Ratio and Proportion

Concept of Ratios

There are situations, when we need to compare two quantities.

For example, Swaminathan and Rohan both are in the same class. Their respective marks in mathematics are 96 and 48.

Marks scored by Swaminathan and Rohan can be compared by two methods.

1. Subtraction method

In this method, we subtract one quantity from other to find that one is how much more than the other.

Now,

Marks scored by Swaminathan – Marks scored by Rohan = 96 – 48 = 48

So, it can be said that Swaminathan scored 48 marks more than Rohan in mathematics.

2. Division method

In this method, we divide one quantity by other to find that one is how many times the other.

Now,

$$\frac{\text{Marks scored by Swaminathan}}{\text{Marks scored by Rohan}} = \frac{96}{48} = \frac{2}{1}$$

So, it can be said that the marks scored by Swaminathan are twice the marks scored by Rohan.

When two quantities are compared using division method, the quotient obtained is called "ratio".

First term of a ratio is called "**antecedent**" and the second term is called "**consequent**".

For example, in the ratio $x : y$, x is antecedent and y is consequent.

Remember

- **Comparison is made between the quantities carrying the same units.**
- **Comparison cannot be made between the quantities which are not similar.**
- **Ratio does not have any unit.**

If *x* and *y* are two quantities in a particular ratio, one should not be confused between *x* : *y* and *y* : *x*.

The ratio *x* : *y* means x/y and *y* : *x* means y/x.

Conversion of a Fractional Ratio into a Whole Number Ratio

Example: Convert 15:1315:13 into ratio in simple form

There are two methods of converting a fractional ratio into a whole number ratio. They are:

Method I: Dividing the first quantity by the second

Solution: We are given the ratio as 15:1315:13.
We simply divide the first quantity by the second.

1513 = 15 × 31 = 35 = 3:51513 = 15 × 31 = 35 = 3:5

Method II:

(i) Find the LCM of the denominators.
So, LCM of 5 and 3 will be 15

(ii) Multiply the terms of the given ratio with the LCM and simplify.

15 × 15:13 × 15 = 3:515 × 15:13 × 15 = 3:5

Let us now look at an example to understand this concept better.

Example:

Identify the cases out of the following in which a comparison can be

made using ratios.

1. **The ratio between the price of a book and the price of a shirt**
2. **The ratio between the age of a person and the amount of money he has**
3. **The ratio of the length of a park to its breadth**

Solution:

1. The price of a book and the price of a shirt are of the same type. Therefore, in this case, comparison can be made using ratios.

2. The age and money are of different types. Therefore, in this case, we cannot compare the quantities.

3. The quantities length and breadth are of the same type. Therefore, in this case, comparison can be made using ratios.

Application of Ratios in Solving Problems

Ratios are used to compare quantities. They are widely applied in many day-to-day situations.

Consider the case where Seema and Sheetal wrote a test and scored 40 marks and 30 marks respectively.

Now, ratio of Seema's and Sheetal's scores $= \frac{\text{Seema's score}}{\text{Sheetal's score}} = \frac{40}{30} = \frac{4}{3}$

The ratio of two quantities is denoted by '**:**'.

Thus, the ratio of Seema's and Sheetal's scores $\left(\frac{4}{3}\right)$ can be written as 4:3.

Now, what information does this ratio give us?

Since the ratio of their scores is 4:3, it tells us that for every 4 marks that Seema scored, Sheetal scored 3 marks. Thus, even if we do not know their actual scores, but only the ratio of the scores, we can still tell who got more marks.

Let us take another case. Let us suppose Meenu has 60 marbles, out of which, 25 are red in colour, and the rest are black in colour. Now, we have to find the ratios of the numbers of red and black marbles out of the total number of marbles.

We know that the total number of marbles is 60.

Number of red marbles = 25

Thus, ratio of the number of red marbles to the total number of marbles

$$= \frac{\text{Number of red marbles}}{\text{Total number of marbles}} = \frac{25}{60} = \frac{5}{12} = 5:12$$

Now, we can find the number of black marbles by subtracting the number of red marbles from the total number of marbles.

Thus, number of black marbles = 60 – 25 = 35

Thus, ratio of the number of black marbles to the total number of marbles

$$= \frac{\text{Number of black marbles}}{\text{Total number of marbles}} = \frac{35}{60} = \frac{7}{12} = 7:12$$

We can, in fact, find the ratio of the number of black marbles to the total number of marbles in a different way. We know that the marbles are either red or black in colour.

Think of ratios as parts of a whole, just like fractions. Thus, the sum of the ratios of the numbers of red and black marbles to the total number of marbles would be 1.

Thus, ratio of the number of black marbles to the total number of marbles

= 1 – ratio of the number of red marbles to the total number of marbles

$$= 1 - \frac{5}{12} = \frac{12-5}{12} = \frac{7}{12} = 7:12$$

If the ratio between two quantities is *a* : *b*, then we cannot write the ratio as *b* : *a*.

$\therefore a : b \neq b : a$

Let us now solve some more problems to understand this concept better.

Example 1:

There are 35 boys and 30 girls in a class. Find the ratio of

1. **the number of boys to the total number of students in the class.**
2. **the total number of students to the number of girls in the class.**
3. **the number of boys to the number of girls in the class.**

Solution:

Number of boys in the class = 35

Number of girls in the class = 30

Thus, total number of students in the class = 35 + 30 = 65

(i) Ratio of number of boys to total number of students $= \frac{35}{65} = \frac{35 \div 5}{65 \div 5} = \frac{7}{13} = 7:13$

(ii) Ratio of total number of students to number of girls $= \frac{65}{30}$

$= \frac{65 \div 5}{30 \div 5} = \frac{13}{6} = 13:6$

(iii) Ratio of number of boys to number of girls = 35/30

$= \frac{35 \div 5}{30 \div 5} = \frac{7}{6} = 7:6$

Example 2:

The length of a table is 1.5 m and its breadth is 75 cm. Find the ratio of the length of the table to its breadth.

Solution:

Length of the table = 1.5 m = (1.5 × 100) cm = 150 cm

Breadth of the table = 75 cm

Thus, ratio of the length of the table to its breadth $= \frac{\text{Length of table}}{\text{Breadth of table}}$

$= \frac{150 \text{ cm}}{75 \text{ cm}} = \frac{2}{1} = 2:1$

Example 3:

Naina has 15 chocolates. She wants to divide these chocolates between Prabha and Priyanka in the ratio 3:2. How many chocolates will each get?

Solution:

There are two parts. One is 3 and another is 2.

Therefore, there are a total of 3 + 2 = 5 parts.

This means that Naina has to divide 15 chocolates into 5 parts. Out of these 5 parts, Prabha will get 3 parts and Priyanka will get 2 parts.

Therefore, number of chocolates that Prabha got $= \frac{3}{5} \times 15 = 9$

Similarly, number of chocolates that Priyanka got $= \frac{2}{5} \times 15 = 6$

Example 4:

The ratio of Nayan's height to Tarun's height is 12:13. Who is taller?

Solution:

The ratio of their heights is 12:13. Now, 12 < 13.

Since the "13" part of the ratio corresponds to Tarun's height, Tarun is taller than Nayan.

Example 5:

Sanju and Rupam started a business. Sanju invested Rs.15000 and Rupam invested Rs.30000 in the business. If the profit of the business was Rs 60000, then divide the profit between them in the ratio of their investment.

Solution:

Amount invested by Sanju = Rs 15000

Amount invested by Rupam = Rs 30000

Thus, ratio of Sanju's investment to Rupam's investment $= \frac{\text{Rs } 15000}{\text{Rs } 30000}$

$= \frac{1}{2} = 1:2$

Therefore, there are a total of 1 + 2 = 3 parts, out of which, Sanju will get $\frac{1}{3}$ part and Rupam will get $\frac{2}{3}$ parts of the total profit.

Thus, Sanju's share of the profit $= \text{Rs}\left(\frac{1}{3}\times 60000\right) = \text{Rs } 20000$

Similarly, Rupam's share of the profit $= \text{Rs}\left(\frac{2}{3}\times 60000\right) = \text{Rs } 40000$

Example 6:

The ratio of milk and water in a 21 L solution is 5:2. Now, 3 L milk and 3 L water is added to the solution. What is the new ratio of milk and water in the solution?

Solution:

The two parts of the ratio are 5 and 2.

Therefore, sum of the parts = 5 + 2 = 7

This means that if there is a 7 L solution, then the amount of milk = 5 L and the amount of water = 2 L.

Out of 1 L solution, the amount of milk is $\frac{5}{7}$ L and the amount of water is $\frac{2}{7}$ L.

Out of 21 L solution, the amount of milk is $\left(\frac{5}{7}\times 21\right)\text{L} = 15\text{ L}$ and the amount of water is $\left(\frac{2}{7}\times 21\right)\text{L} = 6\text{ L}$.

When 3 L of both milk and water is added to the solution, then the amount of milk = (15 + 3) L = 18 L and the amount of water = (6 + 3) L = 9 L.

Therefore, ratio of milk and water in new solution = $\frac{18\text{ L}}{9\text{ L}} = \frac{2}{1} = 2:1$

Example 7:

The population of two countries A and B are 800 lakhs and 1350 lakhs respectively. The respective areas of these countries are 4 lakh sq. km and 5 lakh sq. km. Which of these countries is less populated?

Solution:

The country will be less populated if its population per sq. km (population density) is less.

$$\text{Population density of country A} = \frac{\text{Population of country A}}{\text{Area of country A}}$$

$$= \frac{800 \text{ lakh}}{4 \text{ lakh sq. km}} = 200 \text{ lakh/sq. km}$$

$$\text{Similarly, population density of country B} = \frac{\text{Population of country B}}{\text{Area of country B}}$$

$$= \frac{1350 \text{ lakh}}{5 \text{ lakh sq. km}} = 270 \text{ lakh/sq. km}$$

From the above calculation, we can see that the population density of state A is less than that of state B. Therefore, state A is less populated.

Concept of Proportion

Ravi has 15 pens and Sumit has 10 pens. What is the ratio of the number of pens with Ravi to the number of pens with Sumit?

Ratio = 15:10 = 3/2 = 3:2

This ratio is equivalent to a ratio 6:4 i.e., the ratio 3:2 is same as the ratio 6:4.

The numbers 3, 2, 6, and 4 are said to be in **proportion**.

“**If two ratios are equal, then the numbers or values in the ratios are said to be in proportion”.**

In general, if *a*, *b*, *c*, and *d* are any four numbers and $\frac{a}{b}=\frac{c}{d}$, then *a*, *b*, *c*, and *d* are said to be in proportion.

Proportion is denoted by the symbol '**=**' or '**::**' and is placed between two ratios.

For example, if 2, 4, 5, and 10 are in proportion, then we can denote this by writing

2:4 **=** 5:10

Or, 2:4 **::** 5:10

Now, consider one more example. Suppose there are 6 males and 2 females in a car. And there are 27 males and 9 females in a bus. Now, ratio of the number of males to the number of females in the car $=\frac{6}{2}=\frac{3}{1}$

Ratio of the number of males to the number of females in the bus = 279 = 31= 279 = 31

Therefore, we can write, $\frac{6}{2}=\frac{27}{9}$

Thus, 6, 2, 27, and 9 are in proportion.

Here, the numbers 6, 2, 27, and 9 are called **respective terms.**

The four numbers or values involved in a statement of proportion when taken in order are called respective terms. If the numbers are not taken in order, then the numbers are not called respective terms.

The first and the fourth terms of the respective terms are called extreme terms or extremes. The second and the third terms are called middle terms or means.

For example, in the above example of proportion $\frac{6}{2}=\frac{27}{9}$, the numbers 6, 2, 27, and 9 are respective terms; however, 6, 27, 2, and 9 or 6, 2, 9, and 27 are not respective terms.

The numbers 6 and 9 are called extreme terms, while 2 and 27 are called middle terms.

Remember

- **If *p*, *q*, *r*, and *s* are four numbers and *p*:*q* ≠ *r*:*s*, then *p*, *q*, *r*, and *s* are not in proportion.**

  For example, 2:4 ≠ 3:8, therefore, 2, 4, 3, and 8 are not in proportion.

- **If *x*:*y* :: *a*:*b*, then it is read as *x* is to *y* as *a* is to *b*.**

  Let us look at some more examples to understand this concept better.

  Example 1:

  Determine which of the following numbers are in proportion when taken in the given order.

1. **16, 32, 25, 50**
2. – **22, 55, –35, 48**
3. **72, 24, 12, 36**
4. – **84, – 60, – 56, – 40**

   Solution:

1. 16, 32, 25, 50

$$16:32=\frac{16}{32}=\frac{1}{2}=1:2$$
$$25:50=\frac{25}{50}=\frac{1}{2}=1:2$$

   Therefore, $16:32=25:50$

   Hence, 16, 32, 25, and 50 are in proportion.

2. – 22, 55, –35, 48

$$-22:55=\frac{-22}{55}=-\frac{2}{5}=-2:5$$
$$-35:48=\frac{-35}{48}=-35:48$$

   Therefore, $22:55\neq 35:48$

   Hence, 22, 55, 35, and 48 are not in proportion.

3. 72, 24, 12, 36

$$72:24=\frac{72}{24}=\frac{3}{1}=3:1$$
$$12:36=\frac{12}{36}=\frac{1}{3}=1:3$$

Therefore, $72:24\neq 12:36$

Hence, 72, 24, 12, and 36 are not in proportion.

4. – 84, – 60, – 56, – 40

$$-84:-60=\frac{-84}{-60}=\frac{7}{5}=7:5$$
$$-56:-40=\frac{-56}{-40}=\frac{7}{5}=7:5$$

Therefore, $-84:-60=-56:-40$

Hence, – 84, – 60, – 56, and – 40 are in proportion.

Example 2:

Find the extreme terms and the middle terms of the following proportions.

1. *a*:*b* = *c*:*d*
2. 7:9 :: 28:36
3. $\frac{18}{30}=\frac{36}{60}$

Solution:

1. *a*:*b* = *c*:*d*

   Extreme terms = *a*, *d*

   Middle terms = *b*, *c*

2. 7:9 :: 28:36

   Extreme terms = 7, 36

Middle terms = 9, 28

3. $\frac{18}{30} = \frac{36}{60}$

Extreme terms = 18, 60

Middle terms = 30, 36

Example 3:

The numbers 42, 54, *x*, and 108 are in proportion. Find the value of *x.*

Solution:

42, 54, *x*, and 108 are in proportion.

Therefore, $\frac{42}{54} = \frac{x}{108}$

Multiplication of 54 by 2 gives 108.

To find the value of *x*, we have to multiply 42 by 2.

$\therefore x = 42 \times 2 = 84$

Example 4:

Are the ratios 6 kg to 4800 g and Rs 75 to Rs 60 in proportion?

Solution:

6 kg to 4800 g $= \frac{6\,\text{kg}}{4800\,\text{g}} = \frac{6000\,\text{g}}{4800\,\text{g}} = \frac{6000}{4800} = \frac{5}{4} = 5:4$

Rs 75 to Rs 60 $= \frac{75}{60} = \frac{5}{4} = 5:4$

Therefore, the ratios 6 kg to 4800 g and Rs 75 to Rs 60 are in proportion.

Example 5:

Rita was asked that if a bus travelled 225 km in 5 hours, then what distance will it travel in 15 hours. She replied that the bus will travel 675 km in 15 hours. Was she correct?

Solution:

Rita will be correct, if the ratios 5 hours to 15 hours and 225 km to 675 km are in proportion.

5 hours to 15 hours $= \frac{5}{15} = \frac{1}{3} = 1:3$

225 km to 675 km $= \frac{225}{675} = \frac{1}{3} = 1:3$

Therefore, 5 hours to 15 hours and 225 km to 675 km are in proportion.

Thus, Rita's answer was **correct**.

Example 6:

If the cost of 6 umbrellas is Rs 144, then what is the cost of 8 umbrellas?

Solution:

Let the cost of 8 umbrellas be Rs *x.*

Now, the ratios of 8 umbrellas to 6 umbrellas and Rs *x* to Rs 144 must be in proportion,

i.e., 8 umbrellas to 6 umbrellas = Rs *x* to Rs 144

$$\Rightarrow \frac{8}{6} = \frac{x}{144}$$

Multiplication of 6 with 24 gives 144, therefore, we have to multiply 8 with 24 to find the value of *x.* On multiplying 8 with 24, we obtain

$8 \times 24 = 192$

$\therefore x = 192$

Thus, the cost of 8 umbrellas is Rs 192.

Example 7:

On a sunny day, Preetam observed that a 1 m long stick standing vertically on the ground makes a shadow of length 30 cm. What will be the height of a tree if the length of its shadow is 3 m 60 cm at that time?

Solution:

Let x be the height of the tree.

We know 1 m = 100 cm

$\therefore$ 3 m 60 cm = (3 × 100 + 60) cm = 360 cm

Ratio of height of stick and length of its shadow = 100 cm : 30 cm

Similarly, ratio of height of tree and length of its shadow = x : 360 cm

At the same time, the ratio of the height of the stick and the length of its shadow and the ratio of the height of the tree and the length of its shadow are equal. Thus, the two ratios are in proportion.

$\therefore$ 100 cm: 30 cm = x : 360 cm

$$\frac{100 \text{ cm}}{30 \text{ cm}} = \frac{x}{360 \text{ cm}}$$

$$x = \left(\frac{100 \times 360}{30}\right) \text{ cm}$$

$$x = 1200 \text{ cm}$$

$$x = 12 \text{ m}$$

Therefore, the height of the tree is 12 m.

Continued and Mean Proportion

We know that the numbers in the ratio are said to be in **proportion** if the two ratios are equal.

For example, 2, 5, 10 and 25 are said to be in proportion as 2: 5 = 10: 25

Three quantities of the same kind *a*, *b*, and *c* are said to be in continued proportion, if

$$\frac{a}{b}=\frac{b}{c}$$

i.e., the ratio of *a* to *b* is equal to *b* to *c*.

We can also generalise this for more than three ratios.

Let *a*, *b*, *c*, *d*… are some (non-zero) quantities of the same kind. *a*, *b*, *c*, *d* … are said to be in **continued proportion,** if

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\ldots$$

For example, 2, 6 and 18 are in continued proportion as $\frac{2}{6}=\frac{6}{18}$.

If *a*, *b*, and *c* are in continued proportion, then *a*, *b*, and *c* are called first proportional, mean proportional, and third proportional respectively.

If we know any two proportional, then we can find the third one.

For example: What is the mean proportional to 6 and 96?

Let the mean proportional be *x*.

Since 6, *x*, and 96 are in continued proportion.

$$\frac{6}{x}=\frac{x}{96}$$

$$6\times 96=x\times x$$

$$x^2=6\times 96$$

$$x=\sqrt{6\times 96}$$

$$x=24$$

Thus, 24 is the mean proportional to 6 and 96.

Now, let us solve some more examples to understand the concept better.

Let us understand the concept better with the help of an example.

Example 1:

What are the values of *a* and *b* for which *a*, 5, 35, and *b* are in continued proportion?

Solution:

a, 5, 35, and *b* are in continued proportion.

$$\therefore \frac{a}{5} = \frac{5}{35} = \frac{35}{b}$$

$\frac{a}{5} = \frac{5}{35}$ (Taking first two terms)

$$a = \frac{5 \times 5}{35} = \frac{5}{7}$$

$\frac{5}{35} = \frac{35}{b}$ (Taking last two terms)

$$5 \times b = 35 \times 35$$
$$b = \frac{35 \times 35}{5}$$
$$b = 245$$

Thus, the values of *a* and *b* are 5/7 and 245 respectively.

Example 2:

What number must be added to the numbers 27, 111, and 363 so that they are in continued proportion?

Solution:

Let the required number be *x*.

Then, 27 + *x*, 111 + *x*, and 363 + *x* are in continued proportion.

$$\therefore \frac{27+x}{111+x}=\frac{111+x}{363+x}$$
$$(27+x)(363+x)=(111+x)(111+x)$$
$$x^2+390x+9801=x^2+222x+12321$$
$$390x+9801=222x+12321$$
$$390x-222x=12321-9801$$
$$168x=2520$$
$$x=15$$

Thus, the required number is 15.

Example 3:

If *a*, *b*, *c*, and *d* are in continued proportion, then prove that:

$$(a+c)^3:(b+d)^3::a:d$$

Solution:

Since *a*, *b*, *c*, and *d* are in continued proportion,

$\therefore \frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$ (say)

⇒ $a = bk$, $b = ck$, $c = dk$

Then,

$$(a+c)^3:(b+d)^3$$
$$=\frac{(a+c)^3}{(b+d)^3}$$
$$=\frac{(bk+dk)^3}{(b+d)^3}$$
$$=\frac{k^3(b+d)^3}{(b+d)^3}$$
$$=k^3 \qquad \dots(1)$$

And,

$$\frac{a}{d}=\frac{bk}{d}$$
$$=\frac{(ck)k}{d}$$
$$=\frac{(dk)k^2}{d}$$
$$=k^3 \qquad \ldots(2)$$

From equations (1) and (2), we obtain

$$(a+c)^3:(b+d)^3::a:d$$

Example 4:

Six numbers are in continued proportion. If the first and the fourth terms of proportion are 4 and 108 then find the numbers.

Solution:

Let the required numbers be a, ak, ak^2, ak^3, ak^4 and ak^5.

We have a = 4 and ak^3 = 108

$$\therefore \frac{ak^3}{a}=\frac{108}{4}$$

$\Rightarrow k^3 = 27$

$\Rightarrow k = 3$

Therefore, the remaining numbers can be obtained as follows:

$ak = 4 \times 3 = 12$

$ak^2 = 4 \times 3^2 = 4 \times 9 = 36$

$ak^4 = 4 \times 3^4 = 4 \times 81 = 324$

$ak^5 = 4 \times 3^5 = 4 \times 243 = 972$

Thus, the required numbers are 4, 12, 36, 108, 324 and 972.

Example 5:

If *x*, *y*, *z* are in continued proportion then prove that $\frac{z}{y-z}=\frac{y+z}{x-z}$**.**

Solution:

It is given that *x*, *y*, *z* are in continued proportion.

$\therefore \frac{x}{y}=\frac{y}{z}$

Let $\frac{x}{y}=\frac{y}{z}=k$

$\therefore x = yk$ and $y = zk$

$\Rightarrow x = zk^2$

We have to prove that

$$\frac{z}{y-z}=\frac{y+z}{x-z}$$

$$\text{L.H.S.}=\frac{z}{y-z}=\frac{z}{zk-z}=\frac{z}{z(k-1)}=\frac{1}{k-1}$$

$$\text{R.H.S.}=\frac{y+z}{x-z}=\frac{zk+z}{zk^2-z}=\frac{z(k+1)}{z\left(k^2-1\right)}=\frac{(k+1)}{(k-1)(k+1)}=\frac{1}{k-1}$$

So, L.H.S. = R.H.S.

$\therefore \frac{z}{y-z}=\frac{y+z}{x-z}$

Example 6:
If the cost of 6 wallets is Rs 1500, then find the cost of 20 wallets.

Solution:
Method I:
Let the cost of 20 wallets be Rs *x*.
Ratio of wallets = 6 : 20
Ratio of cost = 1500 : *x*
6 : 20 :: 1500 : *x*

$$\frac{6}{20} = \frac{1500}{x}$$
$$6x = 1500 \times 20$$
$$x = \frac{1500 \times 20}{6}$$
$$x = 5000$$

Thus, the cost of 20 wallets = Rs 5000.

Method II:
This is a case of direct proportion, so it will be indicated by downward arrow.

Number of wallets	Cost
$\downarrow$ 6	$\uparrow$ 1500
20	x

The product of the numbers at the arrow heads is equal to the product of the numbers at the arrow tails.

$$6x = 1500 \times 20$$
$$x = \frac{1500 \times 20}{6}$$
$$x = 5000$$

Method III (Unitary method):
Cost of 6 wallets = Rs 1500

∴ The cost of 1 wallet = $\text{Rs } \frac{1500}{6} = \text{Rs } 250$

∴ Cost of 20 wallets = Rs (250 × 20) = Rs 5000

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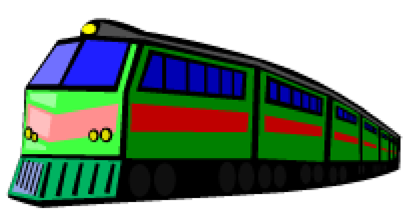
Unitary Method



Suppose train A travels 210 km in 3 hours and train B travels 300 km in 4 hours. **Can we find how much distance train A travels in 5 hours? Can we say which train travels at more speed?**

To answer such types of questions, we will have to learn a method called **unitary method**.

This method requires two steps.

- The value of one unit is evaluated first.
- Then, the value of the required number of units is evaluated.

Let us now view this video and solve the above problem.

Now, while solving the problems based on unitary method, we come across two types of variations:

(a) Direct Variation
(b) Indirect Variation

Let us study about these one by one.

1. **Direct Variation:** Two quantities are said to vary directly, if the increase (or decrease) in one quantity causes increase (or decrease) in the other quantity. For example, more the quantity of sugar, more will be its cost.
2. **Indirect Variation:** Two quantities are said to vary inversely, if the increase (or decrease) in one quantity causes decrease (or increase) in the other quantity. For example, more the duration of a period in a school, less will be the number of periods in the school.

Let us take an example to understand these variations better.

Six men can whitewash a house in 20 days. Calculate the number of men required to whitewash the house in 10 days.
The number of days is decreasing here, so we need to increase the number of men to finish the work on time. When number of days decreases, number of men increases, so this is a case of indirect variation.

Let us find the required number of men now.
From the given condition, we find that the number of men required to whitewash the house in one day is 20×6.

Number of men required to whitewash the house in 10 days = $\frac{20\times 6}{10}$ = 12
Thus, we find that 12 men are required to whitewash the house in 10 days.

Let us take one more example.
Mohan is a taxi driver. He charges Rs 36 as fare for 6 kilometres. Rajesh hired Mohan's taxi for 10 kilometres. How much money will Rajesh pay as fare?
If the distance increases the fare of taxi will also increase, so this is a case of direct

variation.
Let us find the money to be paid as fare by Rajesh.
From the given condition, it is clear that fare for 1 kilometre is Rs $\frac{36}{6}$.

Thus, fare for 10 kilometres = Rs $\frac{36}{6} \times 10$ = Rs 60

Similarly, we can identify the variation in the given conditions and solve the problem.

Let us look at some more examples for a better understanding of this concept.

Example 1:
The cost of 3 shirts is Rs 675. How many shirts can be purchased with Rs 2250?

Solution:

Number of shirts purchased in Rs 675 = 3

Number of shirts purchased in Re 1 = $\frac{3}{675}$

∴ Number of shirts purchased in Rs 2250 = $\frac{3}{675} \times 2250$

$$= \frac{6750}{675}$$
$$= 10$$

Thus, 10 shirts can be purchased with Rs 2250.

Example 2:
Ranveer purchases 5.5 kg apples for Rs 44 and 7 kg mangoes for Rs 52.5. Which of the two fruits costs more per kg?

Solution:
Cost of 5.5 kg apples = Rs 44

Therefore, cost of 1 kg apples = Rs $\frac{44}{5.5}$
= Rs 8

Cost of 7 kg mangoes = Rs 52.5

Therefore, cost of 1 kg mangoes = Rs $\frac{52.5}{7}$
= Rs 7.5

Therefore, the cost of 1 kg apples is more than the cost of 1 kg mangoes.
Hence, apples are costlier than mangoes.

Example 3:
A bus travels 180 km in 6 hours. How much time is required by the bus to travel 225 km?

Solution:
Time required in travelling 180 km = 6 hours

Time required in travelling 1 km $= \frac{6}{180}$ hours

∴ Time required in travelling 225 km $= \frac{6}{180} \times 225$ hours

$= \frac{1350}{180}$ hours

= 7.5 hours

Thus, the time required in travelling 225 km is 7.5 hours i.e., 7 hours 30 minutes.

Example 4:

On a map, the distance between two places A and B is 2.5 cm. However, the actual distance between them is 45 km. If the distance between the other two places X and Y on the same map is 6.5 cm, then find the actual distance between these two places?

Solution:

If the distance on map is 2.5 cm, then actual distance = 45 km

If the distance on map is 1 cm, then actual distance $= \frac{45}{2.5}$ km

Therefore, if the distance on map is 6.5 cm, then actual distance $= \frac{45}{2.5} \times 6.5$ = 117 km

Thus, the actual distance between the places X and Y is 117 km.

Example 5:

A businessman makes a profit of Rs 15400 in 7 days. How much profit will the businessman make in 19 days?

Solution:

In 7 days, businessman makes a profit of Rs 15400.

In 1 day, businessman makes profit = Rs $\frac{15400}{7}$

∴ In 19 days, businessman will make profit = Rs $\frac{15400}{7} \times 19$ = Rs 41800

Thus, in 19 days, the businessman will make a profit of Rs 41800.

Example 6:

If 15 men can do a piece of work in 10 days, then in how many days can 6 men do the same work?

Solution:

This is the case of indirect variation since more the number of men, less will be the number of days required to finish the work.

It is given that 15 men can do the work in 10 days.

∴ One man can do the work in (10 × 15) days.

Hence, 6 men can do the work in $\frac{10 \times 15}{6}$ days = 25 days

Example 7:

A teacher got some number of toffees to distribute amongst the students of her class. If she gives 2 toffees to each student, then all the 45 students of her class get the toffees. Find the number of students who will get the toffees, if the teacher distributes 3 toffees to each student.

Solution:

This is the case of indirect variation since more the number of students, less will be the number of toffees each student will get.

It is given that 2 toffees were distributed to each of the 45 students.
∴ One toffee will be distributed to (45 × 2) students.

∴ Three toffees will be distributed to $\frac{45\times 2}{3}$ students = 30 students

Thus, if the teacher distributes 3 toffees to each student, then the number of students who will get the toffees is 30.

Strengthen this topicTAKE A TOPIC TEST

SCROLL DOWN FOR THE NEXT TOPIC

Solving Problems Based on Time and Work

Consider a situation. Raj is a carpenter. He can make a computer table in 7 days.

Can we find out Raj's one day's work?

We can find it out by using a formula which is given below.

day's work = $\frac{1}{\text{Number of days to complete the work}}$

From the above formula we observe that:

ber of days to complete the work = $\frac{1}{\text{One day's work}}$

So, we can now find Raj's one day's work by using the above formula.

We get that Raj's one day's work $=\frac{1}{7}$

Let us now go through the following video to understand how to solve problems related to time and work.

Let's now look at some more examples to understand this concept better.

Example 1:
A and B can separately complete a work in 48 days and 96 days respectively. They both worked for 16 days, then B took leave for 12 days. During this period A

alone did the work. After B rejoined, A left the job. In how many days will B finish the work?

Solution:

A can complete the work in 48 days.

His/her one day's work $= \frac{1}{48}$

B can complete the work in 96 days.

His/her one day's work $= \frac{1}{96}$

Working together, their one day's work $= \frac{1}{48} + \frac{1}{96} = \frac{2+1}{96} = \frac{3}{96} = \frac{1}{32}$

Working together, their 16 days' work $= 16 \times \frac{1}{32} = \frac{1}{2}$

Work done by A (alone) when B was on leave for 12 days $= \frac{12}{48} = \frac{1}{4}$

So, the work remaining after deducting the joint effort of A and B for 16 days and the work done by A alone for 12 days

$$= 1 - \left(\frac{1}{2} + \frac{1}{4}\right) = 1 - \left(\frac{2+1}{4}\right) = \frac{4-3}{4} = \frac{1}{4}$$

So, time taken by B to complete the remaining work

$$= \frac{\text{Work to be done}}{\text{B's one day's work}}$$

$$= \left(\frac{\frac{1}{4}}{\frac{1}{96}}\right) \text{days}$$

$$= 24 \text{ days}$$

Example 2:

A, B and C together can complete a work in 16 days, where A and C can separately complete the work in 60 days and 80 days respectively. If A and B together did the work for 10 days and the remaining work was done by C, then in how many did C complete the work?

Solution:

A can complete the work in 60 days.

So, his/her one day's work $= \frac{1}{60}$

C can complete the work in 80 days.

So, his/her one day's work $= \frac{1}{80}$

So, one day's work done by A and C $= \frac{1}{60} + \frac{1}{80} = \frac{4+3}{240} = \frac{7}{240}$

It is given that A, B and C can complete the work in 16 days.

So, one day's work done by A, B and C $= \frac{1}{16}$

∴ One day's work of B $= \frac{1}{16} - \frac{7}{240} = \frac{15-7}{240} = \frac{8}{240} = \frac{1}{30}$

This means C can complete the work in 30 days.

Work done by A and B in 1 day $= \frac{1}{60} + \frac{1}{30} = \frac{1+2}{60} = \frac{3}{60} = \frac{1}{20}$

Work done by A and B in 10 days $= 10 \times \frac{1}{20} = \frac{1}{2}$

Remaining work to be done to complete the work $= 1 - \frac{1}{2} = \frac{1}{2}$

So, time taken by C to complete the remaining work $= \left(\dfrac{\frac{1}{2}}{\frac{1}{30}} \right)$ days

$= \left(\frac{1}{2} \times 30 \right) \text{days} = 15\,\text{days}$

Example 3:

Nitika and Ruchika together can type 100 pages in 20 hours. Nitika alone can type 100 pages in 25 hours. How much time would Ruchika take to type 100 pages alone?

Solution:

Nitika and Ruchika together type 100 pages in 20 hours.

∴One hour's work done by Nitika and Ruchika together $= \frac{1}{20}$ hours

Now, Nitika alone can type 100 pages in 25 hours.

∴Nitika's one hour's work $= \frac{1}{25}$ hours

∴Ruchika one hour's work $= \frac{1}{20} - \frac{1}{25}$

$= \frac{5-4}{100} = \frac{1}{100}$

Thus, Ruchika alone would type 100 pages in 100 hours.

Example 4:

A can mend a wall in 14 days. B can mend the wall in 7 days. B started the work and worked for 4 days and then left the work. Find the number of days that A will take to complete the remaining work.

Solution:

It is given that A can mend a wall in 14 days.

∴A's one day's work = $\frac{1}{14}$

B can mend the wall in 7 days.

∴B's one day's work $=\frac{1}{7}$

∴B's four day's work $=\frac{1}{7}\times 4=\frac{4}{7}$

Work left after B has worked for 4 days $=1-\frac{4}{7}=\frac{3}{7}$

Remaining work $=\frac{3}{7}$

∴Number of days taken by A to complete the work

$$=\frac{\text{Work to be done}}{\text{A's one day's work}}$$
$$=\frac{\frac{3}{7}}{\frac{1}{14}}$$
$$=\frac{3}{7}\times\frac{14}{1}=6$$

Hence, A will take 6 days to complete the remaining work.

Strengthen this topicTAKE A TOPIC TEST

SCROLL DOWN FOR THE NEXT TOPIC

Solving Problems Based on Time, Speed and Distance

A cyclist covers a journey of 100 km on bicycle in 5 hours.

Can we find the speed of the cyclist?

Let us go through the given video to understand what we actually mean by the word "speed" and how to find the speed of the cyclist.

If an object covers equal distances in equal intervals of time, then its speed is said to be uniform (or constant), otherwise its speed is said to be variable.

Now, let us again go back and consider the speed of the cyclist, which we had calculated.

Speed = 20 km/hour

Can we also find the speed of cyclist in terms of the unit m/sec?

The answer to this question is 'yes'. Let us see how we do it.

We know that 1 km = 1000 m and 1 hour = 60 min = (60 × 60) sec

Speed = 20 km/hour $= \frac{20\,\text{km}}{1\,\text{hour}} = \frac{20 \times 1000\,\text{m}}{60 \times 60\ \text{sec}} = \frac{50}{9}\,\text{m/sec}$

Thus, the speed of the cyclist can also be written as $\frac{50}{9}\,\text{m/sec}$.

Therefore, we conclude that:

1 km/hour $= \frac{\textbf{1000 metres}}{(\textbf{60} \times \textbf{60})\ \textbf{seconds}} = \frac{\textbf{5}}{\textbf{18}}\,\textbf{m/sec}$

In the same way, if the speed is given in terms of m/sec, then we can convert it into km/hour.

1 m/sec $= \frac{\textbf{1 metre}}{\textbf{1 second}} = \frac{\frac{1}{1000}\ \textbf{km}}{\frac{1}{60 \times 60}\ \textbf{hour}} = \frac{\textbf{18}}{\textbf{5}}\,\textbf{km/hour}$

For example: speed = 25 m/sec

$$= 25 \times \frac{18}{5}\,\text{km/hour}$$
$$= 80\,\text{km/hour}$$

Let us now look at some examples to understand the concept better.

Example 1:

Sonu walks 500 metres in 5 minutes. Find his speed in km/hour as well as in m/sec.

Solution:

Distance travelled by Sonu in 5 minutes = 500 m

$$= \frac{500}{1000}\,\text{km}$$
$$= \frac{1}{2}$$

Time = 5 minutes $= \frac{5}{60}\,\text{hour} = \frac{1}{12}\,\text{hour}$

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{\frac{1}{2}\,\text{km}}{\frac{1}{12}\,\text{hour}} = 6\,\text{km/hour}$$

To convert it into m/sec:

Speed = 6 km/hour

$$= 6 \times \left(\frac{1000}{3600}\right)\ \text{m/sec}$$

[1 km = 1000 metres, 1 hour = 3600 sec]

$$= 6 \times \left(\frac{5}{18}\right)\text{m/sec} = \frac{5}{3}\,\text{m/sec}$$

Example 2:

Aditi travelled certain distance by bus from her office to her house at the rate of 25 km/hour and came back in the bus at the rate of 4 km/hour. The whole journey took $5\frac{4}{5}$ hours. Find the distance between Aditi's office and her house.

Solution:

We know that, $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\therefore \text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

Let the distance be *x*.

Total time $= \dfrac{x}{25} + \dfrac{x}{4}$

$= \dfrac{4x + 25x}{100} = \dfrac{29x}{100}$

However, it is given that the total time is $5\dfrac{4}{5}$ hours.

∴We have,

$$\dfrac{29x}{100} = 5\dfrac{4}{5} = \dfrac{29}{5}$$
$$\Rightarrow x = \dfrac{29}{5} \times \dfrac{100}{29} = 20$$

Thus, the distance between Aditi's house and her office is 20 km.

Example 3:

A car covers a distance of 3 km in 10 minutes. How much time will it take to travel 24 km?

Solution:

We know that, $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$$\text{Speed} = \dfrac{3 \text{ km}}{10 \text{ minute}} = \dfrac{3}{10} \text{km/min}$$

Now, distance = 24 km

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{24\,\text{km}}{\frac{3}{10}\,\text{km/min}}$$

$$= \frac{24 \times 10}{3}\,\text{min}$$

= 80 minutes

= 1 hour 20 minutes ($\because$ 1 hour = 60 minutes)

Thus, the time taken by the car to travel 24 km is 1 hour 20 minutes.