Simultaneous Linear Equations

Substitution Method Of Solving Pairs Of Linear Equations

In a class, the number of boys is 7 more than twice the number of girls.

Can you find the number of boys and girls in the class?

Let the number of boys be x and the number of girls be y.

Now, according to the given condition,

$x - 2y = 7$

We cannot find the unique values of x and y by solving this equation because there are multiple values of x and y for which this equation holds true.

We can write the above equation as $x = 2y + 7$. Thus, by taking different values of y, we will obtain different values of x.

However, if we are given one more condition, then the values of x and y can be evaluated.

To solve a linear equation in two variables, two linear equations in the same two variables are required.

Let us consider one more condition. Suppose the total number of students in the class is 52.

Now, can you find the number of boys and girls in the class?

According to this condition, we can write $x + y = 52$

Therefore, we obtain a pair of linear equations in two variables.

$x - 2y = 7$

$x + y = 52$

Therefore, number of boys = 37

and number of girls = 15

In this method, we have **substituted** the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why this method is known as the ***substitution method***.

Now, let us solve some examples to understand this concept better.

Example 1:

The sum of the digits of a two-digit number is 8. If 18 is added to the number, then the digits interchange their places. Find the number.

Example 2:

Solve the following pair of linear equations.

$7x - 5y + 12 = 0$

$3x + 8y - 5 = 0$

Solution:

Given, $7x - 5y + 12 = 0$ … (i)

$3x + 8y - 5 = 0$ … (ii)

From equation (i), we obtain

$7x - 5y + 12 = 0$

$$\Rightarrow y = \frac{7x+12}{5} \qquad ...(iii)$$

On substituting the value of y in equation (ii), we obtain

$$3x + 8\left(\frac{7x+12}{5}\right) - 5 = 0$$
$$\Rightarrow 3x + \frac{56x+96}{5} - 5 = 0$$
$$\Rightarrow \frac{15x+56x+96-25}{5} = 0$$
$$\Rightarrow 15x+56x+96-25 = 0$$

$\Rightarrow 71x + 71 = 0$

$\Rightarrow 71x = -71$

$\Rightarrow x = -1$

Putting the value of x in equation (iii), we obtain

$$y = \frac{7\times(-1)+12}{5}$$
$$= \frac{-7+12}{5}$$
$$= \frac{5}{5}$$
$$= 1$$

Therefore, $x = -1$ and $y = 1$

Example 3:

Arnab has few coins of Rs 5 and Rs 2 which together amount to Rs 167. If he has a total of 46 coins then find the number of both types of coins with him.

Solution:

Let the number of coins of Rs 2 be x and that of Rs 5 be y.

According to the question, we have

$2x + 5y = 167$ ...(i)

$x + y = 46$ ...(ii)

From equation (ii), we obtain

$x = 46 - y$ ...(iii)

On substituting the value of x from equation (iii) in equation (i), we obtain

$2(46 - y) + 5y = 167$

$\Rightarrow 92 - 2y + 5y = 167$

$\Rightarrow 92 + 3y = 167$

$\Rightarrow 3y = 75$

$\Rightarrow y = 25$

On substituting the value of y in equation (ii), we obtain

$x + 25 = 46$

$\Rightarrow x = 21$

Hence, Arnab has 21 coins of Rs 2 and 25 coins of Rs 5.

Example 4:

A boat can travel 28 km downstream and 20 km upstream in 7 hours. Also, this boat can travel 21 km downstream and 18 km upstream in 6 hours. What is the speed of the boat in still water and the speed of the stream?

Solution:

Let x km/hr and y km/hr be the speed of the boat in still water and speed of the stream respectively.

Therefore,

Speed of the boat downstream = $(x + y)$ km/hr

Speed of the boat upstream = $(x - y)$ km/hr

∴ Time taken by the boat to travel 28 km downstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{28}{x+y}$ hours

And, time taken by the boat to travel 20 km upstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{20}{x-y}$ hours

According to the question, we have

$$\frac{28}{x+y} + \frac{20}{x-y} = 7 \qquad \ldots\text{(i)}$$

Now,

Time taken by the boat to travel 21 km downstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{21}{x+y}$ hours

And, time taken by the boat to travel 18 km upstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{18}{x-y}$ hours

According to the question, we have

$$\frac{21}{x+y}+\frac{18}{x-y}=6 \qquad ...(ii)$$

On substituting $\frac{1}{x+y}=a$ and $\frac{1}{x-y}=b$ in equation (i) and (ii), we obtain

$28a + 20b = 7$ ...(iii)

$21a + 18b = 6$ ...(iv)

From equation (iv), we obtain

$7a + 6b = 2$

$\Rightarrow 7a = 2 - 6b$

$$a=\frac{2-6b}{7} \qquad ...(v)$$

On substituting the value of *a* from (v) in (iii), we obtain

$$28\left(\frac{2-6b}{7}\right)+20b=7$$
$$\Rightarrow 4(2-6b)+20b=7$$
$$\Rightarrow 8-24b+20b=7$$
$$\Rightarrow 8-4b=7$$
$$\Rightarrow 4b=1$$
$$\Rightarrow b=\frac{1}{4}$$

On putting this value of *b* in equation (v), we obtain

$$a=\frac{2-6\left(\frac{1}{4}\right)}{7}=\frac{\left(2-\frac{3}{2}\right)}{7}=\frac{\left(\frac{1}{2}\right)}{7}=\frac{1}{14}$$

On resubstituting the values of *a* and *b*, we obtain

$$\frac{1}{x+y}=\frac{1}{14} \text{ and } \frac{1}{x-y}=\frac{1}{4}$$
$$\Rightarrow x+y=14 \quad ....(vi) \text{ and } x-y=4 \quad ...(vii)$$

From (vii), we obtain $x = y + 4$. On substituting this value of *x* in (vi), we obtain

$y + 4 + y = 14$

$\Rightarrow 2y = 10$

$\Rightarrow y = 5$

On substituting $y = 5$ in (vii), we get

$x - 5 = 4$

$\Rightarrow x = 9$

Thus, the speed of the boat in still water is 9 km/hr and the speed of stream is 5 km/hr.

Elimination Method To Solve A Pair Of Linear Equations

There are many real life situations which can be represented in the form of linear equations. Let us begin with such a situation.

Suppose you go to the market with your friend to buy clothes. You buy 3 shirts and 6 pairs of jeans from a shop and the total cost of your purchase comes out to be Rs 7000. Your friend, on the other hand buys 2 shirts and 3 pairs of jeans and the total cost of his purchase comes out to be Rs 3855.

Now, we can represent this information mathematically as pair of linear equations in two variables. We can then solve the equations and get the cost of each shirt and each pair of jeans. Let’s look at the following video to learn how to mathematically represent the information and then solve them using the elimination method.

Now, let us solve some more problems by elimination method.

Example 1:

Solve the following pair of linear equations by elimination method:

$2x - 3y = 3$

$6x + y = 99$

Solution:

Example 2:

Solve the following equations using elimination method.

$3x + 4y = 0$

$4x - 3y = 50$

Solution:

Given,

$3x + 4y = 0$ … (i)

$4x - 3y = 50$ … (ii)

On multiplying equation (i) by 3 and equation (ii) by 4, we obtain

$9x + 12y = 0$ … (iii)

$16x - 12y = 200$ … (iv)

On adding equation (iii) and (iv), we obtain

$25x = 200$

$\Rightarrow x = \frac{200}{25} = 8$

Putting the value of x in equation (i), we obtain

$3 \times 8 + 4y = 0$

$\Rightarrow 4y = -24$

$\Rightarrow y = \frac{-24}{4} = -6$

Thus, $x = 8$ and $y = -6$.

Example 3:

Solve the following equations using elimination method.

$7x + 5y = 43$

$5x + 7y = 41$

Solution:

Given,

$7x + 5y = 43$ … (i)

$5x + 7y = 41$ … (ii)

It can be observed that coefficients of x and y in equation (i) are interchanged in equation (ii).

Let us add both the equations as follows:

$$\begin{array}{r} 7x + 5y = 43 \\ +\ 5x + 7y = 41 \\ \hline 12x + 12y = 84 \end{array}$$

$\Rightarrow 12(x + y) = 84$

$\Rightarrow x + y = 7$ ...(iii)

Now, let us subtract equation (ii) from equation (i) as follows:

$$\begin{array}{r} 7x + 5y = 43 \\ 5x + 7y = 41 \\ - \quad - \qquad - \\ \hline 2x - 2y = 2 \end{array}$$

$\Rightarrow 2(x - y) = 2$

$\Rightarrow x - y = 1$ ...(iv)

On adding equations (iii) and (iv), we obtain

$$\begin{array}{r} x + y = 7 \\ +\ x - y = 1 \\ \hline 2x \qquad = 8 \end{array}$$

$\Rightarrow x = 4$

On putting the value of x in equation (i), we obtain

$7 \times 4 + 5y = 43$

$\Rightarrow 28 + 5y = 43$

$\Rightarrow 5y = 15$

$\Rightarrow y = 3$

Thus, $x = 4$ and $y = 3$.

Example 4:

The entrance ticket at a fair is Rs 50 for children and Rs 100 for adults. On a certain day, 220 people visited the fair and Rs 12000 were collected. How many children and how many adults visited the fair?

Solution:

Example 5:

Five units of a solution are obtained by mixing two liquids A and B. If the liquids A and B are used in the ratio as 2 : 3 then the total cost comes out to be Rs 50. Also, if the liquids A and B are used in the ratio as 3 : 2 then the total cost increases by 10%. Find the cost of per unit of each of the liquids A and B.

Solution:

Let x and y be costs of per unit of liquids A and B respectively.

According to the question, we have

$2x + 3y = 50$ ...(i)

And

$3x + 2y = 50 + 10\%$ of 50

$3x + 2y = 50 + 5$

$3x + 2y = 55$ ...(ii)

On multiplying equation (i) by 3 and equation (ii) by 2, we obtain

$6x + 9y = 150$ ...(iii)

$6x + 4y = 110$ ...(iv)

On subtracting equation (iv) from equation (iii), we obtain

$5y = 40$

$\Rightarrow y = 8$

On substituting $y = 8$ in equation (ii), we obtain

$3x + 2(8) = 55$

$\Rightarrow 3x + 16 = 55$

$\Rightarrow 3x = 39$

$\Rightarrow x = 13$

Hence, the cost of per unit of liquid A is Rs 13 an that of liquid B is Rs 8.

Example 6:

Some friends ate 5 pizzas and 3 burgers for which they paid Rs 445. One of them said "If the numbers of pizzas and burgers would have been exchanged, we would pay Rs 50 less." What is the cost of a pizza and that of a burger?

Solution:

Let the cost of a pizza be Rs x and that of a burger be Rs y.

$\therefore$ Cost of 5 pizzas = Rs $5x$

And, cost of 3 burgers = Rs $3y$

According to the first condition, we have

$5x + 3y = 445$ ...(i)

Similarly, for the second condition, we have

$3x + 5y = 445 - 50$

$\Rightarrow 3x + 5y = 395$ ...(ii)

On multiplying equation (i) by 3 and equation (ii) by 5, we obtain

$15x + 9y = 1335$ ...(iii)

$15x + 25y = 1975$ ...(iv)

On subtracting equation (iii) from equation (iv), we obtain

$16y = 640$

$\Rightarrow y = 40$

On substituting $y = 40$ in equation (ii), we obtain

$3x + 5(40) = 395$

$\Rightarrow 3x + 200 = 395$

$\Rightarrow 3x = 195$

$\Rightarrow x = 65$

Thus, cost of a pizza is Rs 65 and that of a burger is Rs 40.

Cross-Multiplication Method Of Solving Pairs Of Linear Equations

We can represent many situations in real life as linear equation in two variables. Let us consider such a situation.

Suppose Samay is older than Sumit by 30 years. After 5 years, Samay will be thrice as old as Sumit.

Can we find the present ages of Samay and Sumit?

Yes, we can find the present ages. However, before that, we have to represent this situation in the form of linear equations in two variables. Let us see how we can represent the above given situations as linear equations.

Let the present age of Samay be x years and the present age of Sumit be y years.

It is given that Samay is older than Sumit by 30 years. Thus, according to this condition, we have

$x - y = 30$

After 5 years, the age of Samay will be$(x + 5)$ years and the age of Sumit will be $(y + 5)$ years. However, it is given that after 5 years, Samay will be thrice as old as Sumit. Thus, we have

$x + 5 = 3(y + 5)$

$\Rightarrow x + 5 = 3y + 15$

$\Rightarrow x - 3y = 10$

Now, we obtain the pair of linear equations as follows:

$x - y = 30$ … (1)

$x - 3y = 10$ … (2)

We can find the present ages of Samay and Sumit by solving the above equations for variables x and y.

We know three methods to solve a pair of linear equations.

(i) Substitution method

(ii) Elimination method

(iii) Graphical method

Now, we will follow another method to solve linear equations in two variables. This method is known as **Cross-multiplication method.** Firstly, let us discuss this method and after that we will solve the above equations for x and y. So, go through the following video to understand the cross-multiplication method and how it is applied to solve the above given equations.

Therefore, the present ages of Samay and Sumit are 40 years and 10 years respectively.

In this way, we can solve a pair of linear equations in two variables by cross-multiplication method.

We could solve these linear equations in two variables by other methods also. But why should we use cross-multiplication method to solve linear equations?

The cross-multiplication method is easier as compared to the other methods if the coefficients of variables in the equations are large numbers or numbers that look complex. To understand this concept better, let us see the following examples.

Example 1:

A company placed two orders from two different shops. The first order was for 13 desktops and 4 laptops and the total cost came out to be Rs 487000. The second order was for 6 desktops and 2 laptops and the total cost came out to be Rs 232000. What is the cost of one desktop and of one laptop?

Solution:

Go through this video to look at the solution of the question.

Example 2:

Solve the following pair of linear equations in two variables by cross-multiplication method.

$\boldsymbol{ax + by = a^2 + b^2}$ **and**

$\boldsymbol{x + y = 2a}$

Solution:

The above two equations can be written as

$ax + by - (a^2 + b^2) = 0$ … (1)

$x + y - 2a = 0$ … (2)

From the above two equations, we obtain

$a_1 = a$, $b_1 = b$, $c_1 = -(a^2 + b^2)$ and

$a_2 = 1$, $b_2 = 1$, $c_2 = -2a$

Now, $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \dfrac{x}{b\times(-2a) - 1\times\left[-\left(a^2+b^2\right)\right]} = \dfrac{y}{\left[-\left(a^2+b^2\right)\right]\times 1 - (-2a)\times a} = \dfrac{1}{a\times 1 - 1\times b}$

$\Rightarrow \dfrac{x}{-2ab + a^2 + b^2} = \dfrac{y}{-a^2 - b^2 + 2a^2} = \dfrac{1}{a-b}$

$\Rightarrow \dfrac{x}{(a-b)^2} = \dfrac{y}{a^2 - b^2} = \dfrac{1}{a-b}$

Therefore, $\dfrac{x}{(a-b)^2} = \dfrac{1}{a-b}$ and $\dfrac{y}{a^2-b^2} = \dfrac{1}{a-b}$

$\Rightarrow x = \dfrac{(a-b)^2}{a-b}$ and $y = \dfrac{a^2-b^2}{a-b}$

$\Rightarrow x = a - b$ and $y = \dfrac{(a-b)(a+b)}{a-b}$

$\Rightarrow x = a - b$ and $y = a + b$

Equations Reducible To A Pair Of Linear Equations In Two Variables

Consider the following equations.

$x = y$; $\frac{5}{7}x = 21y + 15$; $m + 20n = -13$; etc.

We can see that the above equations are linear equations in two variables. However, there are many equations, which do not appear to be linear equations at first glance such as

$\dfrac{2}{x} + \dfrac{3}{y} = 20$; $\dfrac{1}{x} - \dfrac{1}{y} = 5$; etc.

Now, can we solve these equations?

Let us look at the given video to understand the method involved in solving such non-linear equations.

Now, let us look at an example in the given video to understand the concept.

Go through the following examples to get a better idea of the concept.

Example1:

Solve the following pair of linear equations.

$$\frac{5}{x}+\frac{1}{y}=7$$

$$\frac{2}{x}-\frac{3}{y}=-4$$

Solution:

$$\frac{5}{x}+\frac{1}{y}=7 \qquad ...(1)$$

$$\frac{2}{x}-\frac{3}{y}=-4 \qquad ...(2)$$

From equation (1), we can write,

$$5\left(\frac{1}{x}\right)+\frac{1}{y}=7 \qquad ...(3)$$

From equation (2), we can write,

$$2\left(\frac{1}{x}\right)-3\left(\frac{1}{y}\right)=-4 \qquad ...(4)$$

Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$

Now, equation (3) becomes

$5a + b = 7$ … (5)

and equation (4) becomes

$2a – 3b = – 4$ … (6)

On multiplying (5) by 3, we obtain

$15a + 3b = 21$ … (7)

On adding equations (6) and (7), we obtain

$17a = 17$

$\Rightarrow a = 1$

On putting the value of *a* in equation (5), we obtain

$5 \times 1 + b = 7$

$\Rightarrow 5 + b = 7$

$\Rightarrow b = 7 - 5 = 2$

Since $\frac{1}{x} = a = 1$

$\Rightarrow x = 1$

and $\frac{1}{y} = b = 2$

$\Rightarrow y = \frac{1}{2}$

Thus, $x = 1$ and $y = \frac{1}{2}$

Example2:

Solve the following pair of linear equations.

$$\frac{2}{x-y} + \frac{5}{x+y} = 7$$
$$\frac{3}{x-y} + \frac{10}{x+y} = 9$$

Solution:

The given equations can be written as

$$2\left(\frac{1}{x-y}\right)+5\left(\frac{1}{x+y}\right)=7 \qquad ...(1)$$

and

$$3\left(\frac{1}{x-y}\right)+10\left(\frac{1}{x+y}\right)=9 \qquad ...(2)$$

Let $\frac{1}{x-y}=a$ and $\frac{1}{x+y}=b$

Thus, the equations become

$2a + 5b = 7$ … (3)

And, $3a + 10b = 9$ … (4)

Now, on multiplying equation (3) by 2, we obtain

$4a + 10b = 14$ … (5)

On subtracting equation (4) from equation (5), we obtain

$a = 14 – 9$

$\Rightarrow a = 5$

Putting the value of a in equation (3), we obtain

$10 + 5b = 7$

$\Rightarrow 5b = 7 – 10$

$\Rightarrow b = \frac{-3}{5}$

Now, $\frac{1}{x-y}=a$ and $\frac{1}{x+y}=b$

$\Rightarrow x-y=\frac{1}{5}$ and $x+y=-\frac{5}{3}$

On adding these two equations, we obtain

$2x = \frac{-22}{15}$

$\Rightarrow x = -\frac{11}{15}$

and, $y = x - \frac{1}{5}$

$\Rightarrow y = -\frac{11}{15} - \frac{1}{5}$

$\Rightarrow y = \frac{-11-3}{15}$

$\Rightarrow y = -\frac{14}{15}$

Thus, $x = -\frac{11}{15}$ and $y = -\frac{14}{15}$