Chapter 5. Linear Inequations

Ex 5.1

Answer 1.

(i) $x + 5 > 11$

$x + 5 > 11$

$\Rightarrow x > 11 - 5$

$\Rightarrow x > 6$

But $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{7,8,9\}$

(ii) $2x + 1 < 17$

$2x + 1 < 17$

$\Rightarrow 2x < 17 - 1$

$\Rightarrow x < \frac{16}{2}$

$\Rightarrow x < 8$

But $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1,2,3,4,5,6,7\}$

(iii) $3x - 5 \leq 7$

$3x - 5 \leq 7$

$\Rightarrow 3x \leq 7 + 5$

$\Rightarrow 3x \leq 12$

$\Rightarrow x \leq 4$

But $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1,2,3,4\}$

(iv) $8 - 3x \geq 2$

$8 - 3x \geq 2$

$\Rightarrow -3x \geq -6$

$\Rightarrow x \geq 2$

But $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1,2\}$

(v) $5 - 2x < 11$

$5 - 2x < 11$

$\Rightarrow -2x < 11 - 5$

$\Rightarrow -2x < 6$

$\Rightarrow x > -3$

But $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1,2,3,4,5,6,7,8,9\}$

Answer 2.

(i) $3x > 12$

$3x > 12$

$x > \frac{12}{3}$

$x > 4$

Since, replacement set is R

Solution set = $\{x: x \in R \text{ and } x > 4\}$

(ii) $2x - 3 < 7$

$2x - 3 < 7$

$2x < 7 + 3$

$2x < 10$

$x < \frac{10}{2}$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x < 5\}$

(iii) $3x + 2 \leq 11$

$3x + 2 \leq 11$

$3x \leq 11 - 2$

$3x \leq 9$

$x \leq \frac{9}{3}$

$x \leq 3$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq 3\}$

(iv) $14 - 3x \geq 5$

$14 - 3x \geq 5$

$-3x \geq 5 - 14$

$3x \leq 9$ (multiplying by -1 changes the sign)

$x \leq \frac{9}{3}$

$x \leq 3$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq 3\}$

(v) $7x + 11 > 16 - 3x$

$7x + 11 > 16 - 3x$

$7x + 3x > 16 - 11$

$10x > 5$

$x > \frac{5}{10} = 0.5$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x > 0.5\}$

(vi) $3x + 25 < 8x - 10$

$3x + 25 < 8x - 10$

$25 + 10 < 8x - 3x$

$35 < 5x$

$x > 7$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x > 7\}$

(vii) $2(3x - 5) \leq 8$

$2(3x - 5) \leq 8$

$6x - 10 \leq 8$

$6x \leq 8 + 10$

$6x \leq 18$

$x \leq 3$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq 3\}$

(viii) $x + 7 \geq 15 + 3x$

$x + 7 \geq 15 + 3x$

$x - 3x \geq 15 - 7$

$-2x \geq 8$

$2x \leq -8$

$x \leq -4$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq -4\}$

(ix) $2x - 7 \geq 5x + 8$

$2x - 7 \geq 5x + 8$

$2x - 5x \geq 8 + 7$

$-3x \geq 15$

$3x \leq -15$

$x \leq -5$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq -5\}$

(x) $9 - 4x \leq 15 - 7x$

$9 - 4x \leq 15 - 7x$

$9 - 15 \leq 4x - 7x$

$-6 \leq -3x$

$6 \geq 3x$ (multiplying by -1 changes the sign)

$x \leq 2$

Since, replacement set is R

Solution set = $\{x : x \in R \text{ and } x \leq 2\}$

Answer 3.

$6 - 10x < 36$

$-10x < 36 - 6$

$-10x < 30$

$10x > -30$

$x > -3$

Solution set = $\{-2, -1, 0, 1, 2\}$

Answer 4.

$3 - 2x \geq x - 12$

$3 + 12 \geq x + 2x$

$15 \geq 3x$

$x \leq 5$

Solution set = {1,2,3,4,5}

Answer 5.

$5x - 9 \leq 15 - 7x$

$5x + 7x \leq 15 + 9$

$12x \leq 24$

$x \leq 2$

Solution set = {0,1,2}

Answer 6.

$7 + 5x > x - 13$

$-x + 5x > -13 - 7$

$4x > -20$

$x > -5$

Solution set = {-4, -3, -2,-1}

Answer 7

$5x - 14 < 18 - 3x$

$5x + 3x < 18 + 14$

$8x < 32$

$x < 4$

Solution set = {0,1,2,3}

Answer 8.

$2x + 7 \geq 5x - 14$

$2x - 5x \geq -14 - 7$

$-3x \geq -21$

$3x \leq 21$

$x \leq 7$

Solution set = {2,3,5,7}

Answer 9.

$$\frac{x}{4} + 3 \leq \frac{x}{3} + 4$$

$$\frac{x+12}{4} \leq \frac{x+12}{3}$$

$3x + 36 \leq 4x + 48$

$3x - 4x \leq 48 - 36$

$-x \leq 12$

$x \geq -12$

Solution set = {-11, -9, -7, -5, -3, -1}

Answer 10.

$$\frac{x+3}{3} \leq \frac{x+8}{4}$$

$4x + 12 \leq 3x + 24$

$4x - 3x \leq 24 - 12$

$x \leq 12$

Solution set = {2,4,6,8,10,12}

Answer 11.

(i) $x \in Z$

$x + 17 \leq 4x + 9$

$x - 4x \leq 9 - 17$

$-3x \leq -8$

$3x \geq 8$

$x \geq \frac{8}{3}$

Since $x \in Z$

Smallest value of $x = [3]$

(ii) $x \in R$

$x + 17 \leq 4x + 9$

$x - 4x \leq 9 - 17$

$-3x \leq -8$

$3x \geq 8$

$x \geq \frac{8}{3}$

Since $x \in R$

Smallest value of $x = \left[2\frac{2}{3}\right]$

Answer 12.

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$2 - 5x + 2x^2 = 0$

$2x^2 - 5x + 2 = 0$

$x^2 - \frac{5}{2}x + 1 = 0$

$x^2 - 2x - \frac{1}{2}x + 1 = 0$

$x(x-2) - \frac{1}{2}(x-2) = 0$

$(x-2)(x-\frac{1}{2}) = 0$

$(x-2) = 0, (x-\frac{1}{2}) = 0$

$x = 2, x = \frac{1}{2}$

Answer 13.

(i) $2x - 11 \leq 7 - 3x,\ x \in N$

$2x - 11 \leq 7 - 3x$

$2x + 3x \leq 7 + 11$

$5x \leq 18$

$x \leq \frac{18}{5}$

$x \leq 3.6$

Since $x \in N$

Solution set $= \{1,2,3\}$

(ii) $3(5x + 3) \geq 2(9x - 17),\ x \in W$

$3(5x + 3) \geq 2(9x - 17)$

$15x + 9 \geq 18x - 34$

$15x - 18x \geq -34 - 9$

$-3x \geq -43$

$3x \leq 43$

$x \leq \frac{43}{3}$

Solution set $= \left[x \leq \frac{43}{3}\right]$

(iii) $2(3x - 5) > 5(13 - 2x),\ x \in W$

$2(3x - 5) > 5(13 - 2x)$

$6x - 10 > 65 - 10x$

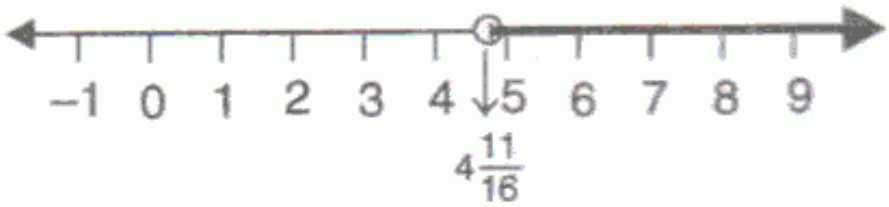
$6x + 10x > 65 + 10$

$16x > 75$

$x > \frac{75}{16}$

$x > 4\frac{11}{16}$

Solution set = $\left[x > 4\frac{11}{16}\right]$



(iv) $3x - 9 \le 4x - 7 < 2x + 5,\ x \in R$

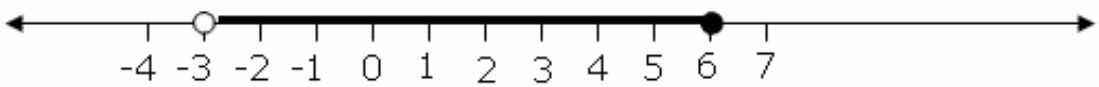
$3x - 9 \le 4x - 7$		$4x - 7 < 2x + 5$
$3x - 4x \le -7 + 9$	and	$4x - 2x < 5 + 7$
$-x \le 2$		$2x < 12$
$x \ge -2$		$x < 6$

Solution set = $[-2 \le x < 6]$

(v) $2x - 7 < 5x + 2 \le 3x + 14,\ x \in R$

$2x - 7 < 5x + 2$		$5x + 2 \le 3x + 14$
$2x - 5x < 2 + 7$	and	$5x - 3x \le 14 - 2$
$-3x < 9$		$2x \le 12$
$x > -3$		$x \le 6$

Solution set = $[-3 < x \le 6]$

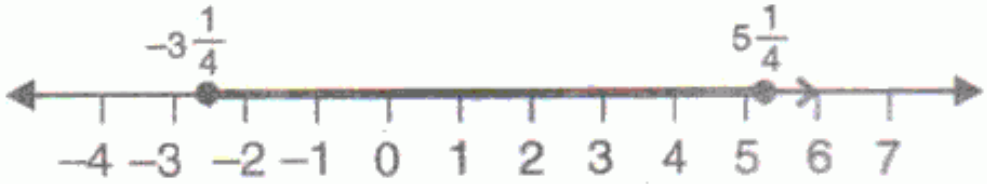


(vi) $-3 \le \frac{1}{2} - \frac{2x}{3} \le 2\frac{2}{3}, x \in N$

$-3 \le \frac{1}{2} - \frac{2x}{3}$		$\frac{1}{2} - \frac{2x}{3} \le 2\frac{2}{3}$
$-3 \le \frac{3 - 4x}{6}$		$\frac{3 - 4x}{6} \le \frac{8}{3}$
$-18 \le 3 - 4x$		$9 - 12x \le 48$
$-18 - 3 \le -4x$	and	$-12x \le 39$
$-21 \le -4x$		$12x \ge -39$
$x \le \frac{21}{4}$		$x \ge -3\frac{1}{4}$
$x \le 5\frac{1}{4}$		

$\left[-3\frac{1}{4} \le x \le 5\frac{1}{4}\right]$

Solution set = $\left[-3\frac{1}{4} \le x \le 5\frac{1}{4}\right]$



(vii) $4\frac{3}{4} \ge x + \frac{5}{6} > \frac{1}{3}, x \in R$

$4\frac{3}{4} \ge x + \frac{5}{6}$ and $x + \frac{5}{6} > \frac{1}{3}$

$\frac{19}{4} \ge \frac{6x+5}{6}$ and $\frac{6x+5}{6} > \frac{1}{3}$

$114 \ge 24x + 20$ and $18x + 15 > 6$

$114 - 20 \ge 24x$ and $18x > 6 - 15$

$94 \ge 24x$ and $18x > -9$

$x \le 3\frac{11}{12}$ and $x > -\frac{1}{2}$

Solution set = $\left[-\frac{1}{2} < x \le 3\frac{11}{12}\right]$

(viii) $\frac{1}{3}(2x-1) < \frac{1}{4}(x+5) < \frac{1}{6}(3x+4), x \in R$

$\frac{1}{3}(2x-1) < \frac{1}{4}(x+5)$ and $\frac{1}{4}(x+5) < \frac{1}{6}(3x+4)$

$4(2x-1) < 3(x+5)$ and $6(x+5) < 4(3x+4)$

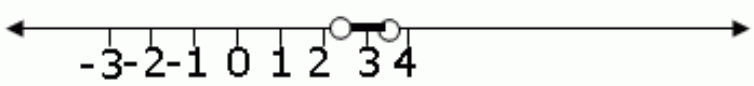
$8x - 4 < 3x + 15$ and $6x + 30 < 12x + 16$

$8x - 3x < 15 + 4$ and $6x - 12x < 16 - 30$

$5x < 19$ and $-6x < -14$

$x < 3\frac{4}{5}$ and $x > 2\frac{1}{3}$

Solution set = $\left[2\frac{1}{3} < x < 3\frac{4}{5}\right]$



(ix) $\frac{1}{3}(5x-8) \ge \frac{1}{2}(4x-7), x \in R$

$\frac{1}{3}(5x-8) \geq \frac{1}{2}(4x-7)$

$2(5x-8) \geq 3(4x-7)$

$10x-16 \geq 12x-21$

$10x-12x \geq -21+16$

$-2x \geq -5$

$x \leq \frac{5}{2}$

$x \leq 2\frac{1}{2}$

Solution set $= \left[x \leq 2\frac{1}{2}\right]$

(x) $\frac{5}{4}x > 1+\frac{1}{3}(4x-1), x \in R$

$\frac{5}{4}x > 1+\frac{1}{3}(4x-1)$

$\frac{5}{4}x > \frac{3+(4x-1)}{3}$

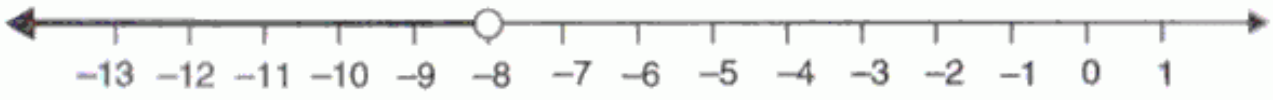
$15x > 12+16x-4$

$15x-16x > 8$

$-x > 8$

$x < -8$

Solution set = [x<-8]



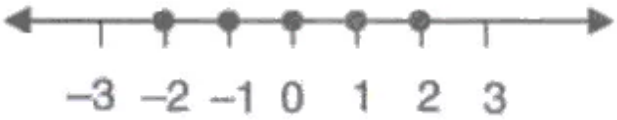
Answer 14.

$P = \{x : -3 < x \leq 7, x \in R\}$ and $Q = \{x : -7 \leq x < 3, x \in R\}$

$P = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$ $Q = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$

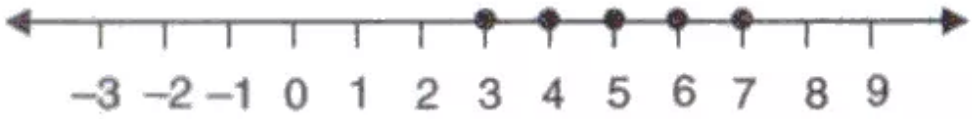
(i) $P \cap Q$

$P \cap Q = \{-2, -1, 0, 1, 2\}$



(ii) $Q' \cap P$

$Q' \cap P = \{3, 4, 5, 6, 7\}$



(iii) P-Q

P-Q = {3,4,5,6,7}

Answer 15.

$P = \{x : 7x - 2 > 4x + 1, x \in R\}$

$7x - 2 > 4x + 1$

$7x - 4x > 1 + 2$

$3x > 3$

$x > 1$

$P = \{2, 3, 4, 5 \ldots\ldots\ldots\ldots\}$

and

$Q = \{x : 9x - 45 \geq 5(x - 5), x \in R\}$

$9x - 45 \geq 5x - 25$

$9x - 5x \geq -25 + 45$

$4x \geq 20$

$x \geq 5$

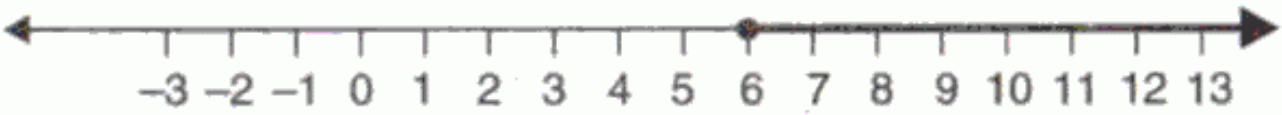
$Q = \{5, 6, 7, 8, 9, \ldots\ldots\ldots\ldots\}$

(i) $P \cap Q$

$P \cap Q = \{2, 3, 4, 5\}$

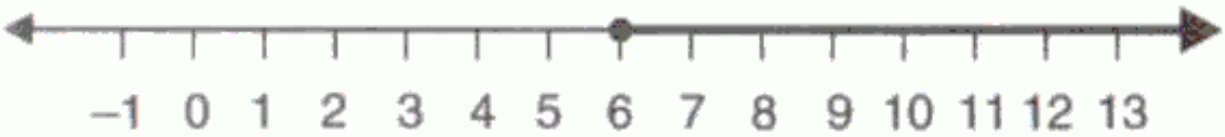
(ii) P-Q

$P\text{-}Q = \{6, 7, 8, 9 \ldots\ldots\ldots\ldots\}$



(iii) $P \cap Q'$

$P \cap Q' = \{6, 7, 8, 9, \ldots\ldots\ldots\ldots\}$



Answer 16.

$P = \{x : 7x - 4 > 5x + 2, x \in R\}$

$7x - 4 > 5x + 2$

$7x - 5x > 2 + 4$

$2x > 6$

$x > 3$

$P = \{4, 5, 6, 7, \ldots\ldots\ldots\ldots\}$

and

$Q = \{x : x - 19 \geq 1 - 3x, x \in R\}$

$x - 19 \geq 1 - 3x$

$x + 3x \geq 1 + 19$

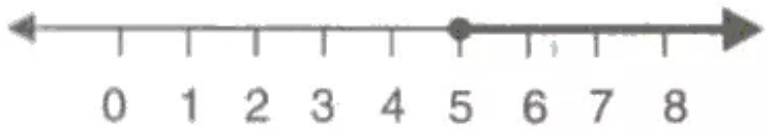
$4x \geq 20$

$x \geq 5$

$Q = \{5, 6, 7, 8, 9\ldots\ldots\ldots\}$

(i) $P \cap Q$

$P \cap Q = \{5, 6, 7, 8, \ldots\ldots\ldots\}$



(ii) $P' \cap Q$

$P' \cap Q = \{\phi\}$