Playing with Numbers

Finding Factors and Multiples of Numbers

Can we exactly divide 26 by 2?

Yes, we can.

Can we exactly divide 26 by 3?

No.

Now, can we say that 2 is a factor of 26 while 3 is not?

Let us find out. Let us first understand the meaning of the term “factor” of a number.

The term “factor” can be defined as follows:

An exact divisor of a number is called a factor of that number.

In the above example, since 2 exactly divides 26 i.e., 2 is an exact divisor of 26, we can say that 2 is a factor of 26. On the other hand, since 3 is not an exact divisor of 26, we can say that 3 is not a factor of 26.

To understand the concept better, look at the following video.

Till now, we were talking about the exact divisors of the numbers (i.e., factors). Let us now look at this from a different point of view.

Let us consider the relation, $24 = 4 \times 6$

We already know that 4 and 6 are the factors of 24. Apart from this, we can also say that 24 is a multiple of both 4 and 6.

Let us now look at some examples to understand the method of finding factors and multiples better.

Example 1:

Write all the factors of the following numbers.

(a) 8 (b) 18 (c) 64

Solution:

(a) We can write 8 as

1 × 8 = 8

2 × 4 = 8

Therefore, the factors of 8 are 1, 2, 4, and 8.

(b) We can write 18 as

1 × 18 = 18

2 × 9 = 18

3 × 6 = 18

Therefore, the factors of 18 are 1, 2, 3, 6, 9, and 18.

(c) We can write 64 as

1 × 64 = 64

2 × 32 = 64

4 × 16 = 64

8 × 8 = 64

Therefore, the factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

Example 2:

Write the first four multiples of the following numbers.

(a) 6 (b) 9

Solution:

(a) The first four multiples of 6 can be calculated as

6 × 1 = 6

6 × 2 = 12

6 × 3 = 18

6 × 4 = 24

Therefore, the first four multiples of 6 are 6, 12, 18, and 24.

(b) The first four multiples of 9 can be calculated as

9 × 1 = 9

9 × 2 = 18

9 × 3 = 27

9 × 4 = 36

Therefore, the first four multiples of 9 are 9, 18, 27, and 36.

Prime and Composite Numbers

Consider the following set of numbers.

2, 3, 5, 7, 11, 13, 17, 19…

Is there any similarity between these numbers?

Each number in the above set of numbers is divisible by 1 and the number itself, or we can say that each number in the above set has only two divisors i.e., 1 and the number itself. Such numbers are called **prime numbers.**

Let us learn more about prime numbers.

A prime number can be written as a product of only two factors. These factors are called **prime factors.**

For example, 23 = 1 × 23

Hence, 1 and 23 are the prime factors of 23.

There are two kinds of prime numbers.

(1) Even prime numbers

(2) Odd prime numbers

Prime numbers which are divisible by 2 are known as **even prime numbers** and those which are not divisible by 2 are known as **odd prime numbers.**

Examples of odd prime numbers are 3, 5, 7 etc.

It is to be **noted** that 2 is the only even prime number.

Now, consider the numbers other than 1 and prime numbers as follows:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18…

All these numbers have more than two divisors. Such numbers are called **composite numbers**.

Like prime numbers, composite numbers are also of two types.

(1) Even composite numbers

(2) Odd composite numbers

Composite numbers which are divisible by 2 are **even composite numbers** and those which are not divisible by 2 are **odd composite numbers.**

For example, 6, 24, 36, 42, etc. are even composite numbers while 9, 25, 39, etc. are odd composite numbers.

Let us go through the following video to have a better understanding about prime and composite numbers.

Note that 3 is the smallest odd prime number.

Twin primes:

If a pair of prime numbers is such that the difference between those numbers is 2 then such prime numbers are known as **twin primes** and the pair is called **twin prime pair**.

For example, the numbers 3 and 5, both are prime numbers and the difference between them is 2, so these are twin primes and make a twin prime pair.

Similarly, few more twin primes are: 5 and 7; 11 and 13; 17 and 19; 29 and 31; 41 and 43; 101 and 103.

There are many more such twin primes.

Sieve of Eratosthenes:

This is a method used for finding the prime and composite numbers from 1 to 100 which was given by the Greek Mathematician **Eratosthenes**, in the third century B.C. Follow the steps given below:

Step 1: Prepare a table of natural numbers from 1 to 100 by taking ten numbers in each row.

Step 2: Cross out 1 because it is not a prime number.

Step 3: Encircle 2 as it is prime number and cross out every multiple of 2.

Step 4: Encircle another prime number which is 3 and cross out every multiple of 3. Crossed numbers are not required to be marked again.

Step 5: Encircle 5 as a prime number and cross out every multiple of 5. Crossed numbers are not required to be marked again.

Step 6: Continue this process till all the numbers from 1 to 100 are encircled or crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~22~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

This method is called the **Sieve of Eratosthenes**.

Let us now have a look at some examples to understand the concept better.

Example 1:

Find the prime numbers among the following numbers.

(i) 2 (ii) 15 (iii) 17 (iv) 27

Solution:

(i) The divisors of 2 are 1 and 2 i.e., 1 and the number 2 itself. Therefore, 2 is a prime number.

(ii) The divisors of 15 are 1, 3, 5, and 15 i.e., 15 has more than two factors. Therefore, 15 is not a prime number.

(iii) The divisors of 17 are 1 and 17. Therefore, 17 is a prime number.

(iv) The divisors of 27 are 1, 3, 9, and 27. Therefore, 27 is not a prime number.

Example 2:

Find an even prime number which is

(i) greater than 1

(ii) less than 100

Solution:

We know that 2 is the only even prime number. Therefore,

(i) A prime number greater than 1 is 2.

(ii) A prime number less than 100 is 2.

Example 3:

Find odd prime numbers less than 50.

Solution:

The odd prime numbers less than 50 are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Example 4:

Find even and odd composite numbers greater than 20 and less than 40.

Solution:

The even composite numbers greater than 20 and less than 40 are 22, 24, 26, 28, 30, 32, 34, 36, and 38.

The odd composite numbers greater than 20 and less than 40 are 21, 25, 27, 33, 35, and 39.

Example 5:

Express 27 and 34 as the sum of three prime numbers.

Solution:

27 can be expressed as follows:

27 = 3 + 11 + 13

And, 34 can be expressed as follows:

34 = 2 + 13 + 19

Example 6:

Write all twin primes lying between 100 and 200.

Solution: Twin primes lying between 100 and 200 are as follows:

101 and 103; 107 and 109; 137 and 139: 149 and 151; 179 and 181; 191 and 193; 197 and 199.

Divisibility of a Number by 2, 4 and 8

Ashok was standing with his friend Nikhil at a bus stop. The first bus that came was of route 432. Ashok looked at it and said to Nikhil that the route number of the bus was divisible by 2. Nikhil wanted to verify what Ashok had said and started calculating in his notebook. This is what he did:

```
   216
2)432
  4
  -
   3
   2
   -
   12
   12
   --
    ×
```

Although he verified what his friend had said, Nikhil was amazed at Ashok's quick calculation. He asked Ashok how he did it.

Ashok said he used a trick to check the divisibility of 432 by 2. He also told Nikhil that he knows divisibility rules for 4 and 8 also.

Let us go through the following video to know these divisibility rules.

Note that **a number is also divisible by 4 if the number formed by its last two digits is 00.**

A number is also divisible by 8, if the number formed by its last three digits is 000.

For example, in the number 200, the number formed by the last two digits is 00. Therefore, 200 is divisible by 4. In the number 2000, the number formed by its last three digits is 000. Therefore, 2000 is divisible by 8.

Now, we do not have to divide a number by 2, 4, and 8 to check whether it is divisible by them or not. We can use the above stated divisibility rules.

Let us now look at some more examples to understand this concept better.

Example 1:

Which of the following numbers are divisible by 2?

(i) 48 (ii) 97 (iii) 345 (iv) 8460

Solution:

(i) The number 48 has 8 at its ones place, which is even. Therefore, 48 is divisible by 2.

(ii) The number 97 has 7 at its ones place, which is odd. Therefore, 97 is not divisible by 2.

(iii) The number 345 has 5 at its ones place, which is odd. Therefore, 345 is not divisible by 2.

(iv) The number 8460 has 0 at its ones place. Therefore, 8460 is divisible by 2.

Example 2:

Which of the following numbers are divisible by 4?

(i) 2348 (ii) 326

Solution:

(i) The given number is 2348.

The number formed by its last two digits is 48, which is divisible by 4.

Thus, the number 2348 is divisible by 4.

(ii) The given number is 326.

The number formed by its last two digits is 26, which is not divisible by 4.

Thus, the number 326 is not divisible by 4.

Example 3:

Find whether 56112 is divisible by 8 or not.

Solution:

The given number is 56112.

The number formed by its last three digits is 112, which is divisible by 8.

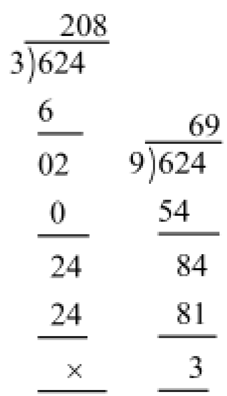
Thus, the given number is divisible by 8.

Divisibility of a Number by 3 and 9

Imagine if you had to check if numbers such as 27, 18, or 33 were divisible by 3 and 9. You will obviously check it mentally and give the answer without performing any long division. But what if you are asked to check if the number 123456789 is divisible by 3 and 9? You will take a lot of time calculating the divisibility of the number with both 3 and 9 before you give the answer.

A teacher asked the students in a class to check whether 624 is divisible by 3 and 9. Mayank said that 624 is divisible by 3 but not divisible by 9 by just looking at the number. Shikhar, another student, started performing the divisions.

This is what Shikhar did.



Although Shikhar and Mayank gave the same answer, Mayank took very little time to give the answer, while Shikhar had to perform the division. Do you know which trick Mayank used to give such a quick answer?

Mayank used two rules of divisibility to get the answer.

Let us go through the following video to learn the rules.

It can be observed that a number which is divisible by 9 is also divisible by 3.

For example, consider the number 252. The sum of its digits is 9, which is divisible by 3 and 9 both.

Let us discuss a few more examples to understand the concept better.

Example 1:

Which of the following numbers are divisible by 3?

(i) 163 (ii) 276

Solution:

(i) Sum of the digits of the number 163 = 1 + 6 + 3 = 10

Here, the sum of the digits (i.e., 10) is not divisible by 3. Therefore, 163 is not divisible by 3.

(ii) Sum of the digits of the number 276 = 2 + 7 + 6 = 15

Here, the sum of the digits (i.e., 15) is divisible by 3. Therefore, 276 is divisible by 3.

Example 2:

Find whether 477918 is divisible by 9 or not.

Solution:

The given number is 477918.

The sum of its digits is 4 + 7 + 7 + 9 + 1 + 8 = 36, which is divisible by 9.

Therefore, the given number is divisible by 9.

Example 3:

Fill in the blank space in the number 6587_41 so that it is divisible by 3.

Solution:

The given number is 6587_41.

The sum of the given digits is 6 + 5 + 8 + 7 + 4 + 1 = 31

We know that a number is divisible by 3 if the sum of its digits is also divisible by 3.

Therefore, the unknown digit can be 2 or 5 or 8 as then, the sum of the digits will be 33 or 36 or 39, and each of these sums is divisible by 3.

Example 4:

Check the divisibility of 388 by 3 and 9.

Solution:

Sum of the digits of the number 388 = 3 + 8 + 8 = 19

Here, the sum of the digits is not divisible by either 3 or 9.

Hence, 388 is not divisible by 3 or 9.

Example 5:

If x is a digit, then what are its possible values if the number 3x38 is divisible by 3?

Solution:

We know that x is a digit (i.e., x is a number between 0 and 9).

We also know that the number 3x38 is divisible by 3.

Thus, the sum of its digits will also be divisible by 3. [Divisibility rule of 3]

Hence, $3 + x + 3 + 8 = 14 + x$ is also divisible by 3.

The numbers larger than 14 and divisible by 3 are

15, 18, 21, and 24

To find the least value of x,

$14 + x = 15$

$x = 15 - 14$

$\therefore x = 1$

Similarly, $14 + x = 18$

$x = 18 - 14$

$\therefore x = 4$

Similarly, $14 + x = 21$

$x = 21 - 14$

$\therefore x = 7$

Similarly, $14 + x = 24$

$x = 24 - 14$

$\therefore x = 10$.

This is a two-digit number.

Hence, x cannot take this value.

Hence, the values of x are 1, 4, and 7 such that $3x38$ is divisible by 3.

Example 6:

The number 10*x*8 is divisible by 9. If *x* is a digit, then find its possible values.

Solution:

$10x8$ is divisible by 9.

Hence, the sum of its digits is also divisible by 9.

$\therefore 1 + 0 + x + 8 = 9 + x$ is divisible by 9

$\therefore 9 + x = 9$ or 18 or 27 ….

If we take

$9 + x = 9$

Then, $x = 0$

And, if we take

$9 + x = 18$

Then, $x = 9$

Here, we cannot take 9 + x = 27 because it gives x = 18, which is a two-digit number.

Hence, the values of x are 0 or 9.

Example 7:

How many numbers from 130 to 200 are divisible by 5 but not by 3?

Solution:

A number is divisible by 3 if and only if the sum of its digits is divisible by 3 whereas a number is divisible by 5 if and only if it ends with 0 or 5.
The numbers from 130 to 200 which are divisible by 5 are 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195 and 200. Now, the sum of digits of these numbers is 4, 9, 5, 10, 6, 11, 7, 12, 8, 13, 9, 14, 10, 15 and 2 respectively. Among these sum of digits, only 6, 9, 12 and 15 are divisible by 3.
Thus, the numbers which are divisible by 5 but not by 3 are 130, 140, 145, 155, 160, 170, 175, 185, 190 and 200.

Hence, there are 10 such number.

Divisibility of a Number by 6

Consider the number 12.

Is it divisible by 6?

Yes, it is divisible by 6. Also note that the number 12 is divisible by both 2 and 3.

Consider some more numbers.

18, 42, 54, 72, 96, 264

Are these numbers divisible by 6?

On dividing by 6, all these numbers give remainder 0. Therefore, all the above numbers are divisible by 6.

Also note that the digit at ones place in each of the above numbers is even. Therefore, each of the above numbers is divisible by 2.

Also the sum of the digits in each of the above numbers is divisible by 3. Therefore, each of the above numbers is divisible by 3.

Now, can you think of any number which is divisible by 6 but not divisible by 2 or 3?

If you think of any number which is divisible by 6, then it will definitely be divisible by both 2 and 3. The reason behind it is that 6 is a multiple of both 2 and 3.

Thus, we can say that any number which is divisible by 6 is also divisible by both 2 and 3.

Using the above fact, we can check the divisibility of a number by 6. Let us state it as a rule to check the divisibility of a number by 6 as follows.

"A number is divisible by 6, if it is divisible by both 2 and 3".

For example, 756, 78, 696, 858, etc. are divisible by 6.

In the number 756, the digit at ones place is 6, which is even. Therefore, the number 756 is divisible by 2. Also the sum of the digits of 756 is 18, which is divisible by 3. Therefore, the number 756 is divisible by 3. Using the above divisibility rule, we can say that 756 is divisible by 6.

In the same way, 78, 696, 858 are divisible by 6.

Let us consider some more examples to understand the concept better.

Example 1:

Which of the following numbers are divisible by 6?

(i) 162 (ii) 286

Solution:

(i) The given number is 162.

We know that a number is divisible by 6 if it is divisible by both 2 and 3.

The digit at ones place is 2, which is even. Therefore, the number 162 is divisible

by 2.

Now, the sum of the digits of the number 162 is 1 + 6 + 2 = 9, which is divisible by 3. Therefore, 162 is divisible by 3.

Thus, the number 162 is divisible by 6.

(ii) The given number is 286.

We know that a number is divisible by 6 if it is divisible by both 2 and 3.

It has 6 at its ones place, which is even. Therefore, 286 is divisible by 2.

And the sum of its digits is 16, which is not divisible by 3. Therefore, 286 is not divisible by 3.

Thus, the number is not divisible by both 2 and 3 and hence, not divisible by 6.

Example 2:

Is the number 174 divisible by both 3 and 6?

Solution:

The given number is 174.

The sum of its digits is 1 + 7 + 4 = 12, which is divisible by 3.

Thus, the number is divisible by 3.

The number has digit 4 at ones place, which is even.

Thus, the number is divisible by 2 as well.

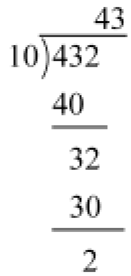
Since the number is divisible by 2 and 3 both, it is divisible by 6.

Thus, the number 174 is divisible by both 3 and 6.

Divisibility of a Number by 5 and 10

Hotel Grand Mansion has 432 rooms and 10 floors. Surbhi, who went to the hotel for the first time with her sister Shweta, was quite amazed by the numbers. She asked Shweta if she could tell whether each of the floors had the same number of rooms. Shweta replied that this is not possible as 432 is not exactly divisible by 10.

Surbhi wanted to check if what Shweta said was true. She quickly divided the numbers as



She found that since the remainder is 2, 432 is not exactly divisible by 10.

Shweta had also said that the number 432 is not divisible by 10.

Do you want to know which trick Shweta used to give such a quick answer?

There is a rule to check if a number is divisible by 10. Similarly, we have a rule to check if a number is divisible by 5 or not. We can save the time used in division by applying this rule.

Go through the following video to learn the rules of divisibility by 5 and 10.

Let us consider a few examples to understand the concept better.

Example 1:

Use divisibility test to find which of the following numbers are divisible by 10.

(i) 11557468 (ii) 98746590 (iii) 95460 (iv) 74684255

Solution:

(i) 11557468 has 8 at its ones place. Therefore, it is not divisible by 10.

(ii) 98746590 has 0 at its ones place. Therefore, it is divisible by 10.

(iii) 95460 has 0 at its ones place. Therefore, it is divisible by 10.

(iv) 74684255 has 5 at its ones place. Therefore, it is not divisible by 10.

Example 2:

Which of the following numbers are divisible by 5 and 10 both?

(i) 8974 (ii) 5540 (iii) 58790 (iv) 57875

Solution:

(i) 8974 has 4 at its ones place. Therefore, it is neither divisible by 5 nor by 10.

(ii) 5540 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iii) 58790 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iv) 57875 has 5 at its ones place. Therefore, it is divisible by 5 but not by 10.

Example 3:

Find the value of *x* and *y* such that 98*x*2*y* is divisible by 10.

Solution:

The given number is $98x2y$.

We know that a number is divisible by 10 if the digit at its units place is 0. Hence, $y = 0$.

According to the rule of divisibility by 10, the digit at the units place of a number must be 0, while the rest of the digits can take any value from 0 to 9. Hence, x may take any value from 0 to 9.

Example 4:

Find the least number which, when

(i) subtracted from 624, gives a number divisible by 10

(ii) added to 624, gives a number divisible by 10

Solution:

(i) The given number is 624.

The number less than 624, which is divisible by 10 (which has a zero at its units place), is 620.

We can see that 624 – 620 = 4.

Hence, 4 is subtracted from 624 to get a number that is divisible by 10.

(ii) Similarly, the number larger than 624, which is divisible by 10 (which has a zero at its units place), is 630.

We can see that 630 – 624 = 6.

Hence, we need to add 6 to 624 to get a number that is divisible by 10.

Example 5:

How many numbers from 201 to 300 are divisible by 5?
Solution:
A number is divisible by 5 if and only if it ends with 0 or 5.
Now, the numbers from 201 to 300 which ends with 0 or 5 are 205, 210, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260, 265, 270, 275, 280, 285, 290, 295 and 300.
Thus, there are 20 such numbers.

Divisibility of a Number by 11

Consider the number 55. Is it divisible by 11?

Yes it is.

Now, consider the number 121. Is it divisible by 11?

Yes it is.

Is the number 868857 divisible by 11?

Now, if we check it by division method, then it will take a long time. Therefore, here we shall learn an easy rule to check whether a number is divisible by 11 or not.

Let us know this rule first and then we shall check the divisibility of this number by 11.

Divisibility of a number by 11 can be checked as follows:

If the difference between the sum of the digits at odd places from the right and the sum of the digits at even places from the right of a number is either 0 or a multiple of 11, then the number is divisible by 11.

Thus, to check the divisibility of a number by 11, we are not required to divide the number by 11; instead, we can use the above mentioned rule.

Let us now have a look at this video and find out whether the number 868857 is divisible by 11 or not.

Let's consider the number 656. If we reverse the digits of this number, then we again get back the same number. Such numbers are palindrome numbers. So, a palindrome number is a number which reads the same from left to right or right to left.

For example, 121 is a 3-digit palindrome number; 2332 is 4-digit palindrome number.

A four-digit palindrome number is always divisible by 11.

Let us now look at a few more examples.

Example 1:

Which of the following numbers is divisible by 11?

(a) 286935

(b) 897562

Solution:

(a) The given number is 286935.

Counting from right, the digits occupying odd places in the number are 5, 9, and 8.

Sum of these digits = 5 + 9 + 8 = 22

Similarly, counting from right, the digits occupying even places in the number are 3, 6, and 2.

Sum of these digits = 3 + 6 + 2 = 11

Thus, difference between the two sums = 22 – 11 = 11

This difference is a multiple of 11. Therefore, the given number is divisible by 11.

(b) The given number is 897562.

Counting from right, the digits occupying odd places in the number are 2, 5, and 9.

Sum of these digits = 2 + 5 + 9 = 16

Similarly, counting from right, the digits occupying even places in the number are 6, 7, and 8.

Sum of these digits = 6 + 7 + 8 = 21

Thus, difference between the two sums = 21 – 16 = 5

This difference is not a multiple of 11. Therefore, the given number is not divisible by 11.

Example 2:

Is 829653 divisible by 11?

Solution:

The given number is 829653.

Counting from right, the digits occupying odd places in the number are 3, 6, and 2.

Sum of these digits = 3 + 6 + 2 = 11

Similarly, counting from right, the digits occupying even places in the number are 5, 9, and 8.

Sum of these digits = 5 + 9 + 8 = 22

Thus, difference between the two sums = 22 – 11 = 11

This difference is a multiple of 11. Therefore, the given number is divisible by 11.

Prime Factorisation of Numbers

Consider the relation, 56 = 7 × 8

We can write 56 as a product of 7 and 8, and also note that 7 and 8 are the factors of 56.

When we write a number as the product of its factors, it is called **factorization.**

Thus, factorization of a number can be defined as follows:

The process of expressing a number as a product of its factors is called factorization.

The above way of writing the number 56 is not the only way.

We can also factorize 56 as $56 = 2 \times 4 \times 7$ or $56 = 2 \times 2 \times 2 \times 7$

These processes are also called factorization.

Watch this video to learn how we can factorize 24.

Thus, prime factorization of a number can be defined as follows:

If a number is expressed as a product of its factors, all of which are prime numbers, then such factorization is known as prime factorization.

For example: Let us write the prime factorization of the number 420.

$420 = 10 \times 42$

$= (2 \times 5) \times (6 \times 7)$

$= 2 \times 5 \times (2 \times 3) \times 7$

$= 2 \times 2 \times 3 \times 5 \times 7$

Here, 2, 3, 5, and 7 are prime numbers and we cannot further factorize this number.

Thus, the prime factorization of 420 is $2 \times 2 \times 3 \times 5 \times 7$.

There are two ways of expressing a number as a product of its prime factors – one is by drawing a **factor tree** and the other is by using the method of **successive division.**

Let us write the prime factorization of the number 56 with the help of a factor tree.
Watch the following video to learn more.

Let us now write the prime factorization of 56 using successive division method. In this method, the number is successively divided by prime numbers, till we obtain 1 as the result. In this method, it is best to start with the smallest prime number i.e., 2.

This process can be shown as follows:

2	56
2	28
2	14
7	7
	1

Thus, by the method of successive division also, we obtain the prime factorization of 56 as 2 × 2 × 2 × 7.

Let us now look at some examples to understand this concept better.

Example 1:

Write the prime factorization of the number 1050 by successive division method.

Solution:

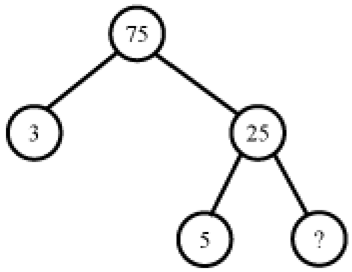
The prime factorization of the number 1050 by successive division method is shown below:

2	1050
3	525
5	175
5	35
7	7
	1

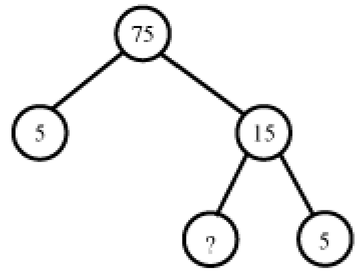
Thus, the prime factorization of 1050 is 2 × 3 × 5 × 5 × 7.

Example 2:

Find the missing values in the following factor trees.



(i)



(ii)

Solution:

(i) 25 can be factorized as 5 × 5. Thus, the missing number is 5.

(ii) 15 can be factorized as 3 × 5. Thus, the missing number is 3.

Example 3:

Find all the prime factors of the smallest 4-digit number.

Solution:

The smallest 4-digit number is 1000.

The prime factorization of the number 1000 by successive division method is given as follows:

2	1000
2	500
2	250
5	125
5	25
5	5
	1

Thus, the prime factorization of 1000 is 2 × 2 × 2 × 5 × 5 × 5.

Example 4:

Find the smallest number having three different prime factors.

Solution:

We have to find the smallest number having three different prime factors.

The three smallest prime numbers are 2, 3, and 5.

Thus, the smallest number having three different prime factors is the product of 2, 3, and 5.

Thus, the required number is 2 × 3 × 5 i.e., 30.

Finding HCF and LCM of Numbers

The concept of highest common factor is derived from common factors and involves finding of the highest number among the common factors of given two or more numbers. Highest common factor (HCF) is also known as greatest common divisor (GCD).

We can find the HCF of given numbers by any of the two methods- **prime factorisation method or common division method.**

We have discussed about highest common factor. In the same way, can we find the highest common multiple of the given numbers?

We know that any given numbers can have infinite common multiples, so we cannot find the highest common multiple but we can find the lowest common multiple (LCM) of the given numbers.

There are two methods of finding the LCM of given numbers also. They are prime factorization method and common division method

Properties of HCF and LCM

(i) HCF of two or more co-prime numbers is 1.
(ii) LCM of co-prime numbers is equal to the product of the co-primes.
(iii) LCM is a multiple of HCF.
(iv) HCF of two or more prime numbers is 1.

Relationship between HCF and LCM

The relation between the HCF and LCM of two given numbers is given by:

Product of LCM and HCF of two numbers = Product of the two numbers

Let us now look at some more examples to understand the concept of HCF and LCM better.

Example 1:

Find the HCF of 90, 108, and 180 using prime factorization method.

Solution:

2	90
3	45
3	15
5	5
	1
2	108
2	54
3	27
3	9
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

Therefore, 90 = 2 × 3 × 3 × 5, 108 = 2× 2 × 3 × 3 × 3, 180 = 2 × 2 × 3 × 3 × 5

Now, the common factors of the given numbers are 2 (occurring once) and 3 (occurring twice).

Therefore, the HCF of the given numbers is 2 × 3 × 3 = 18

Example 2:

Find the LCM of 9, 15, and 27.

Solution:

3	9, 15, 27
3	3, 5, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

Therefore, the LCM of 9, 15, and 27 is 3 × 3 × 3 × 5 = 135

Example 3:

Isha has 32 apples and 48 mangoes. She wants to keep these fruits in separate baskets such that there will be equal number of fruits in each basket and the two types of fruits are not mixed in any basket. Find the greatest number of fruits to be kept in each basket.

Solution:

The number of fruits to be kept in each basket must be an exact divisor of both the numbers 32 and 48.

Also, the number of fruits in each basket should be the greatest. Therefore, the number of fruits to be kept in each basket is the HCF of 32 and 48.

The prime factorization of 32 and 48 is

32 = $\underline{2 \times 2 \times 2 \times 2} \times 2$

48 = $\underline{2 \times 2 \times 2 \times 2} \times 3$

Now, the common factor of 32 and 48 is 2 (occurring four times).

Therefore, the HCF of 32 and 48 is 2 × 2 × 2 × 2 = 16

Thus, the greatest number of fruits to be kept in each basket is 16.

Example 4:

Samay and Sumit start running together around a circular park from the same point at the same time. If Samay completes one round of the park in 6 minutes and

Sumit completes one round of the park in 9 minutes, then after what time

will Samay and Sumit meet at the starting point?

Solution:

The required time would be the common multiple of 6 minutes and 9 minutes. Also, the time should be the least time. Therefore, the required time is the LCM of 6 minutes and 9 minutes.

2	6, 9
3	3, 9
3	1, 3
	1, 1

The LCM of 6 and 9 is 2 × 3 × 3 = 18

Thus, Samay and Sumit will meet at the starting point after 18 minutes.

Example 5:

The HCF and LCM of two numbers are 6 and 900 respectively. One of the two numbers is 150. Find the other number.

Solution:

HCF = 6

LCM = 900

We know that,

HCF × LCM = product of numbers

6 × 900 = 150 × required number

Required number = $\frac{5400}{150} = 36$