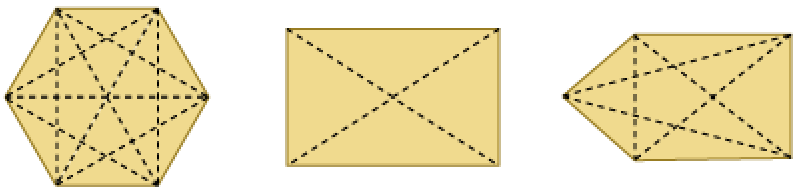
Rectilinear Figures

Classification of Polygons as Convex and Concave

Let us consider some polygons such as a hexagon, a quadrilateral, and a pentagon as shown in the following figure.



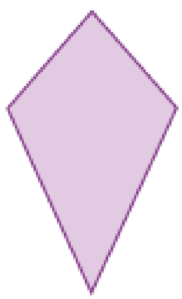
These polygons are known as convex polygons. How can we say these are convex polygons?

Let us now look at some examples.

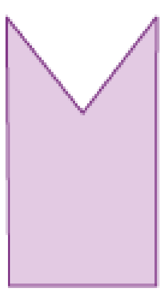
Example:

Classify the following figures as concave or convex polygons.

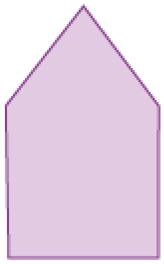
1.



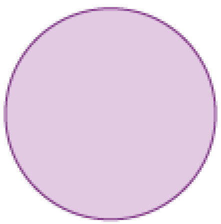
2.



3.



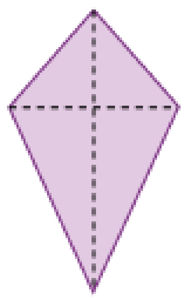
4.



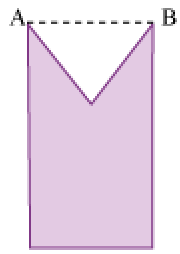
5.

Solution:

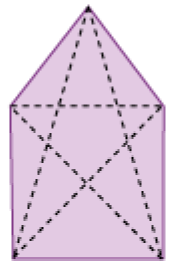
1. In this figure, we can clearly see that all the diagonals of the polygon lie inside the polygon. Therefore, it is a convex polygon.



2. In this polygon, diagonal AB lies in the exterior of the polygon. Therefore, it is a concave polygon.

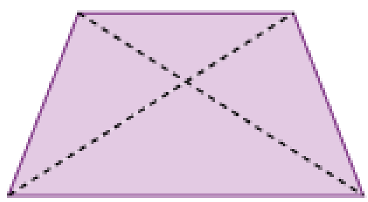


3. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



4. The given figure is a curve. It is not made up of line segments. Therefore, it is not a polygon.

5. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



Angle Sum Property Of Polygons

Let us suppose that we have a quadrilateral and we want to find the sum of all the interior angles made by its sides.

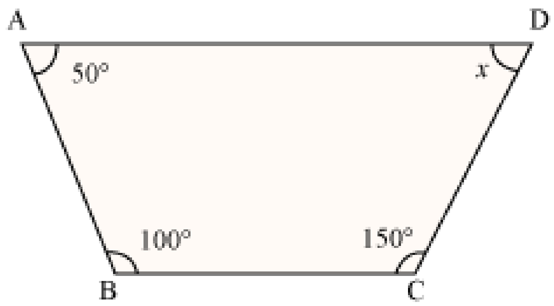
One simple way to find the sum of the angles is to find the measure of the angles and then add them. But how will we find its angles?

Is it possible to find the sum of all the angles of a quadrilateral without finding the measure of each angle? Is the sum of the interior angles of every quadrilateral same?

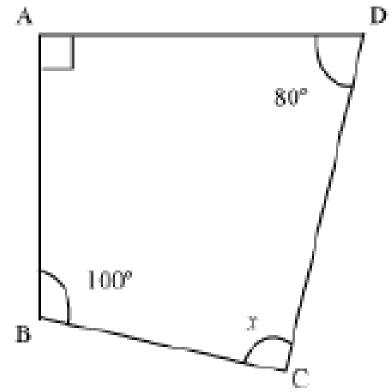
Let us solve some examples now.

Example:

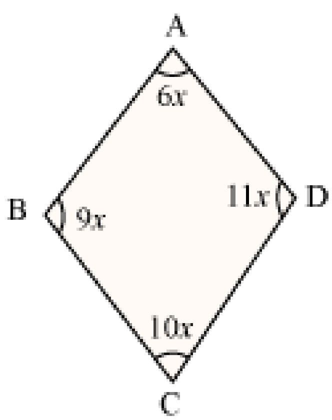
Find the value of *x* in the following figures.


A
D
50°
x
100°
150°
B
C

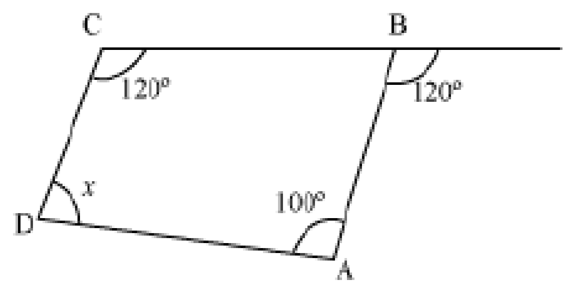
(a)


A
D
80°
100°
x
B
C

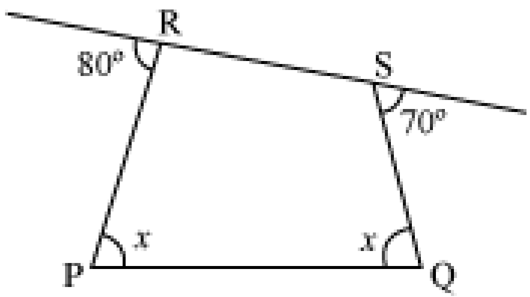
(b)


A
6x
B
9x
11x
D
10x
C

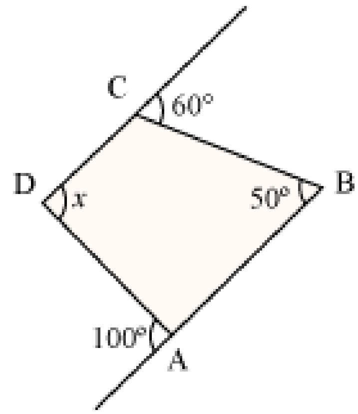
(c)


C
B
120°
120°
x
100°
D
A

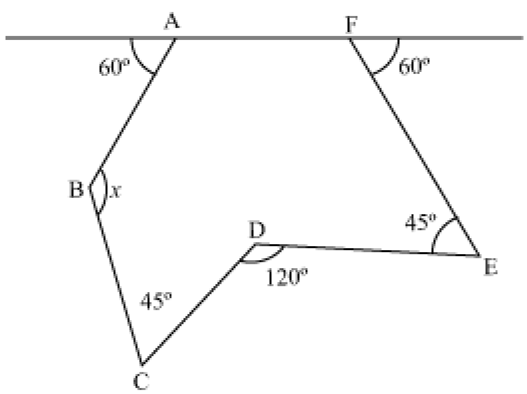
(d)



(e)



(f)



(g)

Solution:

(a) The sum of all the interior angles of a quadrilateral is 360°.

Therefore, from the figure,

100° + 150° + x + 50° = 360°

⇒ 300° + x = 360°

$\Rightarrow x = 360° - 300°$

$\Rightarrow x = 60°$

(b) The sum of all the interior angles of a quadrilateral is 360°.

Therefore,

$90° + 80° + x + 100° = 360°$

$\Rightarrow 270° + x = 360°$

$\Rightarrow x = 360° - 270°$

$\Rightarrow x = 90°$

(c) The sum of all the interior angles of a quadrilateral is 360°. Therefore, from the figure,

$9x + 6x + 11x + 10x = 360°$

$\Rightarrow 36x = 360°$

On dividing both sides by 36, we obtain $x = 10°$

Thus, the angles of the quadrilateral are 90°, 60°, 110°, and 100°.

(d) The sum of the angles which forms a linear pair is 180°.

$\therefore \angle ABC + 120° = 180°$

$\Rightarrow \angle ABC = 180° - 120° = 60°$

Also, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 100° + 60° + 120° = 360°$

$\Rightarrow x + 280° = 360°$

$\Rightarrow x = 360° - 280°$

$\Rightarrow x = 80°$

(e) The sum of the adjacent angles on a straight line is 180°.

$\therefore \angle PSR + 80° = 180°$

$\Rightarrow \angle PSR = 180° - 80°$

$\Rightarrow \angle PSR = 100°$

Also, $\angle SRQ + 70° = 180°$

$\Rightarrow \angle SRQ = 180° - 70°$

$\Rightarrow \angle SRQ = 110°$

The sum of all the interior angles of a quadrilateral is 360°. Therefore,

$100° + 110° + x + x = 360°$

$\Rightarrow 210° + 2x = 360°$

$\Rightarrow 2x = 360° - 210°$

$\Rightarrow 2x = 150°$

$\Rightarrow x = 75°$

(f) The sum of the angles which forms a linear pair is 180°.

$\therefore 100° + \angle DAB = 180°$

$\Rightarrow \angle DAB = 180° - 100°$

$\Rightarrow \angle DAB = 80°$

Similarly, $\angle DCB + 60° = 180°$

$\Rightarrow \angle DCB = 180° - 60°$

$\Rightarrow \angle DCB = 120°$

Now, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 80° + 50° + 120° = 360°$

$\Rightarrow x + 250° = 360°$

⇒ $x = 360° − 250°$

⇒ $x = 110°$

(g) ∠BAF = 180° − 60° = 120°

Similarly, ∠AFE = 120°

∠CDE = 360° − 120° = 240°

The polygon ABCDEF is a hexagon.

∴ Sum of all the interior angles of a hexagon = 180° × (6 − 2)

= 180° × 4

= 720°

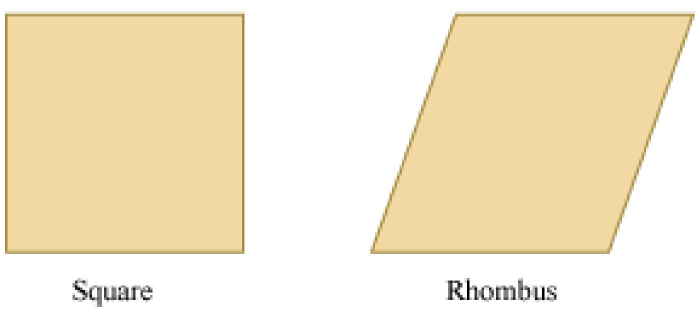
∴ $x = 720° − (120° + 120° + 45° + 240° + 45°)$

⇒ $x = 720° − 570°$

⇒ $x = 150°$

Classification of Polygons as Regular and Irregular

Let us consider a square and a rhombus.



What is the difference between the two figures?

We can see that in a square, all the sides are equal and all the angles are also of equal measure. On the other hand, in a rhombus, all sides are equal; however, the measures of all angles are not equal.

We thus say that a **square is a regular polygon** and a **rhombus is an irregular polygon**.

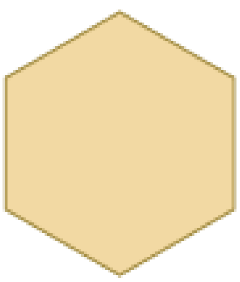
The **regular** and **irregular polygons** can be defined as follows.

"Polygons in which all sides are of equal length and all interior angles of equal measure are known as regular polygons".
"Polygons in which all sides are not of equal length and all angles are not of equal measure are known as irregular polygons".

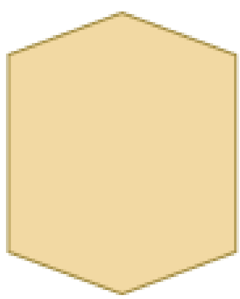
Let us see another example.

A **regular hexagon** has all sides of equal length. Moreover, all the angles are of equal measure 120°.

However, in case of an **irregular hexagon**, all the sides are not of equal length. Also, all the angles are not equal. A regular and an irregular hexagon are shown in the following figure.



Regular hexagon



Irregular hexagon

Formulas Related to Regular Polygons:

(i) The sum of the interior angles of an n sided polygon = $(2n-4) \times 90^\circ$

where each interior angle = $\frac{(2n-4)\times 90^\circ}{n}$

(ii) A regular polygon has all its exterior angles equal.

The sum of its exterior angles = 360°
So, the sum of each exterior angle = $\frac{360^\circ}{n}$
(iii) Number of sides of a regular polygon, $n = \frac{360^\circ}{\text{exterior angle}}$

Note: For a polygon, regardless of the fact whether it is regular or non-regular, at each vertex the sum of exterior and interior angle = 180°180°

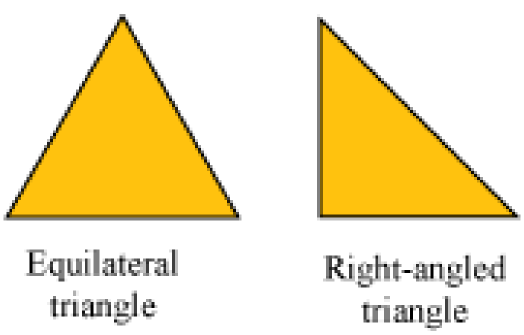
i.e Exterior angle + Interior angle = 180°

Let us now look at some more examples to understand this concept better.

Example 1:

Show that an equilateral triangle is a regular polygon and a right-angled triangle is an irregular polygon.

Solution:



An equilateral triangle is a regular polygon as all the sides of equilateral triangle are of equal length and all angles are of equal measure 60°.

In case of a right-angled triangle, neither all the sides are of equal length nor the measure of all angles are equal. Therefore, right-angled triangle is an example of irregular polygon.

Example 2:

Write the name of a regular polygon having

(i) 3 sides

(ii) 4 sides

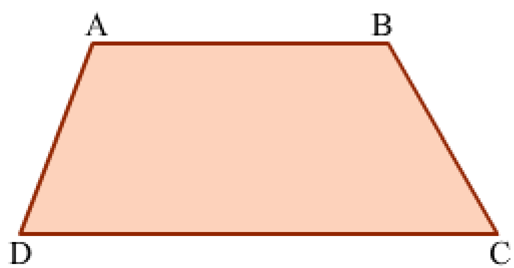
Solution:

A regular polygon is a polygon in which all the sides are of equal length and all interior angles are of equal measure.

Therefore, a regular polygon having 3 sides is an equilateral triangle. A regular polygon having 4 sides is a square.

Properties of a Trapezium

Have a look at the following quadrilateral.



It can be observed that AB || DC.

We know that, a quadrilateral with one pair of parallel sides is known as a trapezium.

Thus, quadrilateral ABCD is a trapezium.

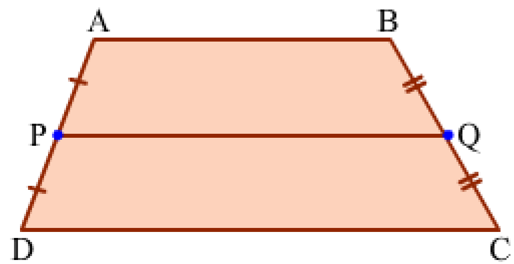
In this lesson, we will discuss some important properties of a trapezium. One such property is stated below.

If the mid-points of the non-parallel sides of a trapezium are joined together, then the obtained line segment is

1. parallel to the parallel sides.

2. half the sum of the lengths of the parallel sides.

Consider a trapezium ABCD with P and Q as the mid-points of sides AD and BC respectively.

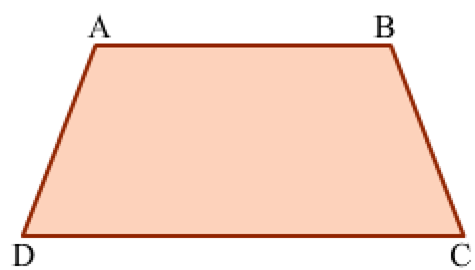


According to the above stated property:

(1) PQ || AB || DC

(2) $PQ = \frac{1}{2}(AB + CD)$

Now, look at the following figure.



Here, AB || DC and AD ≅ BC.

A trapezium whose non-parallel sides are congruent are called **isosceles trapezium**.

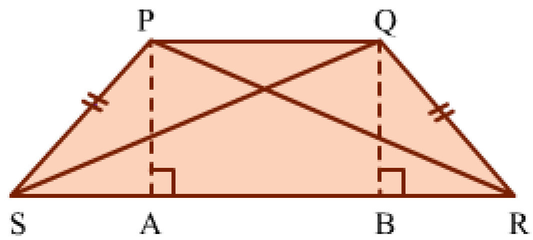
Let us now discuss the properties of isosceles trapeziums.

Diagonals of an isosceles trapezium are congruent (equal).

To prove it, let us consider an isosceles trapezium PQRS such that PQ || SR and PS ≅ QR.

PR and QS are its diagonals.

Construction: Draw two perpendiculars, PA and QB, on the side SR.



In ΔPSA and ΔQRB,
PS ≅ QR (Given)
∠ PAS ≅ ∠QBR= 90° (By construction)
PA ≅ QB (Perpendicular distance between two parallel lines)

∴ ΔPSA ≅ ΔQRB(By RHS congruence criterion)

⇒ ∠PSA ≅ ∠QRB(By c.p.c.t)
⇒∠PSR ≅∠QRS

In ΔPSR and ΔQRS,
PS ≅ QR (Given)
∠PSR ≅ ∠QRS (Proved above)
SR ≅ RS (Common)

∴ ΔPSR ≅ ΔQRS (by SAS congruence criterion)

⇒ PR ≅ QS (by c.p.c.t)
Hence, the diagonals of an isosceles trapezium are always congruent.

Let us try to some examples based on these properties.

Example 1:

ABCD is a trapezium in which AB || DC. P and Q are the mid-points of sides AD and BC respectively. If AB is 9 cm and DC is 11 cm, then find the length of PQ. Also, the length of perpendicular drawn from point A to side DC is 8 cm and it intersects PQ at point N. Find the length of AN.

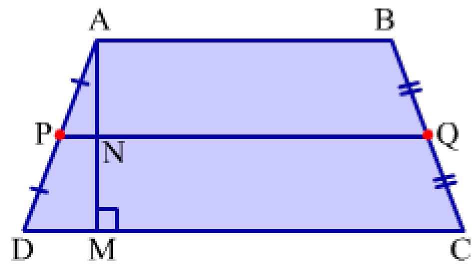
Solution:

Let ABCD be the given trapezium and the perpendicular drawn from point A to DC intersect DC at M.

It is given that AB = 9 cm, DC = 11 cm.

Also, P and Q are mid-points of sides AD and BC.

⇒ AP = PD and BQ = QC



We know that if the mid-points of the non-parallel sides of a trapezium are joined together, then the obtained line segment is parallel to the parallel sides and is equal to half the sum of the lengths of its parallel sides.

$$\begin{aligned}\therefore PQ &= \frac{1}{2}(AB+CD)\\ &= \frac{1}{2}(9\,cm+11\,cm)\\ &= \frac{1}{2}(20\,cm)\\ &= 10\,cm\end{aligned}$$

We know that three parallel lines make congruent intercepts on all transversals.

So, the parallel line segments AB, PQ and CD will make congruent intercepts on the transversals AD, AM and BC.

⇒ AN = NM

Now, AN + NM = 8 cm

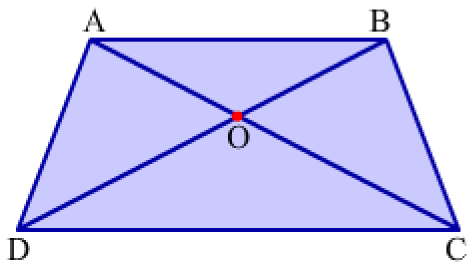
⇒ AN + AN = 8cm

⇒ 2AN = 8 cm

⇒ AN = 4 cm

Example 2:

If ABCD is an isosceles trapezium, then prove that ar (ΔAOD) ≅ ar (ΔBOC).



Solution:

We know that the non-parallel sides and diagonals of an isosceles trapezium are always congruent.

∴ AD ≅ BC and AC ≅ BD

In ΔADC and ΔBCD:

AD ≅ BC (Given)

AC ≅ BD (Given)

DC ≅ DC (Common side)

∴ ΔADC ≅ ΔBCD (By SSS congruence criterion)

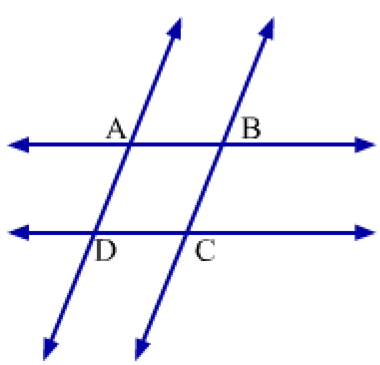
⇒ ar (ΔADC) ≅ ar (ΔBCD)

⇒ ar (ΔODC) + ar (ΔAOD) ≅ ar (ΔODC) + ar (ΔBOC)

⇒ ar (ΔAOD) ≅ ar (ΔBOC)

Property of the Sides of a Parallelogram

Consider the given pairs of parallel lines.



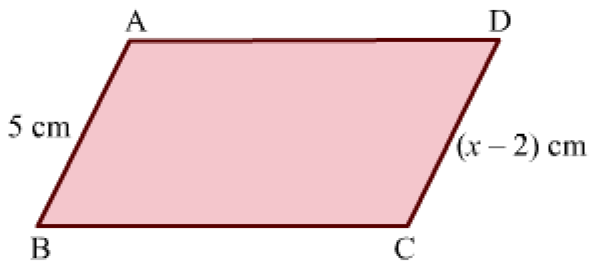
A closed figure ABCD is formed by the intersection of the two pairs of parallel lines. This figure is a parallelogram. A property of parallelograms defines the relation between the sides of a parallelogram as follows:

Opposite sides of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples related to the same.

Opposite Sides of a Parallelogram Are Equal

Consider the given parallelogram ABCD.



We can find the value of x by using the property of parallelograms which states that:

Opposite sides of a parallelogram are equal.

Thus, in the given figure, we have AB = DC and AD = BC.

Since AB = DC, we have:

$x - 2 = 5$

$\Rightarrow x = 7$

Proof of the Property

Concept Builder

- A quadrilateral is a polygon having four sides.
- The sum of the interior angles of a quadrilateral is 360°.

Converse of the Property

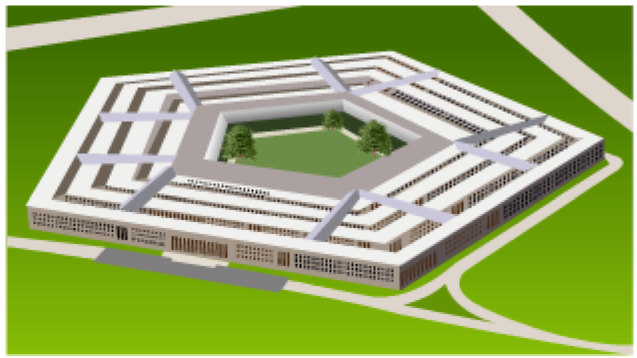
Know More

- A pentadecagon is a fifteen-sided polygon. The sum of its interior angles is 2340°.

- An icosagon is a twenty-sided polygon. The sum of its interior angles is 3240°.

Did You Know?

The headquarters of the US Department of Defense is called 'the Pentagon'. It is one of the world's largest office buildings. It is virtually a city in itself.



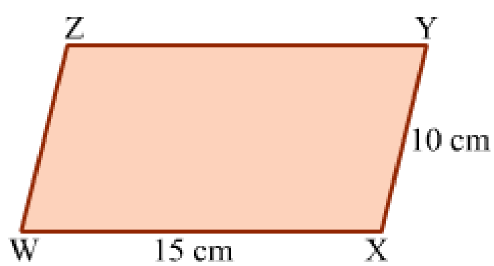
Solved Examples

Easy

Example 1:

What is the perimeter of the given parallelogram WXYZ if WX = 15 cm and XY = 10 cm?

Solution:



We know that the opposite sides of a parallelogram are equal.

∴ WX = ZY = 15 cm and XY = WZ = 10 cm

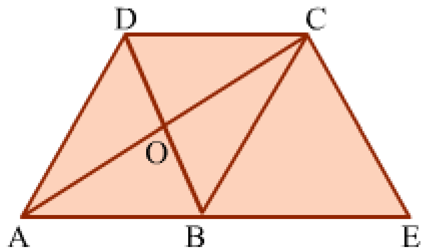
Perimeter of parallelogram WXYZ = WX + XY + YZ + ZW
= (15 + 10 + 15 + 10) cm

= 50 cm

Medium

Example 1:

In the given figure, ABCD is a parallelogram and B is the midpoint of AE. If DB = CE, then prove that BECD is also a parallelogram.



Solution:

We know that the opposite sides of a parallelogram are equal.

∴ AB = DC … (1)

It is given that B is the midpoint of AE.

∴ AB = BE … (2)

From equations 1 and 2, we get:

DC = BE

Also, it is given that DB = CE.

Now, in quadrilateral BECD, the opposite sides are equal (i.e., DC = BE and DB = CE). Therefore, it is a parallelogram.

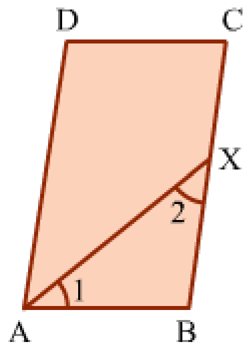
Hard

Example 1:

In a parallelogram ABCD, the bisector of ∠BAD also bisects side BC. Prove that the length of side AD is twice the length of side AB.

Solution:

The parallelogram ABCD according to the given specifications is shown below.



Here, AX is the bisector of ∠BAD.

$\therefore \angle 1 = \frac{1}{2}\angle BAD \qquad \ldots(1)$

Since ABCD is a parallelogram, AD||BC and AB is the transversal between these lines.

∴ ∠BAD + ∠CBA = 180° … (2)

In ΔABX, by the angle sum property of triangles, we have:

∠1 + ∠2 + ∠ABX = 180°

$\Rightarrow \frac{1}{2}\angle BAD + \angle 2 + 180° - \angle BAD = 180° \qquad (\text{Using equations 1 and 2})$

$\Rightarrow \angle 2 + \frac{1}{2}\angle BAD = 0$

$\Rightarrow \angle 2 = \frac{1}{2}\angle BAD$

$\Rightarrow \angle 2 = \angle 1$

We know that the sides opposite equal angles are also equal.

∴ AB = BX … (3)

Since ABCD is a parallelogram, AD = BC.

Now, BC = BX + XC

⇒ AD = BX + XC

⇒ AD = 2BX(∵ AX bisects BC)

⇒ ∴ AD = 2AB (Using equation 3)

Thus, in parallelogram ABCD, the length of side AD is twice the length of side AB.

Properties of The Angles of a Parallelogram

Opposite Angles of a Parallelogram

Look at the postage stamp shown below.

Observe how the stamp is shaped like a parallelogram. What can you say about its opposite angles? Is there any relation between them? Are they equal?

A property of parallelograms relates the opposite angles of a parallelogram as follows:

Opposite angles of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples based on the same.

Proof of the Property

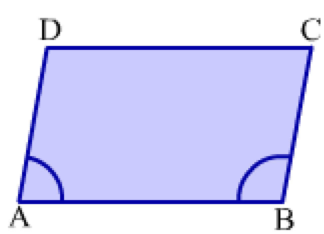
Whiz Kid

The sum of the measures of all the exterior angles of a quadrilateral (i.e., one at each vertex) is equal to the sum of the measures of all the interior angles of the quadrilateral, i.e., 360°.

Concept Builder

Adjacent angles in a parallelogram are supplementary.

In parallelogram ABCD, AD||BC and AB is the transversal intersecting these lines.



Therefore, ∠A and ∠B are **interior angles** on the same side of the transversal and, hence, **supplementary**.

Similarly, we can say that ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A are supplementary angles.

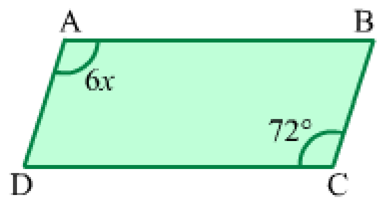
Converse of the Property

Solved Examples

Easy

Example 1:

Find the value of *x* if ABCD is a parallelogram.



Solution:

We know that the opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

⇒ $6x = 72°$

∴ $x = 12°$

Example 2:
Find the measure of all the angles of a parallelogram whose adjacent angles are in the ratio 1:2.

Solution:
In a parallelogram ABCD, let $\angle A = x^\circ$ and $\angle B = 2x^\circ$.
In a parallelogram, the adjacent angles are supplementary.

$$\angle A + \angle B = 180°$$
$$\Rightarrow x° + 2x° = 180°$$
$$\Rightarrow 3x° = 180$$
$$\Rightarrow x° = \frac{180°}{3}$$
$$\Rightarrow x° = 60°$$

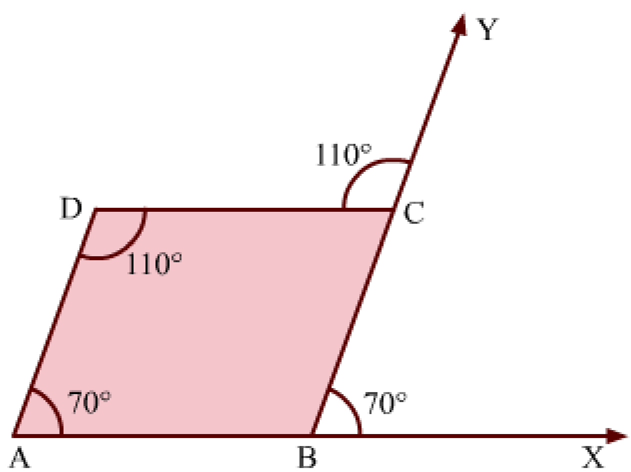
Thus, we get
$\angle A = x°$ = 60°
$\angle B = 2x°$ = 2 × 60° = 120°

In a parallelogram, the opposite angles are equal.
Thus, we get
∠A = ∠C = 60° and ∠B = ∠D = 120°

Medium

Example 1:

Is the shown quadrilateral ABCD a parallelogram?



Solution:

In the given figure, ∠CBX and ∠CBA form a linear pair.

∴ ∠CBX + ∠CBA = 180°

⇒ ∠CBA = 180° − ∠CBX

⇒ ∠CBA = 180° − 70°

⇒ ∠CBA = 110°

⇒ ∠CBA = ∠CDA

Similarly, ∠DCY and ∠BCD form a linear pair.

∴ ∠DCY + ∠BCD = 180°

⇒ ∠BCD = 180° − ∠DCY

⇒ ∠BCD = 180° − 110°

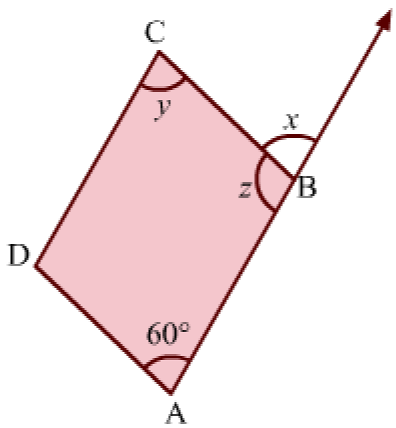
⇒ ∠BCD = 70°

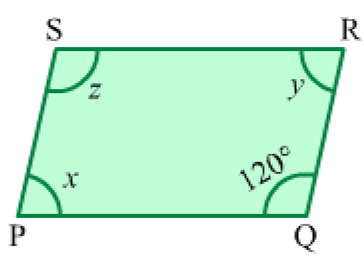
⇒ ∠BCD = ∠BAD

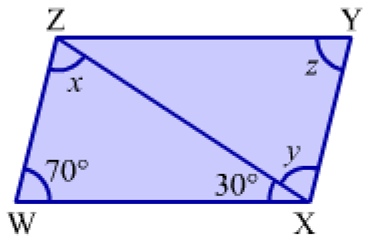
Thus, quadrilateral ABCD has two pairs of equal opposite angles. Hence, it is a parallelogram.

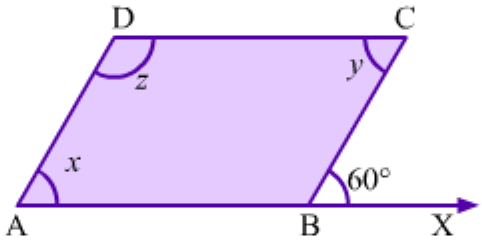
Example 2:

Find the values of *x*, *y* and *z* in the following parallelograms.

1. 

2. 

3. 

4. 

Solution:

1. We know that the opposite angles of a parallelogram are equal.

   So, ∠BCD = ∠DAB

   ∴ $y = 60°$

   We also know that the adjacent angles of a parallelogram are supplementary.

   So, ∠CBA + ∠DAB = 180°

   ⇒ $z + 60° = 180°$

   ⇒ $z = 180° - 60°$

   ⇒ ∴ $z = 120°$

   Now, x and z form a linear pair of angles; so, their sum is 180°.

   So, $x + z = 180°$

   ⇒ $x + 120° = 180°$

   ⇒ ∴ $x = 180° - 120° = 60°$

2. We know that the opposite angles of a parallelogram are equal.

   So, ∠PSR = ∠PQR

$\therefore z = 120°$

∠QPS and ∠PQR are adjacent angles.

So, ∠QPS + ∠PQR = 180°

$\Rightarrow x + 120° = 180°$

$\Rightarrow x = 180° - 120°$

$\Rightarrow \therefore x = 60°$

∠QRS and ∠QPS are opposite angles.

So, ∠QRS = ∠QPS

$\Rightarrow y = x$

$\Rightarrow \therefore y = 60°$

3. We know that the opposite angles of a parallelogram are equal.

So, ∠XYZ = ∠XWZ

$\therefore z = 70°$

∠XYZ and ∠WXY are adjacent angles.

∴ ∠XYZ + ∠WXY = 180°

$\Rightarrow z + 30° + y = 180°$

$\Rightarrow 70° + 30° + y = 180°$

$\Rightarrow 100° + y = 180°$

$\Rightarrow y = 180° - 100°$

$\Rightarrow \therefore y = 80°$

Now, XY||WZ; so, ∠WZX and ∠YXZ are alternate interior angles.

So, ∠WZX = ∠YXZ

$\Rightarrow x = y$

$\Rightarrow \therefore x = 80°$

4. It is given that ∠CBX = 60°.

   ∠CBA and ∠CBX form a linear pair.

   So, ∠CBA + ∠CBX = 180°

   $\Rightarrow$ ∠CBA + 60° = 180°

   $\Rightarrow$ ∠CBA = 180° − 60°

   $\Rightarrow \therefore$ ∠CBA = 120°

   ∠CDA and ∠CBA are opposite angles.

   So, ∠CDA = ∠CBA

   $\Rightarrow z$ = ∠CBA

   $\Rightarrow \therefore z = 120°$

   ∠BCD and ∠CBA are adjacent angles.

   So, ∠BCD + ∠CBA = 180°

   $\Rightarrow y + 120° = 180°$

   $\Rightarrow y = 180° - 120°$

   $\Rightarrow \therefore y = 60°$

   ∠BAD and ∠BCD are opposite angles.

   So, ∠BAD = ∠BCD

   $\Rightarrow x = y$

   $\Rightarrow \therefore x = 60°$

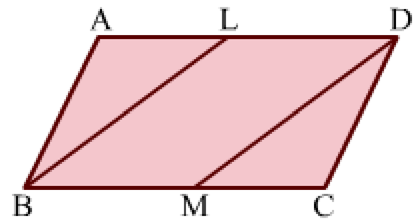
Hard

Example 1:

Show that the bisectors of opposite angles of a parallelogram are parallel to each other.

Solution:

Let ABCD be a parallelogram. Let BL and DM be the bisectors of ∠ABC and ∠ADC respectively.



Since BL and DM are the bisectors of ∠ABC and ∠ADC respectively, we have:

$$\angle LBM = \frac{\angle ABC}{2} \qquad ...(1)$$

$$\angle LDM = \frac{\angle ADC}{2} \qquad ...(2)$$

We know that the opposite angles of a parallelogram are equal.

∴ ∠ABC = ∠ADC

On dividing both sides of the above equation by 2, we obtain:

$$\frac{\angle ABC}{2} = \frac{\angle ADC}{2}$$

Using equations 1 and 2, we obtain:

∠LBM = ∠LDM

Now, LD and BM are parallel.

So, ∠DLB + ∠LBM = 180° (Interior angles on the same side of a transversal)

⇒ ∠DLB = 180° − ∠LBM

Similarly, ∠DMB = 180° − ∠LDM

∴ ∠DLB = ∠DMB (∵ ∠LBM = ∠LDM)

In quadrilateral LDMB, the opposite angles ∠DLB and ∠DMB are equal. Hence, it is a parallelogram.

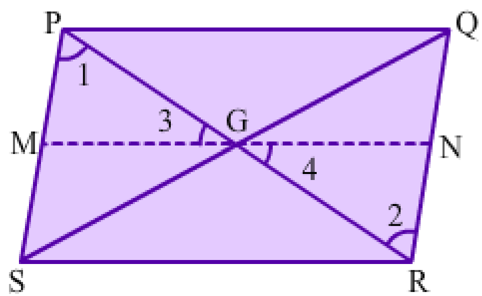
⇒ BL||DM

We know that BL and DM are the bisectors of opposite angles of parallelogram ABCD. Thus, the bisectors of opposite angles of a parallelogram are parallel.

Properties of The Diagonals of a Parallelogram

Relation between the Diagonals of a Parallelogram

Consider the following parallelogram PQRS.



In the figure, GM = GN, but can we prove this?

In order to prove GM = GN, we need to show that ΔGMP is congruent to ΔGNR.

In ΔGMP and ΔGNR, we have two sets of equal angles as follows:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

Now, to apply the ASA congruence rule, we need to show that GP and GR are equal.

A property of parallelograms helps us establish this equality and it can be stated as follows:

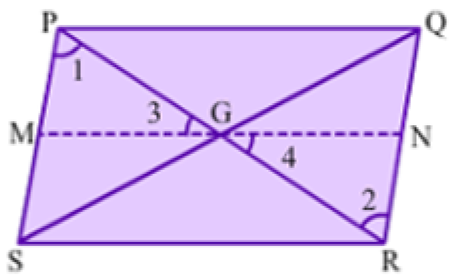
The diagonals of a parallelogram bisect each other.

In this lesson, we will study the above-stated property and solve some problems based on it.

Property of the Diagonals of a Parallelogram

Using the Property

Let us use the property of the diagonals of a parallelogram to solve the problem discussed at the beginning.



Let us once again consider parallelogram PQRS.

We have to prove that GM = GN.

Since diagonals PR and QS bisect each other, we obtain:

GP = GR and GS = GQ… (1)

In ΔGMP and ΔGNR, we have:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

GP = GR(Using 1)

Thus, by the ASA congruence rule, we obtain:

ΔGMP ≅ ΔGNR

⇒ GM = GN(By CPCT)

Similarly, we can use the property of the diagonals of a parallelogram to solve other problems.

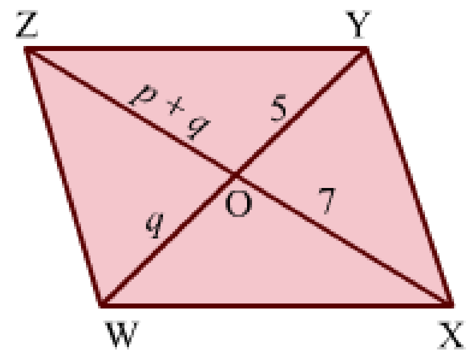
Converse of the Property

Solved Examples

Easy

Example 1:

If the shown quadrilateral WXYZ is a parallelogram, then find the values of *p* and *q*.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ WO = OY

⇒ $q = 5$

Similarly, XO = OZ

∴ $p + q = 7$

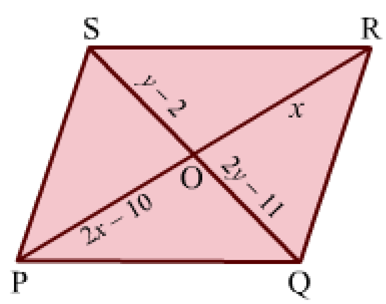
⇒ $p = 7 - q$

⇒ $p = 7 - 5$

⇒ ∴ $p = 2$

Example 2:

In the given parallelogram PQRS, find the lengths of the diagonals PR and QS.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ PO = OR

⇒ $2x - 10 = x$

⇒ $2x - x = 10$

⇒ ∴ $x = 10$

Similarly, QO = OS

⇒ $y - 2 = 2y - 11$

⇒ $2y - y = -2 + 11$

⇒ ∴ $y = 9$

Now, PR = PO + OR

$= 2x - 10 + x$

$= 3x - 10$

$= 3 \times 10 - 10$

$= 30 - 10$

$= 20$

Similarly, QS = QO + OS

$= y - 2 + 2y - 11$

$= 3y - 13$

$= 3 \times 9 - 13$

$= 27 - 13$

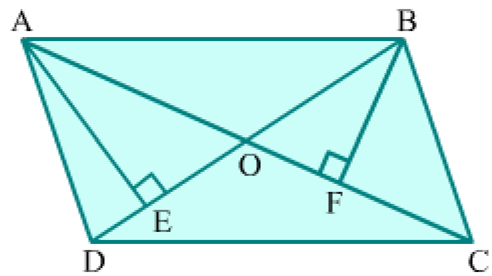
$= 14$

Thus, the lengths of the diagonals PR and QS are 20 units and 14 units respectively.

Medium

Example 1:

ABCD is a parallelogram with diagonals AC and BD of lengths 10 cm and 8 cm respectively. If the perpendiculars on DO and OC are 5 cm each, then find the sum of the areas of ΔAOD and ΔBOC.



Solution:

We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, we have:

AO = OC = $\frac{AC}{2}$ and BO = OD = $\frac{BD}{2}$

⇒ AO = OC = $\frac{10}{2}$ cm = 5 cm and BO = OD = $\frac{8}{2}$ cm = 4 cm

Now, area of ΔAOD= $\frac{1}{2}$ × Base × Height

= $\frac{1}{2}$ × OD × AE

= $\frac{1}{2}$ × 4 × 5 cm^2

= 10 cm^2

Similarly, area of ΔBOC= 1/2 × OC × BF

$= \frac{1}{2} \times 5 \times 5$ cm^2

= 12.5 cm^2

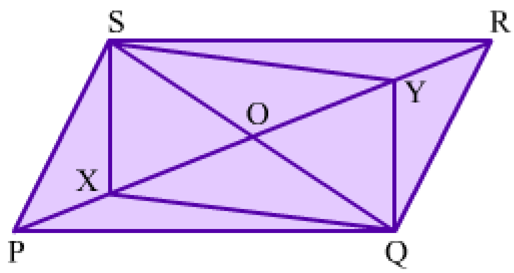
Therefore, sum of the areas of ΔAOD and ΔBOC = (10 + 12.5) cm^2 = 22.5 cm^2

Hard

Example 1:

In parallelogram PQRS, X and Y are points on PR such that PX = YR. Prove that:

1. **XQYS is a parallelogram**
2. **ΔSXP ≅ ΔQYR**



Solution:

1. We know that the diagonals of a parallelogram bisect each other.

∴ OS = OQ … (1)

And OP = OR… (2)

Also, PX = YR… (3) [Given]

On subtracting equation 3 from equation 2, we obtain:

OP − PX = OR − YR

⇒ OX = OY … (4)

In quadrilateral XQYS, XY and QS are the diagonals.

We know from equations 1 and 4 that the diagonals bisect each other.

Thus, XQYS is a parallelogram.

2. In ΔSXP and ΔQYR, we have:

   PS = QR (Opposite sides of parallelogram PQRS)

   SX = QY (Opposite sides of parallelogram XQYS)

   PX = YR (Given)

   ∴ ΔSXP ≅ ΔQYR (By the SSS congruence rule)

Simulation

Properties Of Rectangles

Let us suppose that we draw a parallelogram with 90° as the measure of each angle. Then we will obtain the following figure.

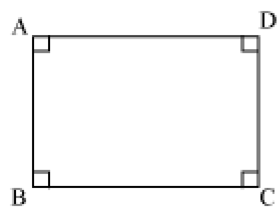


Figure ABCD is called a **rectangle.**

Thus, we can define a rectangle as follows:

“***A rectangle is a parallelogram in which each interior angle is a right angle”.***

Being a parallelogram, **a rectangle has opposite sides of equal length that are parallel to each other and its diagonals bisect each other.**

Is there any other property of the diagonals of a rectangle?

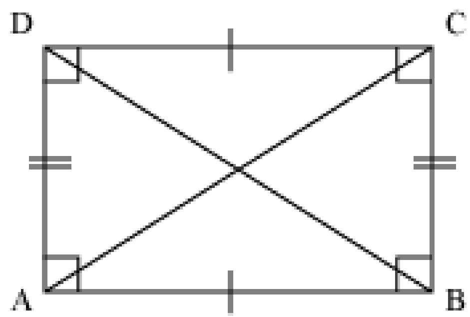
The given video will help you understand another important property related to diagonals of a rectangle.

In this video, we have seen that the diagonals of a rectangle are equal and bisect each other. Can we also say that if the diagonals of a parallelogram are equal, then the parallelogram is a rectangle?

To know the answer of this question, look at the following video.

Now, we can write:
"A parallelogram is a rectangle, if its diagonals are equal."

Thus, we can summarize the properties of rectangles as follows:

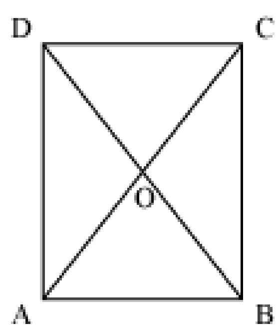


Opposite sides are equal.
Opposite sides are parallel.
Each angle is 90°.
Diagonals are equal and bisect each other.

Let us look at some examples now.

Example 1:

In the given rectangle ABCD, find the value of x, if OB = $x - 2$ and OC = $2x - 10$.



Solution:

Since ABCD is a rectangle, its diagonals are equal and they bisect each other.

i.e., BD = AC

Also, AO = OC and OB = OD

Now, BD = AC

⇒ BO + OD = AO + OC

⇒ 2OB = 2OC

⇒ OB = OC

⇒ $x - 2 = 2x - 10$

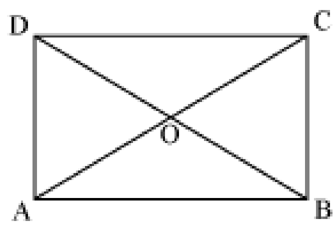
⇒ $2x - x = -2 + 10$

⇒ $x = 8$

Thus, the value of x is 8.

Example 2:

In a rectangle ABCD, find the length of diagonal BD, if CO = 20 cm.



Solution:

We know that the diagonals of a rectangle bisect each other, therefore

OA = OC

It is given that OC = 20 cm

∴ OA = 20 cm

From figure,

AC = AO + OC

AC = 20 + 20

AC = 40 cm

In a rectangle, both the diagonals are of equal length.

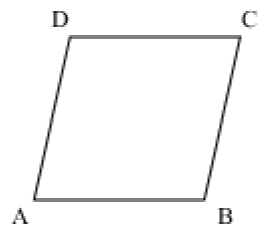
∴ AC = BD

⇒ BD = 40 cm

Thus, the length of diagonal BD is 40 cm.

Properties Of Rhombuses

Let us consider a parallelogram ABCD.



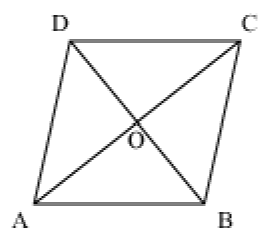
The above parallelogram has been drawn such that all its sides are of equal length. Therefore, can we give any special name to the parallelogram ABCD?

Yes, ABCD is called a **rhombus.**

"***A rhombus is a parallelogram in which all sides are of equal length".***

Being a parallelogram, ***the diagonals of a rhombus bisect each other.***

Therefore, OA = OC and OB = OD.



Can we say anything else about the diagonals of a rhombus?

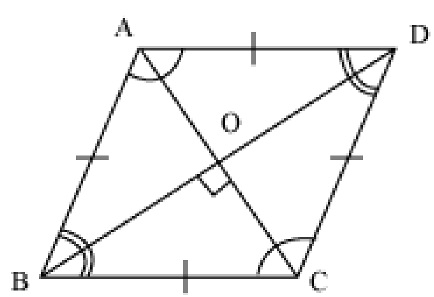
Look at the given video to find out the answer.

In this video, we have seen that the diagonals of a rhombus act as perpendicular bisectors. Therefore, can we say that if we have a quadrilateral whose diagonals are perpendicular bisectors, then the quadrilateral is a rhombus?

So, we can conclude that:

“A quadrilateral is a rhombus, if its diagonals bisect each other at right angles.”

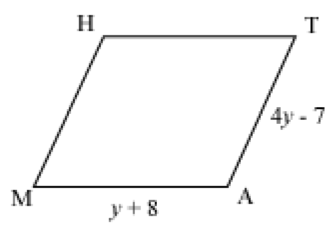
We can thus summarize the properties of a rhombus as follows:



All sides are equal.
Opposite sides are parallel.
Opposite angles are equal.
Diagonals are perpendicular bisectors of each other.
The two diagonals divide the rhombus into four congruent right angled triangles.
Diagonals bisect the angles.

Example 1:

In a rhombus MATH, MA = *y* + 8 and AT = 4*y* − 7. Find the length of MA. Also, find the perimeter of the rhombus.



Solution:

All the sides of a rhombus are equal. Therefore,

MA = AT = TH = HM

Now, MA = AT

⇒ $y + 8 = 4y - 7$

⇒ $y - 4y = -7 - 8$

⇒ $-3y = -15$

Dividing both sides by 3, we obtain

$y = 5$ units

Now, MA = $y + 8$

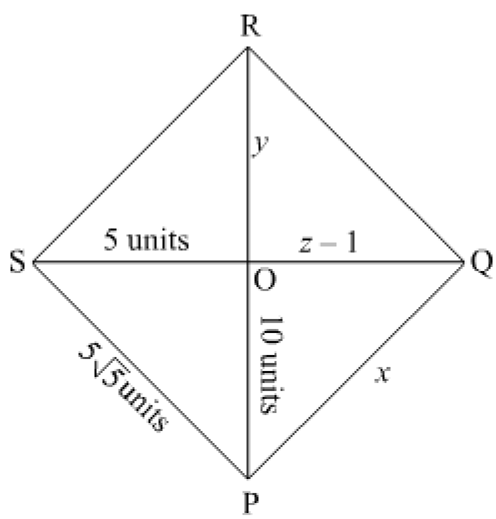
⇒ MA = 5 + 8 (Since $y = 5$ units)

⇒ MA = 13 units

∴ Perimeter of rhombus = MA + AT + TH + HM = 4MA
= 4 × 13
= 52 units

Example 2:

Find the values of *x*, *y*, and *z* from the given figure where PQRS is a rhombus.



Solution:

Since PQRS is a rhombus, all sides are equal.

∴ PQ = PS

⇒ $x = 5\sqrt{5}$ units

The diagonals of a rhombus bisect each other.

∴ OP = OR

⇒ y = 10 units

Also, OS = OQ

⇒ z − 1 = 5

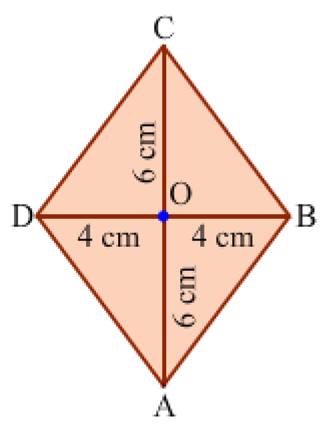
⇒ z = 5 + 1

⇒ z = 6 units

Example 3:
The diagonals of a rhombus are 12 cm and 8 cm. Find the area of the rhombus.

Solution:

We have a rhombus *ABCD* with diagonals are 12 cm and 8 cm.
It is known that the diagonals of a rhombus bisect each other at right angles.
So we get OB = OD = 4 cm and OA = OC = 6 cm.

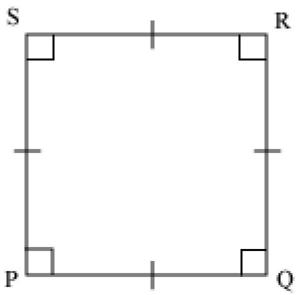


Now, area of right triangle BOC = $\frac{1}{2} \times OB \times OC = \frac{1}{2} \times 4 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$

Diagonals of a rhombus divides it into four congruent right triangles.
∴ Area of the rhombus = 4 × Area of ΔBOC = 4 × 12 cm^2 = 48 cm^2

Properties Of Squares

If we draw a rectangle PQRS with all sides of equal measure, then we will obtain the following figure.



The quadrilateral PQRS so formed is called a **square.**

A square is a special case of parallelogram in which all the sides are equal in length and the measure of each angle is 90°.

Being a parallelogram, the diagonals of a square bisect each other, but there are some properties of diagonals of a square, which are exclusive for the square.

The first property is that the diagonals of a square act as perpendicular bisectors. The following video will show you how this property can be proved.

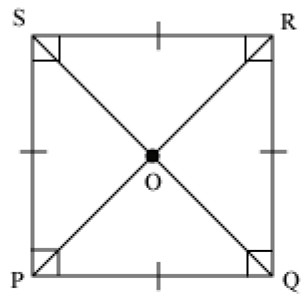
One more property about the diagonals of a square is that they are equal in length. You can see the proof of this property in the following video.

We have studied that the diagonals of a square are equal and act as perpendicular bisectors. Its converse is also true. If the diagonals of a quadrilateral are equal and act as perpendicular bisectors, then the quadrilateral is a square.

To prove this converse, look at the following video.

Now, we can state this result as follows:
"A quadrilateral is a square, if its diagonals are equal and bisect each other at right angles."

We can thus summarize the properties of squares as follows:

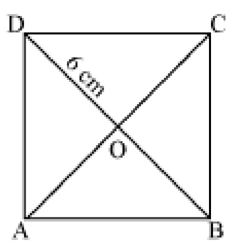


All sides are equal.
The measure of each angle is 90°.
The opposite sides are parallel.
Diagonals are of equal lengths and perpendicular bisectors of each other.

A **square** can also be thought of a **rectangle** in which adjacent sides are equal or a **rhombus** in which each angle is 90º.

Example:

In the given figure, find the length of the sides of the square ABCD.



Solution:

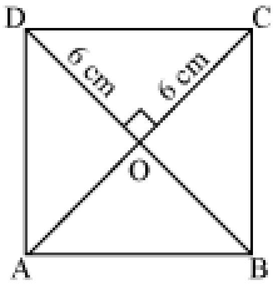
We know that the diagonals of a square are equal.

Therefore, BD = AC

Also, the diagonals bisect each other.

∴ OD = OC

Also, the diagonals are perpendicular to each other.



Now, applying Pythagoras Theorem in ΔDOC, we obtain

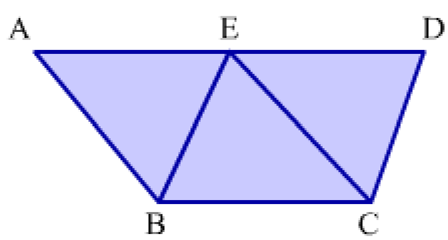
$$OD^2 + OC^2 = DC^2$$
$$6^2 + 6^2 = DC^2$$
$$DC^2 = 36 + 36$$
$$DC^2 = 72$$
$$DC = \sqrt{72}$$
$$DC = 6\sqrt{2}\text{cm}$$

Thus, the length of each side of square ABCD is $6\sqrt{2}$ cm.

A Quadrilateral is a Parallelogram if a Pair of Opposite Sides is Equal and Parallel

Necessary Condition for a Quadrilateral to Be a Parallelogram

Consider the given figure.



Here, ABCE is a parallelogram and AE is extended to D such that AE = ED.

Using an important property of parallelograms, we can prove that BCDE is also a parallelogram.

The property used for proving the above can be stated as follows:

A quadrilateral is a parallelogram if it has one pair of parallel and equal (or congruent) opposite sides.

Since ABCE is a parallelogram, AE||BC and AE = BC.

Also, AE = ED ⇒ ED = BC

And AE||BC ⇒ AD||BC ⇒ ED||BC

Hence, by the above-stated property, BCDE is a parallelogram.

In this lesson, we will understand and prove this property of parallelograms. We will also solve some examples based on the same.

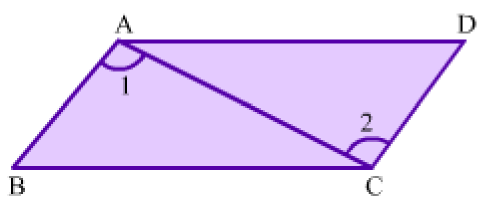
Proof of the Property

Solved Examples

Easy

Example 1:

In the given figure, ∠1 = ∠2 and AB = DC. Is quadrilateral ABCD a parallelogram?



Solution:

In quadrilateral ABCD, ∠1 = ∠2(Given)

These angles are alternate angles made by the transversal AC between lines AB and DC.

We know that if equal alternate angles are made by a transversal between two lines, then the lines intersected by the transversal are parallel.

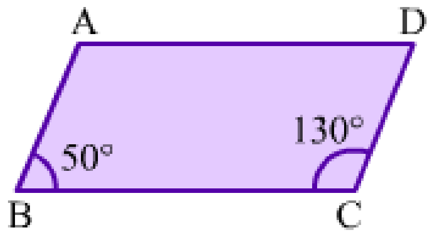
∴ AB||DC

Also, AB = DC (Given)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

Example 2:

In the given figure, AB = DC. Prove that AD||BC.



Solution:

In quadrilateral ABCD, ∠B + ∠C = 50° + 130° = 180°

We know that if the interior angles on the same side of a transversal are supplementary, then the lines intersected by the transversal are parallel.

⇒ AB||DC

Also, AB = DC (Given)

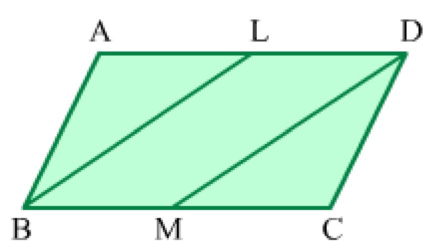
We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

⇒ AD||BC (∵ Opposite sides of a parallelogram are parallel)

Medium

Example 1:

ABCD is a parallelogram in which L and M are the midpoints of AD and BC respectively. Prove that BMDL is a parallelogram.



Solution:

It is given that L and M are the midpoints of AD and BC respectively.

$\therefore BM = \frac{1}{2}BC$ and $LD = \frac{1}{2}AD$

ABCD is a parallelogram. (Given)

So, BC = AD (∵ Opposite sides of a parallelogram are equal)

$\therefore \frac{1}{2}BC = \frac{1}{2}AD$

⇒ BM = LD … (1)

Also, BC||AD (∵ Opposite sides of a parallelogram are parallel)

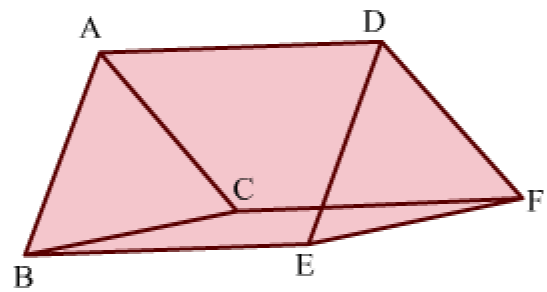
⇒ BM||LD … (2)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. From 1 and 2, we conclude that BMDL is a parallelogram.

Hard

Example 1:

Sides AB and BC of ΔABC are parallel and equal to the corresponding sides DE and EF of ΔDEF. Prove that ACFD is a parallelogram.



Solution:

Consider quadrilateral ABED.

We have AB = DE and AB||DE (Given)

Hence, ABED is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)

⇒ AD = BE and AD||BE… (1)

Now, consider quadrilateral BCFE.

We have BC = EF and BC||EF (Given)

Hence, BCFE is a parallelogram. (∵ There is one pair of equal and parallel opposite sides) ⇒ CF = BE and CF||BE… (2)

From 1 and 2, we have:

AD = CF and AD||CF

Hence, ACFD is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)

The Diagonal of a Parallelogram Divides It into Two Congruent Triangles

Look at the two triangular sandwiches obtained by cutting a sandwich along the **diagonal**.

Are the two divided parts of the sandwich equal in **area**? What can we say about their congruency? Does the diagonal divide the sandwich into two triangles of equal area?

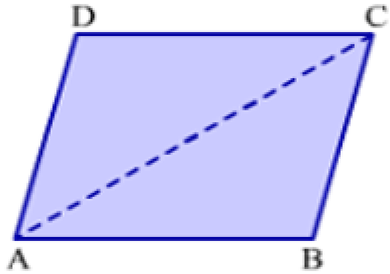
The answers to these questions are based on a property of **parallelograms**. It can be stated as follows:

A diagonal of a parallelogram divides it into two congruent triangles.

In this lesson, we will understand and prove the above-stated property of parallelograms. We will also solve some examples related to the same.

The Diagonal of a Parallelogram Divides It into Two Congruent Triangles

Consider a parallelogram ABCD. AC is a diagonal of this parallelogram.



Suppose the area of ΔABC is 15 cm^2.

We can find the area of the given parallelogram with the help of the area of ΔABC by using a property of parallelograms which states that:

A diagonal of a parallelogram divides it into two congruent triangles.

In the given figure, diagonal AC divides the parallelogram into two congruent triangles, ΔABC and ΔCDA.

We know that congruent figures are equal in area.

So, ar (ΔABC) = ar (ΔCDA)

∴ ar (parallelogram ABCD) = 2 × ar (ΔABC)

= 2 × 15 cm^2

= 30 cm^2

Concept Builder

- A trapezium is a quadrilateral having one pair of parallel opposite sides.
- A rectangle is a quadrilateral having two pairs of equal opposite sides. Also, each angle in a rectangle is equal to 90°.
- A rhombus is a quadrilateral having all sides equal.
- A square is a quadrilateral having all sides equal and all angles equal to 90°.

Did You Know?

- The quadrilateral formed by joining the midpoints of a quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral formed by joining the midpoints of the quadrilateral is a rectangle.
- The sum of any three sides of a quadrilateral is always greater than the fourth side.
- The sum of all sides of a quadrilateral is greater than the sum of its diagonals.

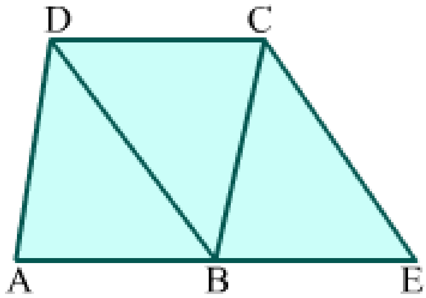
Proof of the Property

Solved Examples

Easy

Example 1:

In the given figure, ABCD and BECD are two parallelograms. Prove that ΔABD ≅ ΔBEC.



Solution:

ABCD is a parallelogram with BD as a diagonal and BECD is a parallelogram with BC as a diagonal. We know that a diagonal of a parallelogram divides it into two congruent triangles.

∴ ΔABD ≅ ΔCDB

And ΔCDB ≅ ΔBEC

⇒ ΔABD ≅ ΔBEC

Example 2:

ABCD is a parallelogram with area 60 cm^2. Find the area of ΔACD.

Solution:

We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

So, we have:

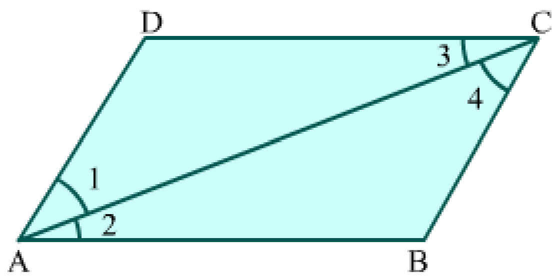
$$\text{ar}(\Delta ACD) = \text{ar}(\Delta CAB) = \frac{\text{ar (parallelogram ABCD)}}{2}$$

$\Rightarrow$ ar (ΔACD) = $\frac{60\,\text{cm}^2}{2}$

$\Rightarrow \therefore$ ar (ΔACD) = 30 cm^2

Example 3:

ABCD is a parallelogram in which ∠1 = ∠2. Prove that ∠3 = ∠4.



Solution:

ABCD is a parallelogram with AC as a diagonal. We know that a diagonal of a parallelogram divides it into two congruent triangles.

$\therefore \Delta ABC \cong \Delta CDA$

$\Rightarrow \angle 2 = \angle 3$ and $\angle 1 = \angle 4$ … (1)

It is given that ∠1 = ∠2. … (2)

From 1 and 2, we get:

$\angle 3 = \angle 4$

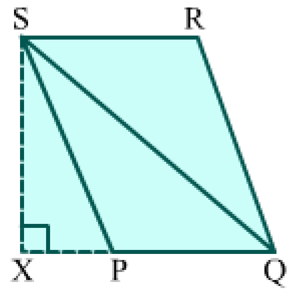
Medium

Example 1:

The area of a parallelogram PQRS is 50 cm^2. Find the distance between PQ and SR if the length of PQ is 6 cm.

Solution:

Construction: Draw diagonal SQ of the given parallelogram PQRS. Extend PQ and draw a line perpendicular to it from point S.



We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

∴ ar (ΔPQS) = ar (ΔRSQ)

Now, ar (parallelogram PQRS) = ar (ΔPQS) + ar (ΔRSQ)

⇒ ar (parallelogram PQRS) = 2 × ar (ΔPQS)

⇒ ar (ΔPQS) = $\frac{1}{2} \times \text{ar (parallelogram PQRS)}$

⇒ ar (ΔPQS) = $\frac{50}{2} \text{cm}^2$

⇒ ∴ ar (ΔPQS) = 25 cm^2

Also, ar (ΔPQS) = $\frac{1}{2} \times PQ \times SX$

⇒ $\frac{1}{2} \times PQ \times SX$ = 25 cm^2

⇒ $PQ \times SX$ = 50 cm^2

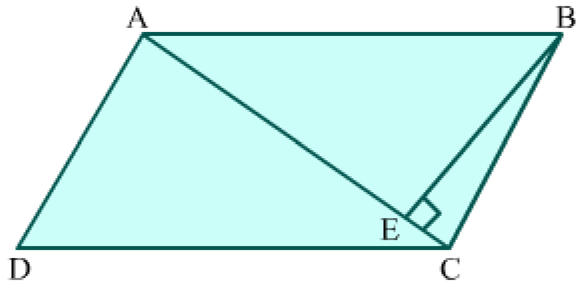
⇒ SX = $\frac{50}{6}$ cm (∵ It is given that PQ = 6 cm)

⇒ ∴ SX = 8.3 cm

Thus, the distance between PQ and SR is 8.3 cm.

Example 2:

In the given parallelogram ABCD, altitude BE on AC is of length 4 cm. If the length of diagonal AC is 6 cm, then find the area of the parallelogram.



Solution:

We know that a diagonal of a parallelogram divides it into two congruent triangles.

Also, congruent figures are equal in area.

∴ ar (ΔABC) = ar (ΔCDA)

Now, ar (ΔABC) = $\frac{1}{2}$ × Base × Height

$= \frac{1}{2} \times AC \times BE$

$= \frac{1}{2} \times 6\,cm \times 4\,cm$

$= 12\,cm^2$

Therefore, ar (parallelogram ABCD) = ar (ΔABC) + ar (ΔCDA)

= 2 × ar (ΔABC)

= 2 × 12 cm^2

= 24 cm^2