31. Mathematical Reasoning

Exercise 31.1

1 A. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Listen to me, Ravi!

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The sentence “Listen to me, Ravi ! “ is an exclamatory sentence.

Hence, it is not a statement.

1 B. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Every set is a finite set.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

So, This sentence is always false, because there are sets which are not finite.

Hence, it is a statement.

1 C. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Two non-empty sets have always a non-empty intersection.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

So, This sentence is always false, because there are non-empty sets whose intersection is empty.

Hence, it is a statement.

1 D. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

The cat pussy is black.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

Here, Some cats are black, and some cat is not black, So, the given sentence may or may not be true.

Hence, it is not a statement.

1 E. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Are all circles round?

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

Here, The given sentence is an interrogative sentence.

Hence, it is not a statement.

1 F. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

All triangles have three sides.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is a true declarative sentence.

Hence, it is a true statement.

1 G. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Every rhombus is a square.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

So, This sentence is always false, because Rhombuses are not a square.

Hence, it is a statement.

1 H. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

$x^2 + 5|x| + 6 = 0$ has no real roots.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

Here, If $x>0$,

$x^2+5|x|+6=0$

$x^2+5x+6=0$

$x = -3$ or $x = -2$

But, Since $x>0$, the equation has no roots.

If $x<0$,

$x^2+5|x|+6=0$

$x^2 - 5x+6=0$

But, Since $x<0$, the equation has no real roots.

Hence,it is a statement.

1 I. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

This sentences is a statement

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

Here, We cannot find the truth values of this sentence, because either value contradict the sense of the sentence.

Hence, it is not a statement.

1 J. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Is the earth round?

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is an interrogative sentence.

Hence, it is not a statement.

1 K. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Go!

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence “Go!” is an exclamatory sentence.

Hence, it is not a statement.

1 L. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

The real number x is less than 2.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

It is not a statement because its truth and false value cannot be determined without knowing the value of x.

Hence, It is not a statement.

1 M. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

There are 35 days in a month.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The sentence “There are 35 days in a month” is a false declarative statement, so it is a false statement.

Hence, It is a statement.

1 N. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

Mathematics is difficult.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is true for some people, and it is not true for some people, So the given sentence may or may not be true.

Hence, It is not a statement.

1 O. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

All real numbers are complex numbers.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is always true.

Hence, It is a statement.

1 P. Question

Find out which of the following sentences are statements and which are not. Justify your answer.

The product of (-1) and 8 is 8.

Answer

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is always false because,

$(-1) \times (8) = -8$

Hence, It is a statement.

2. Question

Give three examples of sentences which are not statements. Give reasons for the answers.

Answer

(i) “Who is the Chancellor of your University”?

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is an interrogative sentence.

Hence, It is not a statement.

(ii) There are 31 days in a month

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is true for some particular months, and it is not true for others, so it may be true of false.

Hence, It is a not statement.

(iii) Hurray! We won the match.

Concept Used: A statement is an assertive (declarative) sentence if it is either true or false but not both.

The given sentence is in an exclamatory sentence.

Hence, It is a not statement.

Exercise 31.2

1 A. Question

Write the negation of the following statement:

Banglore is the capital of Karnataka.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement is “Banglore is not the capital of Karnataka.”

1 B. Question

Write the negation of the following statement:

It rained on July 4, 2005.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement is “It didn’t rain on July 4, 2005”.

1 C. Question

Write the negation of the following statement:

Ravish is honest.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So,

The negation of the statement is “Ravish is not honest.”

1 D. Question

Write the negation of the following statement:

The earth is round.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “The earth is not round.”

1 E. Question

Write the negation of the following statement:

The sun is cold.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “The sun is not cold.”

2 A. Question

All birds sing.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “Not all birds sing.”

2 B. Question

Some even integers are prime.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “No Even integer is prime.”

2 C. Question

There is a complex number which is not a real number.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement is “All complex number are real numbers.”

2 D. Question

I will not go to school

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “I will go to school.”

2 E. Question

Both the diagonals of a rectangle have the same length.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “There is at least one rectangle whose both diagonals do not have the same length.”

2 F. Question

All policemen are thieves.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

So, The negation of the statement is “No policemen is thief”.

3. Question

Are the following pairs of statements are a negation of each other:

(i) The number x is not a rational number.

The number x is not an irrational number.

(ii) The number x is not a rational number.

The number x is an irrational number.

Answer

(i) The number x is not a rational number.

= The number x is an irrational number.

Since The statement “The number x is not a rational number.” is a negation of the first statement.

(ii) The number x is not a rational number.

= The number x is an irrational number.

Since The statement “The number x is a rational number.” Is not a negation of the first statement.

4 A. Question

Write the negation of the following statements:

p : For every positive real number x, the number (x – 1) is also positive.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement:

p : For every positive real number x, the number (x – 1) is also positive.

is

~p : There exists a positive real number x, such that the number (x – 1) is not positive.

4 B. Question

Write the negation of the following statements:

q : For every real number x, either x > 1 or x < 1.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement:

q : For every real number x, either x > 1 or x < 1.

is

~q : There exists a real number such that neither x>1 or x<1.

4 C. Question

Write the negation of the following statements:

r : There exists a number x such that 0 < x < 1.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement:

r : There exists a number x such that 0 < x < 1.

is

~r : For every real number x,either x ≤ 0 or x ≥ 1.

5. Question

Check whether the following pair of statements is a negation of each other. Give reasons for your answer.

(i) a + b = b + a is true for every real number a and b.

(ii) There exist real numbers a and b for which a + b = b + a.

Answer

Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement:

p : a + b = b + a is a true for every real number a and b.

is

~p : There exist real numbers are a and b for which a+b≠b+a.

So, the given statement is not the negation of the first statement.

Exercise 31.3

1. Question

Find the component statements of the following compound statements:

(i) The sky is blue, and the grass is green.

(ii) The earth is round, or the sun is cold.

(iii) All rational numbers are real, and all real numbers are complex.

(iv) 25 is a multiple of 5 and 8.

Answer

(i) The components of the compound statement are:

P: The sky is blue.

Q: The grass is green.

(ii) The components of the compound statement are:

P: The earth is round.

Q: The sun is cold.

(iii) The components of the compound statement are:

P: All rational number is real.

Q: All real number are complex.

(iv) The components of the compound statement are:

P: 25 is multiple of 5.

Q: 25 is multiple of 8.

2. Question

For each of the following statements, determine whether an inclusive “OR” o exclusive “OR” is used. Give reasons for your answer.

(i) Students can take Hindi or Sanskrit as their third language.

(ii) To entry a country, you need a passport or a voter registration card.

(iii) A lady gives birth to a baby boy or a baby girl.

(iv) To apply for a driving license, you should have a ration card or a passport.

Answer

(i) In the given statement

“Students can take Hindi or Sanskrit as their third language” an exclusive “OR” is used because a student cannot take both Hindi and Sanskrit as the third language.

(ii) In the given statement “To entry a country, you need a passport or a voter registration card” an inclusive “OR” is used because

A person can have both a passport and a voter registration card to enter a country.

(iii) In the given statement “A lady gives birth to a baby boy or a baby girl.” An exclusive “OR” is used because A lady cannot give birth to a baby who is both a boy and a girl.

(iv) In the given statement “To apply for a driving license, you should have a ration card or a passport” an inclusive “OR” is used because

A person can have both a ration card and passport to apply for a driving license.

3. Question

Write the component statements of the following compound statements and check whether the compound statement is true or false:

(i) To enter into a public library children need an identification card from the school or a letter from the school authorities.

(ii) All rational numbers are real and all real numbers are not complex.

(iii) Square of an integer is positive or negative.

(iv) x = 2 and x = 3 are the roots of the equation $3x^2 – x – 10 = 0$.

(v) The sand heats up quickly in the sun and does not cool down fast at night.

Answer

(i) The components of the compound statement are:

P: To get into a public library children need an identity card.

Q: To get into a public library children need a letter from the school authorities.

We know if P and Q are true then P and Q must also be true.

Hence, The compound statement is true.

(ii) The components of the compound statement are:

P: All rational number is real.

Q: All real numbers are not complex.

We know, P is true and Q is False then P and Q is False.

Hence, The compound statement is False

(iii) The components of the compound statement are:

P: Square of an integer is positive.

Q: Square of an integer is negative.

We know that, if P and Q are true then P or Q is True.

Hence, The compound statement is True.

(iv) The components of the compound statement are:

P: x=2 is a root of the equation $3x^2-x-10=0$

Q: x=3 is a root of the equation $3x^2-x-10=0$

Here, P is true, but Q is False then P and Q is False.

Hence, The compound statement is False.

(v) The components of the compound statement are:

P: The sand heats up quickly in the sun.

Q: The sand does not cool down fast at night.

Here, P is false and Q is also False then P and Q is False.

Hence, The compound statement is False.

4. Question

Determine whether the following compound statements are true or false:

(i) Delhi is in India and 2 + 2 = 4

(ii) Delhi is in England and 2 + 2 = 4

(iii) Delhi is in India and 2 + 2 = 5

(iv) Delhi is in England and 2 + 2 = 5

Answer

(i) The components of the compound statement are:

P: Delhi is in India.

Q: 2+2 = 4

Here, P and Q are true then P or Q is True.

Hence, The compound statement is True.

(ii) The components of the compound statement are:

P: Delhi is in England.

Q: 2+2 = 4

Here, P is false, and q is true . So, P and Q must be false.

Hence, The compound statement is False.

(iii) The components of the compound statement are:

P: Delhi is in India.

Q: 2+2 = 5

Here, P is True, and q is False . So, P and Q must be false.

Hence, The compound statement is False.

(iv) The components of the compound statement are:

P: Delhi is in England.

Q: 2+2 = 5

Here, P and q are False . So, P and Q must be false.

Hence, The compound statement is False.

Exercise 31.4

1. Question

Write the negation of each of the following statements:

(i) For every $x \in N$, $x + 3 > 10$

(ii) There exists $x \in N$, $x + 3 = 10$

Answer

(i) Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement

For every $x \in N$, $x + 3 > 10$

is

There exist $x \in N$, such that $x + 3 \geq 10$.

(ii) Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement

For every $x \in N$, $x + 3 = 10$

is

There exist $x \in N$, such that $x + 3 \neq 10$.

2. Question

Negate each of the following statements:

(i) All the students complete their homework.

(ii) There exists a number which is equal to its square.

Answer

(i) Negation of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.

The negation of the statement

All the students complete their homework.

is

Some of the students did not complete their homework.

(ii) The negation of the statement

There exists a number which is equal to its square.

is

For every real number x, $x^2 \neq x$.

Exercise 31.5

1. Question

Write each of the following statements in the form “if p, then q.”

(i) You can access the website only if you pay a subscription fee.

(ii) There is traffic jam whenever it rains.

(iii) It is necessary to have a passport to log on to the server.

(iv) It is necessary to be rich in order to be happy.

(v) The game is canceled only if it is raining.

(vi) It rains only if it is cold.

(vii) Whenever it rains, it is cold.

(viii) It never rains when it is cold.

Answer

(i) If you access the website, then you pay a subscription fee.

(ii) If it rains, then there is a traffic jam.

(iii) If you log on the server, then you must have a passport.

(iv) If he is happy, then he is rich.

(v) If it is raining, then the game is canceled.

(vi) If it rains, then it is cold.

(vii) If it rains, then it is cold.

(viii) If it is cold, then it never rains.

2 A. Question

State the converse and contrapositive of each of the following statements:

If it is hot outside, then you feel thirsty.

Answer

Definition of Converse: A conditional statement is not logically equivalent to its converse.

Definition of contrapositive: A conditional statement is logically equivalent to its contrapositive.

Converse: If you feel thirsty, then it is hot outside.

Contrapositive: If you do not feel thirsty, then it is not hot outside.

2 B. Question

State the converse and contrapositive of each of the following statements:

I go to a beach whenever it is a sunny day.

Answer

Definition of Converse: A conditional statement is not logically equivalent to its converse.

Definition of contrapositive: A conditional statement is logically equivalent to its contrapositive.

Converse: If I go to a beach, then it is a sunny day.

Contrapositive: If I do not go to a beach, then it is not a sunny day.

2 C. Question

State the converse and contrapositive of each of the following statements:

A positive integer is prime only if it has no divisions other than 1 and itself.

Answer

Definition of Converse: A conditional statement is not logically equivalent to its converse.

Definition of contrapositive: A conditional statement is logically equivalent to its contrapositive.

Converse: If an integer has no divisor other that 1 and itself, then it is prime.

Contrapositive: If an integer has some divisor other than 1 and itself, then it is prime.

2 D. Question

State the converse and contrapositive of each of the following statements:

If you live in Delhi, then you have winter clothes.

Answer

Definition of Converse: A conditional statement is not logically equivalent to its converse.

Definition of contrapositive: A conditional statement is logically equivalent to its contrapositive.

Converse: If you have winter clothes, then you live in Delhi.

Contrapositive: If you do not have winter clothes, then you do not live in Delhi.

2 E. Question

State the converse and contrapositive of each of the following statements:

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Answer

Definition of Converse: A conditional statement is not logically equivalent to its converse.

Definition of contrapositive: A conditional statement is logically equivalent to its contrapositive.

Converse: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Contrapositive: If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

3. Question

Rewrite each of the following statements in the form "p if and only is q."

(i) p : If you watch television, then your mind is free, and if your mind is free, then you watch television.

(ii) q : If a quadrilateral is equiangular, then it is a rectangle, and if a quadrilateral is a rectangle, then it is equiangular.

(iii) r : For you to get an A grade, it is necessary and sufficient that you do all the homework you regularly.

(iv) s : If a tumbler is half empty, then it is half full, and if a tumbler is half full, then it is half empty.

Answer

(i) You watch television if and only if your mind is free.

(ii) A quadrilateral is a rectangle if and only if it is equiangular.

(iii) You get an A grade if and only if you do all the homework regularly.

(iv) A tumbler is half empty if and only if it is half full.

4. Question

Determine the contrapositive of each of the following statements:

(i) If Mohan is a poet, then he is poor.

(ii) Only if Max studies will he pass the test.

(iii) If she works, she will earn money.

(iv) If it snows, then they do not drive the car.

(v) It never rains when it is cold.

(vi) If Ravish skis, then it snowed.

(vii) If x is less than zero, then x is not positive.

(viii) If he has courage he will win.

(ix) It is necessary to be strong in order to be a sailor.

(x) Only if he does not tire will he win.

(xi) If x is an integer and x^2 is odd, then x is odd.

Answer

(i) Statement: If Mohan is a poet, then he is poor.

Contrapositive : If Mohan is not poor, then he is not a poet.

(ii) Statement: Only if Max studies will he pass the test.

Contrapositive : If Max does not study, then he will not pass the test.

(iii) Statement: If she works, she will earn money.

Contrapositive : If she does not earn money, then she does not work.

(iv) Statement: If it snows, then they do not drive the car.

Contrapositive : If then they do not drive the car, then there is no snow.

(v) Statement: It never rains when it is cold.

Contrapositive : If it rains, then it is not cold.

(vi) Statement: If Ravish skis, then it snowed.

Contrapositive : If it did not snow, then Ravish will not ski.

(vii) Statement: If x is less than zero, then x is not positive.

Contrapositive : If x is positive, then x is not less than zero.

(viii) Statement: If he has courage he will win.

Contrapositive : If he does not win, then he does not have courage.

(ix) Statement: It is necessary to be strong in order to be a sailor.

Contrapositive :If he is not strong, then he is not a sailor

(x) Statement: Only if he does not tire will he win.

Contrapositive : If he tries, then he will not win.

(xi) Statement: If x is an integer and x^2 is odd, then x is odd.

Contrapositive : If x is even, then x^2 is even.

Exercise 31.6

1. Question

Check the validity of the following statements:

(i) p : 100 is a multiple of 4 and 5.

(ii) q : 125 is a multiple of 5 and 7.

(iii) r : 60 is a multiple of 3 or 5.

Answer

(i) Statement: 100 is a multiple of 4 and 5.

Here, we know that 100 is a multiple of 4 as well as 5. So, statement p is true.

Hence, The statement is true, Therefore The statement “p” is a valid statement.

(ii) Statement: 125 is a multiple of 5 and 7

Since 125 is a multiple of 5, but it is not a multiple of 7. So, The statement “q” is not a true statement.

Hence, The statement “q” id not a valid statement.

(iii) Statement: 60 is a multiple of 3 or 5.

Here, we know that 60 is a multiple of 3 as well as 5. So, statement r is true.

Hence, The statement is true. Therefore The statement “r” is a valid statement.

2 A. Question

Check whether the following statement is true or not:

p : If x and y are odd integers, then x + y is an even integer.

Answer

Let us Assume that p and q be the statements given by

p: x and y are odd integers.

q: x + y is an even integer

since the given statement can be written as :

if p, then q.

Let p be true . then,

x and y are odd integers

x = 2m+1, y = 2n+1 for some integers m,n

x+y = (2m+1)+(2n+1)

x+y = (2m+2n+2)

x+y = 2(m+n+1)

x + y is an integer

q is true.

Therefore, p is true ⇒ q is true

Hence, "if p, then q " is a true statement."

2 B. Question

Check whether the following statement is true or not:

q : if x, y are integer such that xy is even, then at least one of x and y is an even integer.

Answer

Let us Assume that p and q be the statements given by

p: x and y are integers and xy is an even integer.

q: At least one of x and y is even.

Let p be true, then

xy is an even integer. So,

xy = 2(n + 1)

Now,

Let x = 2(k + 1)

Now as x is an even integer, xy = 2(k + 1). y is also an even integer.

Now take x = 2(k + 1) and y = 2(m + 1)

xy = 2(k + 1).2(m + 1) = 2.2(k + 1)(m + 1)

Therefore, it is also true.

Hence, the statement is true.

3. Question

Show that the statement

p : "If x is a real number such that $x^3 + x = 0$, then x is 0" is true by

(i) Direct method

(ii) method of contrapositive

(iii) method of contradiction

Answer

(i) Direct Method:

Let us Assume that q and r be the statements given by

q: x is a real number such that $x^3+x=0$.

r: x is 0.

since, the given statement can be written as :

if q, then r.

Let q be true . then,

x is a real number suc that $x^3+x = 0$

x is a real number such that $x(x^2+1) = 0$

$x = 0$

r is true

Thus, q is true

Therefore, q is true $\Rightarrow$ r is true

Hence, p is true.

(ii). Method of contrapositive

Let r be not true. then,

R is not true

$x \neq 0, x \in R$

$x(x^2+1) \neq 0, x \in R$

q is not true

Thus, -r = -q

Hence, p : q $\Rightarrow$ r is true

(iii) Method of Contradiction

If possible, let p be not true. Then,

P is not true

-p is true

$-p(p \Rightarrow r)$ is true

q and –r is true

x is a real number such that $x^3+x = 0$and $x \neq 0$

$x =0$ and $x \neq 0$

This is a contradiction

Hence, p is true

4. Question

Show that the following statement is true by the method of the contrapositive

p : “If x is an integer and x^2 is odd, then x is also odd.”

Answer

Let us Assume that q and r be the statements given

q: x is an integer and x^2 is odd.

r: x is an odd integer.

since the given statement can be written as :

p: if q, then r.

Let r be false . then,

x is not an odd integer, then

x is an even integer

x = (2n) for some integer n

$x^2 = 4n^2$

x^2 is an even integer

Thus, q is False

Therefore, r is false ⇒ q is false

Hence, p: " if q, then r" is a true statement.

5. Question

Show that the following statement is true

"The integer n is even if and only if n^2 is even"

Answer

Let the statements,

p: Integer n is even

q: If n^2 is even

Let p be true.

Then

Let n = 2k

Squaring both the sides, we get,

$n^2 = 4k^2$

$n^2 = 2.2k^2$

Therefore, n^2 is an even number.

So, q is true when p is true.

Hence, the statement is true.

6. Question

By giving a counter example, show that the following statement is not true.

p : "If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle."

Answer

let us consider a triangle ABC with all angles equal,

Then, each angle of the triangle is equal to 60.

So, ABC is not an obtuse angle triangle.

Therefore, The statement “p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle” is False.

7. Question

Which of the following statements are true and which are false? In each case give a valid reason for saying so

(i) p : Each radius of a circle is a chord of the circle.

(ii) q : The centre of a circle bisect each chord of the circle.

(iii) r : Circle is a particular case of an ellipse.

(iv) s : If x and y are integers such that x > y, then – x < - y.

(v) $t : \sqrt{11}$ is a rational number.

Answer

(i) This statement is False

Because the Radius of the circle is not it chord

(ii) This statement is False

Because A chord does not have to pass through the center.

(iii) This statement is true,

Because a circle can be an ellipse in a particular case when the circle has equal axes.

(iv) This statement is true,

Because for any two integer, if x – y id positive then –(x-y) is negative

(v) This statement is False

Because square root of prime numbers are irrational numbers.

8. Question

Determine whether the argument used to check the validity of the following statement is correct:

p: “If x^2 is irrational, then x is rational.”

The statement is true because the number $x^2 = \pi^2$ is irrational, therefore $x = \pi$ is irrational.

Answer

Argument Used: $x^2 = \pi^2$ is irrational, therefore $x = \pi$ is irrational.

p: “If x^2 is irrational, then x is rational.”

Let us take an irrational number given by $x = \sqrt{k}$, where k is a rational number.

Squaring both sides, we get,

$x^2 = k$

Therefore, x^2 is a rational number and contradicts our statement.

Hence, the given argument is wrong.