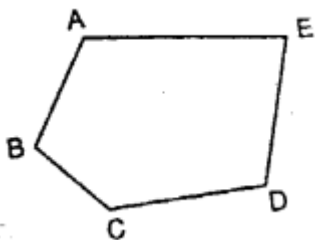


Polygons

IMPORTANT POINTS

1. Polygon : A closed plane geometrical figure, bounded by at least three line segments, is called a polygon.

The adjoining figure is a polygon as it is :



(i) Closed

(ii) bounded by five line segments AB, BC, CD, DE and AE.

Also, it is clear from the given polygon that:

(i) the line segments AB, BC, CD, DE and AE intersect at their end points.

(ii) two line segments, with a common vertex, are not collinear i.e. the angle at any vertex is not 180° .

A polygon is named according to the number of sides (line-segments) in it:

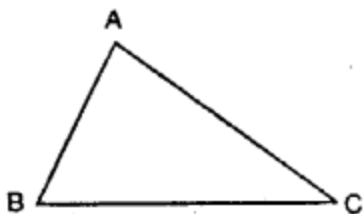
Note : No. of sides :	3	4	5	6
Name of polygon :	Triangle	Quadrilateral	Pentagon	Hexagon

2. Sum of Interior Angles of a Polygon

1. Triangle : Students already know that the sum of interior angles of a triangle is always 180° .

i.e. for $\triangle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180^\circ$

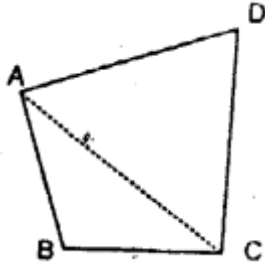
$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$



2. Quadrilateral : Consider a quadrilateral ABCD as shown alongside.

If diagonal AC of the quadrilaterals drawn, the quadrilateral will be divided into two triangles ABC and ADC.

Since, the sum of interior angles of a triangle is 180° .



$\therefore$ In $\triangle ABC$, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

And, in $\triangle ADC$ $\angle DAC + \angle ADC + \angle ACD = 180^\circ$

Adding we get:

$$\angle ABC + \angle BAC + \angle ACB + \angle DAC + \angle ADC + \angle ACD = 180^\circ + 180^\circ$$

$$\Rightarrow (\angle BAC + \angle DAC) + \angle ABC + (\angle ACB + \angle ACD) + \angle ADC = 360^\circ$$

$$\Rightarrow \angle BAD + \angle ABC + \angle BCD + \angle ADC = 360^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

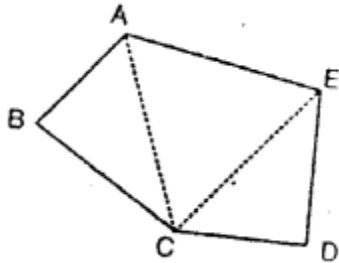
Alternative method : On drawing the diagonal AC, the given quadrilateral is divided into two triangles. And, we know the sum of the interior angles of a triangle is 180° .

$\therefore$ **Sum of interior angles of the quadrilateral ABCD**

$$= \text{Sum of interior angles of } \triangle ABC + \text{sum of interior angles of } \triangle ADC = 180^\circ + 180^\circ = 360^\circ$$

3. Pentagon : Consider a pentagon ABCDE as shown alongside.

On joining CA and CE, the given pentagon is divided into three triangles ABC, CDE and ACE.



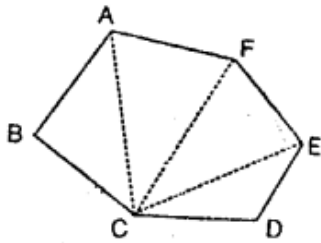
Since, the sum of the interior angles of a triangle is 180°

$$\text{Sum of the interior angles of the pentagon ABCDE} = \text{Sum of interior angles of } (\triangle ABC + \triangle CDE + \triangle ACE)$$

$$= 180^\circ + 180^\circ + 180^\circ = 540^\circ$$

4. Hexagon :

It is clear from the given figure that the sum of the interior angles of the hexagon ABCDEF.



= Sum of interior angles of
 $(\triangle ABC + \triangle ACF + \triangle FCE + \triangle ECD)$
 $= 180^\circ + 180^\circ + 180^\circ + 180^\circ = 720^\circ$

3. Using Formula : The sum of interior angles of a polygon can also be obtained by using the following formula:

Note : Sum of interior angles of a polygon $= (n - 2) \times 180^\circ$
 where, n = number of sides of the polygon.

$\therefore$ (i) **For a triangle :**

$n = 3$ (a triangle has 3 sides)

and, sum of interior angles $= (2n - 4) \times 90^\circ$
 $= (6 - 4) \times 90^\circ = 180^\circ$

(ii) **For a quadrilateral :**

$n = 4$

and, sum of interior angles $= (2n - 4) \times 90^\circ$
 $= (8 - 4) \times 90^\circ = 360^\circ$

(iii) **For a pentagon :**

$n = 5$

and, sum of interior angles $= (2n - 4) \times 90^\circ$
 $= (10 - 4) \times 90^\circ = 6 \times 90^\circ = 540^\circ$

(iv) **For a hexagon :**

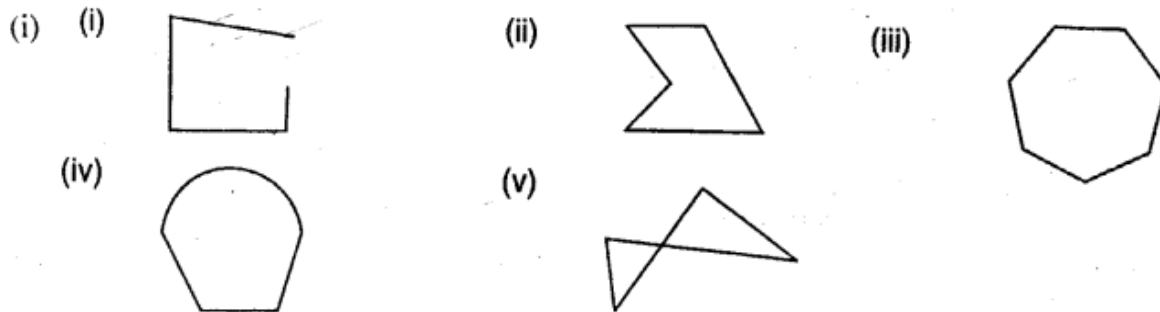
$n = 6$

and, sum of interior angles $= (2n - 4) \times 90^\circ$
 $= (12 - 4) \times 90^\circ = 8 \times 90^\circ = 720^\circ$

EXERCISE 28 (A)

Question 1.

State, which of the following are polygons :



Solution:

Only figure (ii) and (iii) are polygons.

Question 2.

Find the sum of interior angles of a polygon with :

(i) 9 sides

(ii) 13 sides

(iii) 16 sides

Solution:

(i) 9 sides

No. of sides $n = 9$

$\therefore$ Sum of interior angles of polygon $= (2n - 4) \times 90^\circ$

$= (2 \times 9 - 4) \times 90^\circ$

$= 14 \times 90^\circ = 1260^\circ$

(ii) 13 sides

No. of sides $n = 13$

$\therefore$ Sum of interior angles of polygon $= (2n - 4) \times 90^\circ = (2 \times 13 - 4) \times 90^\circ = 1980^\circ$

(iii) 16 sides

No. of sides $n = 16$

$\therefore$ Sum of interior angles of polygon $= (2n - 4) \times 90^\circ$

$= (2 \times 16 - 4) \times 90^\circ$

$= (32 - 4) \times 90^\circ = 28 \times 90^\circ$

$= 2520^\circ$

Question 3.

Find the number of sides of a polygon, if the sum of its interior angles is :

(i) 1440°

(ii) 1620°

Solution:

(i) 1440°

Let no. of sides = n

$\therefore$ Sum of interior angles of polygon = 1440°

$$\therefore (2n - 4) \times 90^\circ = 1440^\circ$$

$$\Rightarrow 2n - 4 = \frac{1440^\circ}{90^\circ}$$

$$\Rightarrow 2(n - 2) = \frac{1440^\circ}{90^\circ}$$

$$\Rightarrow n - 2 = \frac{1440^\circ}{2 \times 90^\circ}$$

$$\Rightarrow n - 2 = 8$$

$$\Rightarrow n = 8 + 2$$

$$\Rightarrow n = 10$$

(ii) 1620°

Let no. of sides = n

$\therefore$ Sum of interior angles of polygon = 1620°

$$\therefore (2n - 4) \times 90^\circ = 1620^\circ$$

$$\Rightarrow 2(n - 2) = \frac{1620^\circ}{90^\circ}$$

$$\Rightarrow n - 2 = \frac{1620^\circ}{2 \times 90^\circ}$$

$$\Rightarrow n - 2 = 9$$

$$\Rightarrow n = 9 + 2 \Rightarrow n = 11$$

Question 4.

Is it possible to have a polygon, whose sum of interior angles is 1030° .

Solution:

Let no. of sides be $= n$

Sum of interior angles of polygon =
 1030°

$$\therefore (2n - 4) \times 90^\circ = 1030^\circ$$

$$\Rightarrow 2(n - 2) = \frac{1030^\circ}{90^\circ}$$

$$\Rightarrow (n - 2) = \frac{1030^\circ}{2 \times 90^\circ}$$

$$\Rightarrow (n - 2) = \frac{103}{18}$$

$$\Rightarrow n = \frac{103}{18} + 2$$

$$\Rightarrow n = \frac{139}{18}$$

Which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is 1030° .

Question 5.

- (i) If all the angles of a hexagon are equal, find the measure of each angle.
- (ii) If all the angles of an octagon are equal, find the measure of each angle,

Solution:

(i) No. of sides of hexagon, $n = 6$

Let each angle be $= x^\circ$

$$\therefore \text{Sum of angles} = 6x^\circ$$

$$\therefore (2n - 4) \times 90^\circ = \text{Sum of angles}$$

$$(2 \times 6 - 4) \times 90^\circ = 6x^\circ$$

$$(12 - 4) \times 90^\circ = 6x^\circ$$

$$\Rightarrow \frac{8 \times 90^\circ}{6} = x^\circ$$

$$\Rightarrow x = 120^\circ$$

$$\therefore \text{Each angle of hexagon} = 120^\circ$$

(ii) No. of sides of octagon $n = 8$

Let each angle be $= x^\circ$

$$\therefore \text{Sum of angles} = 8x^\circ$$

$$\therefore (2n - 4) \times 90^\circ = \text{Sum of angles}$$

$$(2 \times 8 - 4) \times 90^\circ = 8x^\circ$$

$$12 \times 90^\circ = 8x^\circ$$

$$\Rightarrow x^\circ = \frac{90^\circ \times 12^\circ}{8} \quad \Rightarrow x^\circ = 135^\circ$$

$$\therefore \text{Each angle of octagon} = 135^\circ$$

Question 6.

One angle of a quadrilateral is 90° and all other angles are equal ; find each equal angle.

Solution:

Let the angles of a quadrilateral be x° , x° , x° , and 90°

$$\therefore \text{Sum of interior angles of quadrilateral} = 360^\circ$$

$$\Rightarrow x^\circ + x^\circ + x^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow 3x^\circ = 360^\circ - 90^\circ$$

$$\Rightarrow x = \frac{270^\circ}{3}$$

$$\Rightarrow x = 90^\circ$$

Question 7.

If angles of quadrilateral are in the ratio $4 : 5 : 3 : 6$; find each angle of the quadrilateral.

Solution:

Let the angles of the quadrilateral be $4x$, $5x$, $3x$ and $6x$

$$\therefore 4x + 5x + 3x + 6x = 360^\circ$$

$$18x = 360^\circ$$

$$x = \frac{360^\circ}{18} = 20^\circ$$

$$\therefore \text{First angle} = 4x = 4 \times 20^\circ = 80^\circ$$

$$\text{Second angle} = 5x = 5 \times 20^\circ = 100^\circ$$

$$\text{Third angle} = 3x = 3 \times 20^\circ = 60^\circ$$

$$\text{Fourth angle} = 6x = 6 \times 20^\circ = 120^\circ$$

Question 8.

If one angle of a pentagon is 120° and each of the remaining four angles is x° , find the magnitude of x .

Solution:

One angle of a pentagon = 120°

Let remaining four angles be x , x , x and x

Their sum = $4x + 120^\circ$

But sum of all the interior angles of a pentagon = $(2n - 4) \times 90^\circ$

$$= (2 \times 5 - 4) \times 90^\circ = 540^\circ$$

$$= 3 \times 180^\circ = 540^\circ$$

$$\therefore 4x + 120^\circ = 540^\circ$$

$$4x = 540^\circ - 120^\circ$$

$$4x = 420$$

$$x = \frac{420}{4} \Rightarrow x = 105^\circ$$

$\therefore$ Equal angles are 105° (Each)

Question 9.

The angles of a pentagon are in the ratio $5 : 4 : 5 : 7 : 6$; find each angle of the pentagon.

Solution:

Let the angles of the pentagon be $5x$, $4x$, $5x$, $7x$, $6x$

Their sum $= 5x + 4x + 5x + 7x + 6x = 27x$

Sum of interior angles of a polygon

$$= (2n - 4) \times 90^\circ$$

$$= (2 \times 5 - 4) \times 90^\circ = 540^\circ$$

$$\therefore 27x = 540 \Rightarrow \frac{540}{27} \Rightarrow x = 20^\circ$$

$$\therefore \text{Angles are } 5 \times 20^\circ = 100^\circ$$

$$4 \times 20^\circ = 80^\circ$$

$$5 \times 20^\circ = 100^\circ$$

$$7 \times 20^\circ = 140^\circ$$

$$6 \times 20^\circ = 120^\circ$$

Question 10.

Two angles of a hexagon are 90° and 110° . If the remaining four angles are equal, find each equal angle.

Solution:

Two angles of a hexagon are 90° , 110°

Let remaining four angles be x , x , x and x

Their sum $= 4x + 200^\circ$

But sum of all the interior angles of a hexagon

$$= (2n - 4) \times 90^\circ$$

$$= (2 \times 6 - 4) \times 90^\circ = 8 \times 90^\circ = 720^\circ$$

$$\therefore 4x + 200^\circ = 720^\circ$$

$$\Rightarrow 4x = 720^\circ - 200^\circ = 520^\circ$$

$$\Rightarrow x = \frac{520^\circ}{4} = 130^\circ$$

$\therefore$ Equal angles are 130° (each)

EXERCISE 28 (B)

Question 1.

Fill in the blanks :

In case of regular polygon, with

Number of sides	Each exterior angle	Each interior angle
(i) 6		
(ii) 8		
(iii)	36°	
(iv)	20°	
(v)		135°
(vi)		165°

Solution:

Number of sides	Each exterior angle	Each interior angle
(i) 6	60°	120°
(ii) 8	45°	135°
(iii) 10	36°	144°
(iv) 18	20°	160°
(v) 8	45°	135°
(vi) 24	15°	165°

(i) Each exterior angle = $\frac{360^\circ}{6} = 60^\circ$

Each interior angle = $180^\circ - 60^\circ = 120^\circ$

(ii) Each exterior angle = $\frac{360^\circ}{8} = 45^\circ$

Each interior angle = $180^\circ - 45^\circ = 135^\circ$

(iii) Since each exterior angles = 36°

$\therefore$ Number of sides = $\frac{360^\circ}{36^\circ} = 10$

Also, interior angle = $180^\circ - 20^\circ = 160^\circ$

(iv) Since each exterior angles = 20°

$\therefore$ Number of sides = $\frac{360^\circ}{20^\circ} = 18$

Also, interior angle $180^\circ - 20^\circ = 160^\circ$

(v) Since interior angle = 135°

$\therefore$ Exterior angle = $180^\circ - 135^\circ$

$\therefore$ Number of sides = $\frac{360^\circ}{45^\circ} = 8$

(vi) Since interior angle = 165°

$\therefore$ Exterior angle = $180^\circ - 165^\circ = 15^\circ$

$\therefore$ Number of sides = $\frac{360^\circ}{15^\circ} = 24$

Question 2.

Find the number of sides in a regular polygon, if its each interior angle is :

(i) 160°

(ii) 150°

Solution:

(i) 160°

Let no. of sides of regular polygon be n

Each interior angle = 160°

$$\therefore \frac{(2n-4) \times 90^\circ}{n} = 160^\circ$$

$$180n - 360^\circ = 160n$$

$$180n - 160n = 360^\circ$$

$$n = \frac{360^\circ}{20}$$

$$n = 18$$

(ii) 150°

Let no. of sides of regular polygon be n

Each interior angle = 150°

$$\therefore \frac{(2n-4) \times 90^\circ}{n} = 150^\circ$$

$$180n - 360^\circ = 150n$$

$$180n - 150n = 360^\circ$$

$$30n = 360^\circ$$

$$n = \frac{360^\circ}{30}$$

$$n = 12$$

Question 3.

Find number of sides in a regular polygon, if its each exterior angle is :

(i) 30°

(ii) 36°

Solution:

(i) 30°

Let number of sides = n

$$\therefore \frac{360^\circ}{n} = 30^\circ$$

$$n = \frac{360^\circ}{30^\circ}$$

$$n = 12$$

(ii) 36°

Let number of sides = n

$$\therefore \frac{360^\circ}{n} = 36^\circ$$

$$n = \frac{360^\circ}{36^\circ}$$

$$n = 10$$

Question 4.

Is it possible to have a regular polygon whose each interior angle is :

(i) 135°

(ii) 155°

Solution:

(i) 135°

No. of sides = n

Each interior angle = 135°

$$\therefore \frac{(2n-4) \times 90^\circ}{n} = 135^\circ$$

$$180n - 360^\circ = 135n$$

$$180n - 135n = 360^\circ$$

$$n = \frac{360^\circ}{45^\circ}$$

$$n = 8$$

Which is a whole number.

Hence, it is possible to have a regular polygon whose interior angle is 135° .

(ii) 155°

No. of sides = n

Each interior angle = 155°

$$\therefore \frac{(2n-4) \times 90^\circ}{n} = 155^\circ$$

$$180n - 360^\circ = 155n$$

$$180n - 155n = 360^\circ$$

$$25n = 360^\circ$$

$$n = \frac{360^\circ}{25^\circ}$$

$$n = \frac{72^\circ}{5}$$

Which is not a whole number.

Hence, it is not possible to have a regular polygon having interior angle is of 155° .

Question 5.

Is it possible to have a regular polygon whose each exterior angle is :

(i) 100°

(ii) 36°

Solution:

(i) 100°

Let no. of sides = n

Each exterior angle = 100°

$$= \frac{360^\circ}{n} = 100^\circ$$

$$\therefore n = \frac{360^\circ}{100^\circ}$$

$$n = \frac{18}{5}$$

Which is not a whole number.

Hence, it is not possible to have a regular polygon whose each exterior angle is 100° .

(ii) 36°

Let number of sides = n

Each exterior angle = 36°

$$= \frac{360^\circ}{n} = 36^\circ$$

$$\therefore n = \frac{360^\circ}{36^\circ}$$

$$n = 10$$

Which is a whole number.

Hence, it is possible to have a regular polygon whose each exterior angle is of 36° .

Question 6.

The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :

(i) each exterior angle of this polygon.

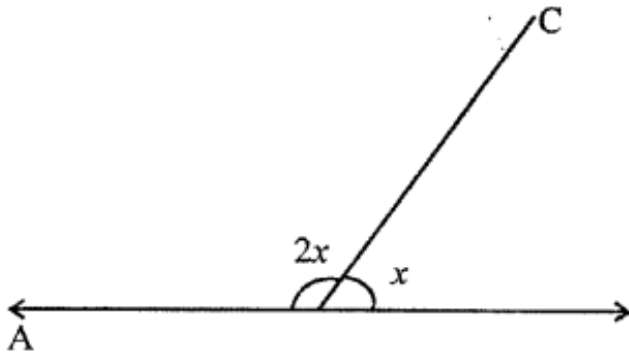
(ii) number of sides in the polygon.

Solution:

(i) Interior angle : exterior angle = 2 : 1

∴ Let interior angle = $2x^\circ$

and exterior angle = x°



$$\therefore 2x^\circ + x^\circ = 180^\circ$$

$$3x^\circ = 180^\circ \Rightarrow x = \frac{180^\circ}{3} = 60^\circ$$

(ii) $x = 60$

∴ Each exterior angle = 60°

$$\therefore \frac{360^\circ}{n} = 60^\circ$$

$$n = \frac{360^\circ}{60^\circ} = 6 \text{ sides}$$