle $\angle B = 120^\circ$.
- (4) AE and BD intersect each other at C.
- (5) $\therefore \triangle ABC$ is the required triangle.
- (6) On measuring length of AC = 12cm and BC = 7 cm respectively.

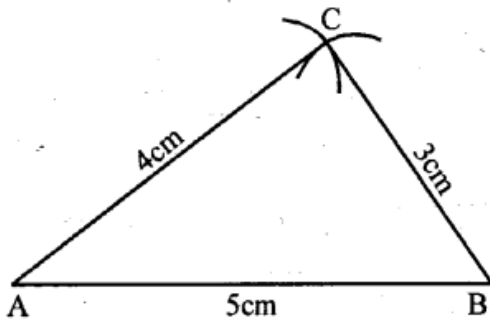
Question 10.

BC = 3 cm, AC = 4 cm and AB = 5 cm. Measure angle ACB. Give a special name to this triangle.

Solution:

Steps of Construction :

- (1) Draw a line segment AB = 5 cm



- (2) From A, with the help of compass cut arc AC = 4cm
 - (3) From point B cut an arc BC = 3 cm.
 - (4) Join AC and BC.
 - (5) $\triangle ABC$ is required right-angled triangle.
- Measuring $\angle ACB = 90^\circ$

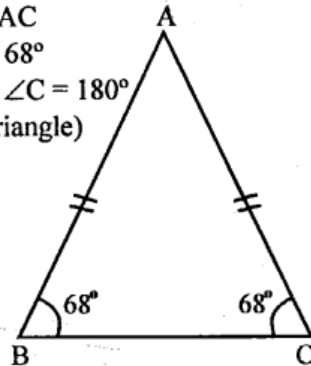
REVISION EXERCISE

Question 1.

If each of the two equal angles of an isosceles triangle is 68° , find the third angle.

Solution:

Let $\triangle ABC$ is an isosceles triangle
 $\therefore$ In $\triangle ABC$, $AB = AC$
and $\angle B = \angle C = 68^\circ$
But $\angle A + \angle B + \angle C = 180^\circ$
(Sum of angles of a triangle)



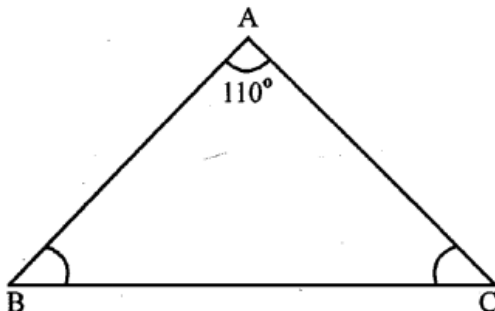
$$\begin{aligned}\Rightarrow \angle A + 68^\circ + 68^\circ &= 180^\circ \\ \Rightarrow \angle A + 136^\circ &= 180^\circ \Rightarrow \angle A = 180^\circ - 136^\circ \\ \therefore \angle A &= 44^\circ \\ \text{Hence third angle} &= 44^\circ\end{aligned}$$

Question 2.

One of the angles of a triangle is 110° , the two other angles are equal. Find their value.

Solution:

Let in $\triangle ABC$,
 $\angle A = 110^\circ$ and $\angle B = \angle C$
But $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 110^\circ + \angle B + \angle B = 180^\circ$
 $\Rightarrow 2\angle B = 180^\circ - 110^\circ \Rightarrow 2\angle B = 70^\circ$
 $\Rightarrow \angle B = \frac{70^\circ}{2} = 35^\circ$
 $\therefore \angle C = \angle B = 35^\circ$
Hence each of two equal angles is 35°



Question 3.

The angles of a triangle are in the ratio 3:5: 7. Find each angle.

Solution:

Ratio in angles of a triangle is 3 : 5 : 7

But sum of angles of a triangle = 180°

Sum of ratios = $3 + 5 + 7 = 15$

$$\therefore \text{First angle} = \frac{180^\circ}{15} \times 3 = 36^\circ$$

$$\text{Second angle} = \frac{180^\circ \times 5}{15} = 60^\circ$$

$$\text{and third angle} = \frac{180^\circ \times 7}{15} = 84^\circ$$

$\therefore$ Angles of the triangle are 36° , 60° and 84°

Question 4.

The angles of a triangle are $(2x - 30^\circ)$, $(3x - 40^\circ)$ and $(\frac{5}{2}x + 10^\circ)$ Find the value of x .

Solution:

$\therefore$ The sum of angles of a triangle = 180°

$$\therefore (2x - 30^\circ) + (3x - 40^\circ) + \left(\frac{5}{2}x + 10^\circ\right) = 180^\circ$$

$$\Rightarrow 2x - 30^\circ + 3x - 40^\circ + \frac{5}{2}x + 10^\circ = 180^\circ$$

$$\Rightarrow 2x + 3x + \frac{5}{2}x - 70^\circ + 10^\circ = 180^\circ$$

$$\Rightarrow \frac{4x + 6x + 5x}{2} - 60^\circ = 180^\circ$$

$$\Rightarrow \frac{15}{2}x = 180^\circ + 60^\circ = 240^\circ$$

$$x = \frac{240^\circ \times 2}{15} = 32$$

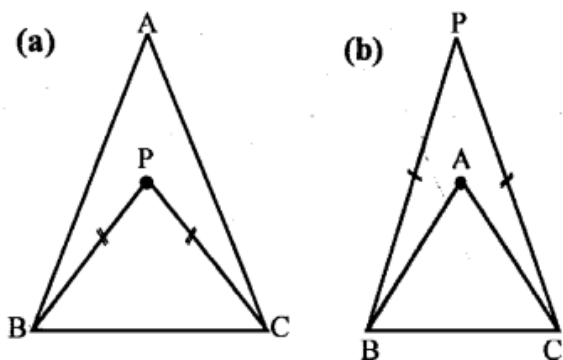
$$\therefore x = 32^\circ$$

Question 5.

In each of the following figures, triangle ABC is equilateral and triangle PBC is isosceles. If $\angle PBA = 20^\circ$; find in each case:

(a) angle PBC.

(b) angle BPC.



Solution:

(a) In figure (a)

$\triangle ABC$ is an equilateral triangle and $\triangle PBC$ is an isosceles triangle and $\angle PBA = 20^\circ$

$\therefore \triangle ABC$ is an equilateral triangle

$\therefore \angle ABC = 60^\circ$

$\Rightarrow \angle PBA + \angle PBC = 60^\circ \Rightarrow 20^\circ + \angle PBC = 60^\circ$

$\Rightarrow \angle PBC = 60^\circ - 20^\circ = 40^\circ$

$\therefore \triangle PBC$ is an isosceles triangle

$\therefore \angle PBC = \angle PCB = 40^\circ$

Now in $\triangle BPC$,

$\angle PBC + \angle PCB + \angle BPC = 180^\circ$

(Sum of angles of a triangle)

$\Rightarrow 40^\circ + 40^\circ + \angle BPC = 180^\circ$

$\Rightarrow 80^\circ + \angle BPC = 180^\circ$

$\Rightarrow \angle BPC = 180^\circ - 80^\circ = 100^\circ$

(b) In figure (b)

$\angle PBA = \angle ABC + \angle PBA = 60^\circ + 20^\circ = 80^\circ$

But $\triangle PBC$ is an isosceles triangle

$\therefore \angle PCB = \angle PBC = 80^\circ$

But $\angle PBC + \angle PCB + \angle BPC = 180^\circ$

(Sum of angles of a triangle)

$\Rightarrow 80^\circ + 80^\circ + \angle BPC = 180^\circ$

$\Rightarrow 160^\circ + \angle BPC = 180^\circ$

$\angle BPC = 180^\circ - 160^\circ = 20^\circ$

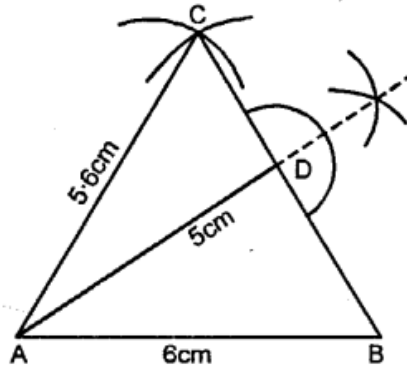
Question 6.

Construct a triangle ABC given $AB = 6$ cm, $BC = 5$ cm and $CA = 5.6$ cm. From vertex A draw a perpendicular on to side BC. Measure the length of this perpendicular.

Solution:

Steps of Construction :

- (1) Draw a line $AB = 6$ cm.
- (2) Using compass, taking A and B as centre draw arcs of 5 cm and 5.6 cm respectively, which cut each other at C.
- (3) Join AC and BC.
- (4) Now, from vertex A draw a bisector AD towards BC. On measuring length $AD = 5$ cm.

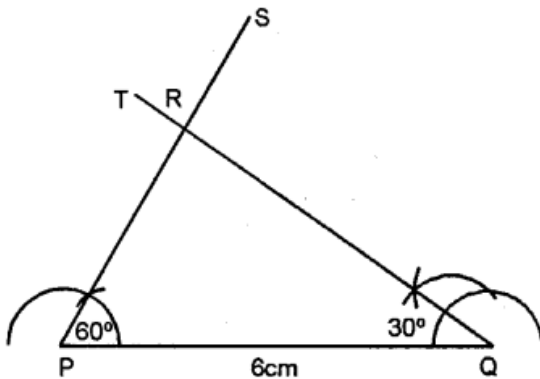
**Question 7.**

Construct a triangle PQR, given $PQ = 6$ cm, $\angle P = 60^\circ$ and $\angle Q = 30^\circ$. Measure angle R and the length of PR.

Solution:

Steps of Construction :

- (1) Draw a line $PQ = 6$ cm.



- (2) Using compass taking P as centre draw an angle $\angle P = 60^\circ$.
- (3) Using compass taking Q as a centre draw an angle $\angle Q = 30^\circ$.
- (4) PS and QT intersect each other R.
- (5) $\triangle RPQ$ is the required triangle.

On measuring; $\angle R = 90^\circ$, length of $PR = 3$ cm.

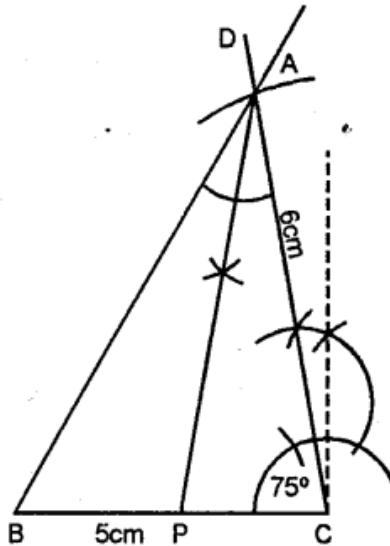
Question 8.

Construct a triangle ABC given $BC = 5$ cm, $AC = 6$ cm and $\angle C = 75^\circ$. Draw the bisector of the interior angle at A. Let this bisector meet BC at P ; measure BP.

Solution:

Steps of Construction :

- (1) Draw $BC = 5$ cm.
- (2) With the help of compass from centre C. Draw an angle $\angle C = 75^\circ$.
- (3) From CD, cut an arc $AC = 6$ cm.



- (4) Join AB.
- (5) From A draw an bisector AP.
- (6) On measuring $BP = 2.6$ cm.

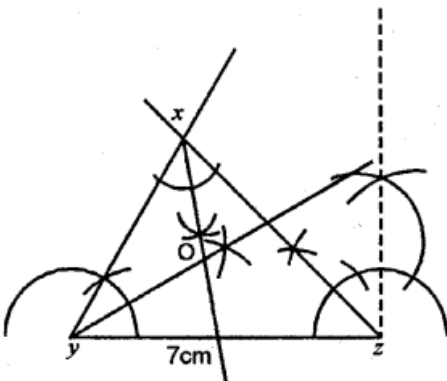
Question 9.

Using ruler and a pair compass only, construct a triangle XYZ given $YZ = 7$ cm, $\angle XYZ = 60^\circ$ and $\angle XZY = 45^\circ$. Draw the bisectors of angles X and Y.

Solution:

Steps of Construction :

- (1) Draw a line $YZ = 7$ cm.



- (2) Y as a centre draw an $\angle XYZ = 60^\circ$.

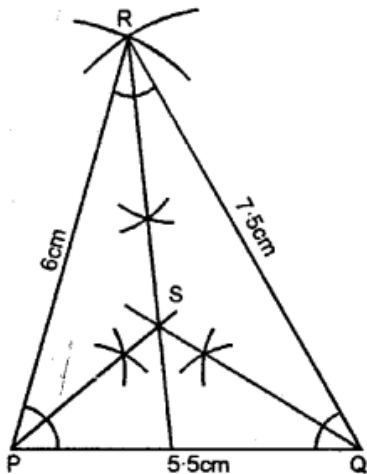
(3) Z as a centre draw an $\angle XZY = 45^\circ$.

(4) From X and Y as centre draw bisector of $\angle X$ and $\angle Y$, which meet at point O.

Question 10.

Using ruler and a pair compass only, construct a triangle PQR, given $PQ = 5.5$ cm, $QR = 7.5$ cm and $RP = 6$ cm. Draw the bisectors of the interior angles at P, Q and R. Do these bisectors meet at the same point ?

Solution:



Steps of Construction :

(1) Draw a line $PQ = 5.5$ cm.

(2) From Q as a centre draw an arc $QR = 7.5$ cm.

(3) From P as a centre draw an arc $PR = 6$ m, which intersects previous arc at R.

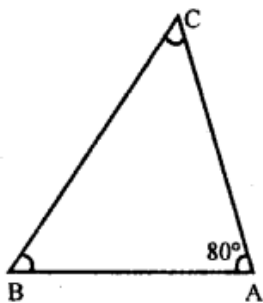
(4) Join PR and QR.

(5) Now, draw interior bisectors of $\angle P$, $\angle Q$ and $\angle R$ which meet each other at point S.

Question 11.

One angle of a triangle is 80° and the other two are in the ratio 3 : 2. Find the unknown angles of the triangle.

Solution:



Let angle $\angle A$ of a triangle $ABC = 80^\circ$

But sum of three angles of a triangle = 180°

Sum of remaining two angles = $180^\circ - 80^\circ = 100^\circ$

Ratio in their two angles = 3:2

Let second angle = $3x$

and third angle = $2x$

$$3x + 2x = 100^\circ$$

$$5x = 100 \Rightarrow x = \frac{100}{5} = 20^\circ$$

$$\text{second angle } \angle B = 3x = 3 \times 20^\circ = 60^\circ$$

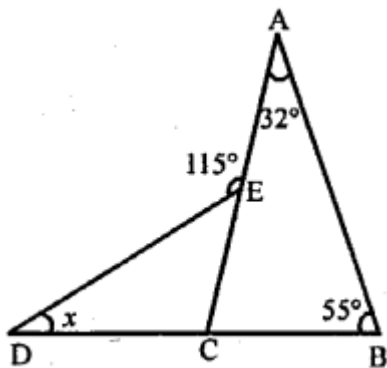
$$\text{and third angle } \angle C = 2x = 2 \times 20^\circ = 40^\circ$$

Hence other two angles are 60° and 40° .

Question 12.

Find the value of x if $\angle A = 32^\circ$, $\angle B = 55^\circ$ and obtuse angle $\angle AED = 115^\circ$.

Solution:



In the figure, $\angle A = 32^\circ$, $\angle B = 55^\circ$

$$\angle AED = 115^\circ$$

In $\triangle ABC$

$$\text{Exterior } \angle ACD = \angle A + \angle B = 32^\circ + 55^\circ = 87^\circ$$

Similarly in $\triangle CDE$

$$\text{Ext. } \angle AED = \angle D + \angle ACD$$

$$\Rightarrow 115^\circ = x + 87^\circ \Rightarrow x = 115^\circ - 87^\circ = 28^\circ$$

$$\text{Hence } x = 28^\circ$$