

Chapter 9. Ratio and Proportion

Ex 9.1

Answer 1.

d 2.40

$$= \frac{560}{240} = \frac{56}{24} = \frac{28}{12} = \frac{14}{6} = \frac{7}{3}$$

! and 120

$$\frac{216}{60} = \frac{108}{30} = \frac{54}{15} = \frac{18}{5}$$

5.40 and 180 paise

) = 540 paise

$$\frac{54}{18} = \frac{3}{1}$$

- b^4 and $a^3 - b^3$

$$= \frac{(a-b)(a^2+ab^2+a^2b+b^3)}{(a-b)(a^2+ab+b^2)} = \frac{a^2+ab^2+a^2b+b^3}{a^2+ab+b^2}$$

$a^2+a^2b+b^3$: a^2+ab+b^2

$x+4$ and x^2-x-6

$$\frac{x+4}{x-6} = \frac{(x+2)^2}{(x-3)(x+2)} = \frac{(x+2)}{(x-3)}$$

) : (x-3)

Answer 3.

$$\frac{3}{8}$$

$$2n) (5m+n) = \frac{(3m+2n)}{(5m+n)}$$

$$\frac{3}{8} + 2$$

$$\frac{3}{8} + 1$$

$$\frac{3}{8} + 2$$

$$\frac{3}{8} + 1$$

$$+ 2$$

$$+ 1$$

$$= 25:23$$

$$5m+n)=25:23$$

Answer 4.

(i) Annual expenses to annual income.

$$\text{Annual income} = \text{monthly income} \times 12 = \text{Rs } 5,000 \times 12 = \text{Rs } 60,000$$

$$\text{Monthly expenses} = \text{Rs } 5,000 - 800 = \text{Rs } 4,200$$

$$\text{Annual expenses} = \text{monthly expenses} \times 12 = \text{Rs } 4,200 \times 12 = \text{Rs } 50,400$$

$$\frac{\text{Annual expenses}}{\text{Annual income}} = \frac{50,400}{60,000} = \frac{504}{600} = \frac{21}{25}$$

Therefore, Annual expenses : annual income = 21:25

(ii) monthly savings to monthly expenses.

$$\text{Monthly expenses} = \text{Rs } 5,000 - 800 = \text{Rs } 4,200$$

$$\frac{\text{Monthly savings}}{\text{Monthly expenses}} = \frac{800}{4,200} = \frac{8}{42} = \frac{4}{21}$$

Therefore, monthly savings : monthly expenses = 4 : 21

Answer 5.

$$a+b : a-b = 11 : 8$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{11}{8}$$

$$\Rightarrow 8a+8b = 11a-11b$$

$$\Rightarrow 3a = 19b$$

$$\Rightarrow \frac{a}{b} = \frac{19}{3}$$

$$a : b = 19 : 3$$

Answer 6.

$$p : q = 2 : 5 \Rightarrow \frac{p}{q} = \frac{2}{5}$$

$$q : r = 4 : 3 \Rightarrow \frac{q}{r} = \frac{4}{3}$$

$$\Rightarrow \frac{p}{q} \times \frac{q}{r} = \frac{2}{5} \times \frac{4}{3}$$

$$\Rightarrow \frac{p}{r} = \frac{8}{15}$$

$$p : r = 8 : 15$$

Answer 7.

$$a : e = 5 : 12 \Rightarrow \frac{a}{e} = \frac{5}{12}$$

$$e : i = 8 : 3 \Rightarrow \frac{e}{i} = \frac{8}{3}$$

$$i : u = 9 : 16 \Rightarrow \frac{i}{u} = \frac{9}{16}$$

$$\Rightarrow \frac{a}{e} \times \frac{e}{i} \times \frac{i}{u} = \frac{5}{12} \times \frac{8}{3} \times \frac{9}{16}$$

$$\Rightarrow \frac{a}{u} = \frac{10}{16} = \frac{5}{8}$$

$$a : u = 5 : 8$$

Answer 8.

: 16 and 8 : 5

ounded ratio of 15:16 and 8:5

$$\frac{15}{16} \times \frac{8}{5} = \frac{3}{2}$$

ounded ratio = 3:2

$(a^2 - b^2) : (a^2 + b^2)$ and $(a^4 - b^4) : (a + b)^4$

ounded ratio of $(a^2 - b^2) : (a^2 + b^2)$ and $(a^4 - b^4) : (a + b)^4$

$$\begin{aligned} & \frac{(a^2 - b^2)}{(a^2 + b^2)} \times \frac{(a^4 - b^4)}{(a + b)^4} \\ & \frac{(a + b)(a - b)}{(a^2 + b^2)} \times \frac{(a^2 + b^2)(a^2 - b^2)}{(a + b)^2(a + b)^2} \\ & \frac{(a - b)(a + b)(a - b)(a + b)}{(a + b)^2(a + b)^2} \\ & \frac{(a - b)^2}{(a + b)^2} \end{aligned}$$

ounded ratio = $(a - b)^2 : (a + b)^2$

7:9 and 15: 28

ounded ratio of 3:5, 7:9 and 15: 28

$$\begin{aligned} & \frac{3}{5} \times \frac{7}{9} \times \frac{15}{28} \\ & \frac{1}{4} \end{aligned}$$

pounded ratio = 1:4

(iv) $\sqrt{8} : 4, 3 : \sqrt{5}$ and $\sqrt{20} : \sqrt{27}$

Compounded ratio of $\sqrt{8} : 4, 3 : \sqrt{5}$ and $\sqrt{20} : \sqrt{27}$

$$\begin{aligned}
 &= \frac{\sqrt{8}}{4} \times \frac{3}{\sqrt{5}} \times \frac{\sqrt{20}}{\sqrt{27}} \\
 &= \frac{2\sqrt{2}}{4} \times \frac{3}{\sqrt{5}} \times \frac{2\sqrt{5}}{3\sqrt{3}} \\
 &= \frac{\sqrt{2}}{\sqrt{3}}
 \end{aligned}$$

Compounded ratio = $\sqrt{2} : \sqrt{3}$

(v) $(m-n):(m+n)$, $(m+n)^2:(m^2+n^2)$ and $(m^4-n^4):(m^2-n^2)^2$

Compounded ratio of $(m-n):(m+n)$, $(m+n)^2:(m^2+n^2)$ and $(m^4-n^4):(m^2-n^2)^2$

$$\begin{aligned}
 &= \frac{m-n}{m+n} \times \frac{(m+n)^2}{(m^2+n^2)} \times \frac{(m^4-n^4)}{(m^2-n^2)^2} \\
 &= \frac{m-n}{1} \times \frac{m+n}{(m^2+n^2)} \times \frac{(m^2+n^2)(m^2-n^2)}{(m^2-n^2)^2} \\
 &= \frac{(m^2-n^2)}{(m^2+n^2)} \times \frac{(m^2+n^2)(m^2-n^2)}{(m^2-n^2)^2} \\
 &= \frac{1}{1}
 \end{aligned}$$

Compounded ratio = $1 : 1$

Answer 15.

Let the two numbers be $7x$ and $10x$; since the ratio between numbers is $7 : 10$.

Now,

$$\begin{aligned}
 \frac{7x+8}{10y+8} &= \frac{3}{4} \\
 \Rightarrow 28x+32 &= 30x+24 \\
 \Rightarrow 2x &= 8 \\
 \Rightarrow x &= 4
 \end{aligned}$$

$$\therefore 7x = 28$$

$$10x = 40$$

Therefore, two numbers are 28 and 40.

Answer 16.

Let the two numbers be $3x$ and $4x$, since the ratio between them is 3:4.

Now,

$$\begin{aligned}(3x)^2 + (4x)^2 &= 1225 \\ \Rightarrow 9x^2 + 16x^2 &= 1225 \\ \Rightarrow 25x^2 &= 1225 \\ \Rightarrow x^2 &= 49 \\ \Rightarrow x &= 7\end{aligned}$$

$$\therefore 3x = 21$$

$$4x = 28$$

Therefore, the two numbers are 21 and 28.

Answer 17.

Let the two numbers be $5x$ and $7x$, since the ratio between them is 5:7.

Now,

$$\begin{aligned}(7x)^2 - (5x)^2 &= 600 \\ \Rightarrow 49x^2 - 25x^2 &= 600 \\ \Rightarrow 24x^2 &= 600 \\ \Rightarrow x^2 &= 25 \\ \Rightarrow x &= 5\end{aligned}$$

$$\therefore 5x = 25$$

$$7x = 35$$

Therefore, the two numbers are 25 and 35.

Answer 18.

Let x be subtracted from each term such that

$$\frac{39 - x}{89 - x} = \frac{2}{5}$$

$$\Rightarrow 195 - 5x = 178 - 2x$$

$$\Rightarrow 3x = 17$$

$$\Rightarrow x = \frac{17}{3}$$

$\frac{17}{3}$ should be subtracted from each term.

Answer 19.

Let x be added from each term such that

$$\frac{19 + x}{51 + x} = \frac{3}{7}$$

$$\Rightarrow 133 + 7x = 153 + 3x$$

$$\Rightarrow 4x = 20$$

$$\Rightarrow x = 5$$

5 should be added to each term.

Answer 20.

Let x be added from each term such that

$$\frac{(p+q)+x}{(p-q)+x} = \frac{(p+q)^2}{(p-q)^2}$$

$$\Rightarrow (p+q+x)(p^2-2pq+q^2) = (p^2+2pq+q^2)(p-q+x)$$

$$\Rightarrow p^3 + p^2q + p^2x + q^2p + q^3 + q^2x - 2p^2q - 2pq^2 - 2pqx$$

$$= p^3 - p^2q + p^2x + q^2p - q^3 + q^2x + 2p^2q - 2pq^2 + 2pqx$$

$$\Rightarrow p^2q + q^3 - 2p^2q - 2pqx = -p^2q - q^3 + 2p^2q + 2pqx$$

$$\Rightarrow 2q^3 - p^2q = 4pqx$$

$$\Rightarrow 2q(q^2 - p^2) = 4pqx$$

$$\Rightarrow x = \frac{(q^2 - p^2)}{2p}$$

$\frac{(q^2 - p^2)}{2p}$ should be added to each term.

Answer 21.

$$\frac{3x - 4}{2x + 5} = \frac{(3)^2}{(4)^2}$$

$$\Rightarrow \frac{3x - 4}{2x + 5} = \frac{9}{16}$$

$$\Rightarrow 48x - 64 = 18x + 45$$

$$\Rightarrow 30x = 109$$

$$\Rightarrow x = \frac{109}{30}$$

$$x = \frac{109}{30}$$

Answer 22.

$$\frac{5x + 3}{3x + 1} = \frac{(4)^3}{(3)^3}$$

$$\Rightarrow \frac{5x + 3}{3x + 1} = \frac{64}{27}$$

$$\Rightarrow 135x + 81 = 192x + 64$$

$$\Rightarrow 57x = 17$$

$$\Rightarrow x = \frac{17}{57}$$

$$x = \frac{17}{57}$$

Answer 23.

$$\frac{p}{q} = \frac{(p+r)^2}{(q+r)^2}$$

$$\Rightarrow \frac{p}{q} = \frac{p^2 + r^2 + 2pr}{q^2 + r^2 + 2qr}$$

$$\Rightarrow \frac{p}{q} = \frac{p^2 + pq + 2pr}{q^2 + pq + 2qr}$$

$$\Rightarrow pq^2 + p^2q + 2pqr = p^2q + pq^2 + 2pqr$$

Since LHS = RHS, therefore p:q is the duplicate ratio of (p+r) : (q+r)

Answer 24.

$$\frac{2}{3} = \frac{x}{7,47,300}$$
$$\Rightarrow 14,94,600 = 3x$$
$$\Rightarrow x = 4,98,200$$

Sum distributed = Rs (7,47,300+4,98,200) = Rs 12,45,500

Answer 25.

$$\frac{5}{7} = \frac{60}{x}$$
$$\Rightarrow 5x = 420$$
$$\Rightarrow x = 84$$

Number = 84+60 = 144

Answer 26.

Let the number of Re 1, Rs 2 and Rs 5 coins be $3x$, $7x$ and $11x$

The sum of their value =

$$3x + (2 \times 7x) + (5 \times 11x) = \text{Rs}1800$$
$$3x + 14x + 55x = 1800$$
$$72x = 1800$$
$$x = 25$$

$$3x = 3 \times 25 = 75$$
$$7x = 7 \times 25 = 175$$
$$11x = 11 \times 25 = 275$$
$$\text{total} = 75 + 175 + 275 = 525$$

Total number of coins = 525

Answer 27.

Let present ages be $4x$ and $3x$.

$$\begin{aligned}\frac{4x + 9}{3x + 9} &= \frac{23}{18} \\ \Rightarrow 72x + 162 &= 69x + 207 \\ \Rightarrow 3x &= 45 \\ \Rightarrow x &= 15\end{aligned}$$

$$\begin{aligned}\therefore 3x &= 45 \\ 4x &= 60\end{aligned}$$

Hence, their present ages are 60 years and 45 years.

Answer 28.

Let money saved by the boy and her sister be $5x$ and $4x$ and let the sister need to save Rs y more

$$\begin{aligned}\frac{5x + 100}{4x + y} &= \frac{5}{4} \\ \Rightarrow 20x + 400 &= 20x + 5y \\ \Rightarrow 5y &= 400 \\ \Rightarrow y &= 80\end{aligned}$$

The sister needs to save Rs 80 more.

Answer 29.

$$\text{Quantity of milk} = 60 \times \frac{11}{15} = 44 \text{ litres}$$

$$\text{Quantity of water in it} = 60 - 44 = 16 \text{ litres}$$

$$\text{New ratio} = 11:6$$

Let quantity of water to be added further be x litres.

$$\text{Then, milk : water} = \frac{44}{16 + x}$$

Now,

$$\begin{aligned}\frac{44}{16 + x} &= \frac{11}{6} \\ \Rightarrow 264 &= 176 + 11x \\ \Rightarrow 11x &= 88 \\ \Rightarrow x &= 8\end{aligned}$$

Therefore, 8 litres of water need to be added further.

Answer 30.

Let successful candidates be $7x$ and unsuccessful candidates be $5x$.

Now,

$$\frac{7x + 10}{5x + 20} = \frac{4}{3}$$

$$\Rightarrow 21x + 30 = 20x + 80$$

$$\Rightarrow x = 50$$

$$7x = 7 \times 50 = 350$$

$$5x = 5 \times 50 = 250$$

$$\therefore 7x + 5x = 350 + 250 = 600$$

Therefore, there were 600 candidates originally.

Ex 9.2**Answer 5.**

Since x , 12 and 16 are in continued proportion

$$\Rightarrow x : 12 :: 12 : 16$$

$$\Rightarrow 16x = 12 \times 12$$

$$\Rightarrow 16x = 144$$

$$\Rightarrow x = 9$$

$$x = 9$$

Answer 6.

Since $\frac{1}{12}$, x and $\frac{1}{75}$ are in continued proportion

$$\Rightarrow \frac{1}{12} : x :: x : \frac{1}{75}$$

$$\Rightarrow x^2 = \frac{1}{12} \times \frac{1}{75}$$

$$\Rightarrow x = \sqrt{\frac{1}{900}}$$

$$\Rightarrow x = \frac{1}{30}$$

$$x = \frac{1}{30}$$

Answer 7.

Since y is the mean proportion between x and z

$$y^2 = xz$$

LHS

$$xy^2(x + y + z)^3$$

$$= yy^2(x + y + z)^3$$

$$= y^3(x + y + z)^3$$

$$= [y(x + y + z)]^3$$

$$= (xy + y^2 + yz)^3$$

$$= (xy + yz + xz)^3 = \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

Answer 8.

Since y is the mean proportion between x and z

Therefore, $y^2 = xz$

Now, we have to prove that $xy + yz$ is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$.

$$(xy + yz)^2 = (x^2 + y^2)(y^2 + z^2)$$

$$\text{LHS} = (xy + yz)^2$$

$$= [y(x + z)]^2$$

$$= y^2(x + z)^2$$

$$= xz(x + z)^2$$

$$\text{RHS} = (x^2 + y^2)(y^2 + z^2)$$

$$= (x^2 + xz)(xz + z^2)$$

$$= x(x + z)z(x + z)$$

$$= xz(x + z)^2$$

$$\text{LHS} = \text{RHS}$$

Answer 9.

Let x, y and z are the three quantities which are in continued proportion.

Then, $x:y::y:z \Rightarrow y^2 = xz$

Now, we have to prove that

$$x:z = x^2:y^2$$

$$\Rightarrow xy^2 = x^2z$$

$$\text{LHS}$$

$$= xy^2 = x(xz) = x^2z = \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

Answer 11.

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{ax}{b} = \frac{cx}{d}$$

Adding y to both sides:

$$\Rightarrow \frac{ax}{b} + y = \frac{cx}{d} + y$$

$$\Rightarrow \frac{ax + by}{b} = \frac{cx + dy}{d}$$

$$\Rightarrow (ax + by) : b = (cx + dy) : d$$

Proved.

Answer 12.

$$\frac{a+c}{b} = \frac{5}{1}$$

$$\Rightarrow a + c = 5b$$

$$\frac{bc+cd}{bd} = \frac{5}{1}$$

$$\Rightarrow bc + cd = 5bd$$

$$\Rightarrow bc + cd = (a + c)d$$

$$\Rightarrow bc + cd = ad + cd$$

$$\Rightarrow bc = ad$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Hence $a:b = c:d$

Answer 13.

Let x be subtracted from each number so that 20-x, 29-x, 84-x and 129-x are in proportion.

$$\therefore \frac{20-x}{29-x} = \frac{84-x}{129-x}$$

$$\Rightarrow (20-x)(129-x) = (29-x)(84-x)$$

$$\Rightarrow 2580 - 129x - 20x + x^2 = 2436 - 84x - 29x + x^2$$

$$\Rightarrow 2580 - 149x = 2436 - 113x$$

$$\Rightarrow 36x = 144$$

$$\Rightarrow x = 4$$

Hence, 4 is to be subtracted from 20, 29, 84 and 129 for them to be in proportion.

Answer 14.

Let a and b be the two numbers , whose mean proportional is 12.

$$\therefore ab = 12^2 \Rightarrow ab = 144 \Rightarrow b = \frac{144}{a} \dots\dots\dots(i)$$

Now, third proportional is 324

$$\therefore a : b :: b : 324$$

$$\Rightarrow b^2 = 324a$$

$$\Rightarrow \left(\frac{144}{a}\right)^2 = 324a$$

$$\Rightarrow \frac{(144)^2}{a^2} = 324a$$

$$\Rightarrow a^3 = \frac{144 \times 144}{324}$$

$$\Rightarrow a^3 = 64$$

$$\Rightarrow a = 4$$

$$b = \frac{144}{a} = \frac{144}{4} = 36$$

Therefore, numbers are 4 and 36

Answer 15.

Let a and b be the two numbers , whose mean proportional is 18.

$$\therefore ab = 18^2 \Rightarrow ab = 324 \Rightarrow b = \frac{324}{a} \dots\dots\dots(i)$$

Now, third proportional is 486

$$\therefore a : b :: b : 486$$

$$\Rightarrow b^2 = 486a$$

$$\Rightarrow \left(\frac{324}{a}\right)^2 = 486a$$

$$\Rightarrow \frac{(324)^2}{a^2} = 486a$$

$$\Rightarrow a^3 = \frac{324 \times 324}{486}$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

$$b = \frac{324}{a} = \frac{324}{6} = 54$$

Therefore, numbers are 6 and 54