

## Chapter 10. Remainder And Factor Theorems

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### Ex 10.1

#### Answer 1.

$5x^2 - 9x + 4$  is divided by  $(x-2)$

Putting  $x-2=0$ , we get:  $x=2$

Substituting this value of  $x$  in the equation, we get

$$\begin{aligned} 5 \times 2 \times 2 - 9 \times 2 + 4 &= 20 - 18 + 4 \\ &= 6 \end{aligned}$$

$5x^3 - 7x^2 + 3$  is divided by  $(x-1)$

Putting  $x-1=0$ , we get:  $x=1$

Substituting this value of  $x$  in the equation, we get

$$\begin{aligned} 5 \times 1 \times 1 \times 1 - 7 \times 1 \times 1 + 3 \\ &= 5 - 7 + 3 \\ &= 1 \end{aligned}$$

$8x^2 - 2x + 1$  is divided by  $(2x+1)$

Putting  $2x+1=0$ , we get:  $x=-\frac{1}{2}$

Substituting this value of  $x$  in the equation, we get

$$\begin{aligned} 8 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) - 2 \times \left(-\frac{1}{2}\right) + 1 \\ &= 2 + 1 + 1 \\ &= 4 \end{aligned}$$

$x^3 + 8x^2 + 7x - 11$  is divisible by  $(x+4)$

Putting  $x+4=0$ , we get:  $x=-4$

Substituting this value of  $x$  in the equation, we get

$$\begin{aligned} (-4) \times (-4) \times (-4) + 8 \times (-4) \times (-4) + 7 \times (-4) - 11 \\ &= -64 + 128 - 28 - 11 \\ &= 25 \end{aligned}$$

$2x^3 - 3x^2 + 6x - 4$  is divisible by  $(2x-3)$

Putting  $2x-3=0$ , we get:  $x=\frac{3}{2}$

Substituting this value of  $x$  in the equation, we get

$$\begin{aligned} 2 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} - 3 \times \frac{3}{2} \times \frac{3}{2} + 6 \times \frac{3}{2} - 4 \\ &= \frac{27}{4} - \frac{27}{4} + 9 - 4 \\ &= 5 \end{aligned}$$

**Answer 2.**

$(x-2)$  is a factor of  $2x^3 - 7x - 2$

$$x-2=0 \Rightarrow x=2$$

Substituting this value, we get

$$f(2) = 2 \times 2 \times 2 - 7 \times 2 - 2 = 0$$

Hence  $(x-2)$  is a factor of  $2x^3 - 7x - 2$

$(2x+1)$  is a factor of  $4x^3 + 12x^2 + 7x + 1$

$$2x+1=0 \Rightarrow x=-\frac{1}{2}$$

Substituting this value, we get

$$\begin{aligned} f\left(-\frac{1}{2}\right) &= 4 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) + 12 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) + 7 \times \left(-\frac{1}{2}\right) + 1 \\ &= 0 \end{aligned}$$

Hence  $(2x+1)$  is a factor of  $4x^3 + 12x^2 + 7x + 1$

$(3x-2)$  is a factor of  $18x^3 - 3x^2 + 6x - 8$

$$3x-2=0 \Rightarrow x=\frac{2}{3}$$

Substituting this value, we get

$$\begin{aligned} f\left(\frac{2}{3}\right) &= 18 \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) - 3 \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) + 6 \times \left(\frac{2}{3}\right) - 8 \\ &= \frac{16}{3} - \frac{4}{3} + 4 - 8 \\ &= 4 + 4 - 8 \\ &= 0 \end{aligned}$$

Hence  $(3x-2)$  is a factor of  $18x^3 - 3x^2 + 6x - 8$ .

$(2x-1)$  is a factor of  $6x^3 - x^2 - 5x + 2$

$$2x-1=0 \Rightarrow x=\frac{1}{2}$$

Substituting this value, we get

$$\begin{aligned} f\left(\frac{1}{2}\right) &= 6 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{1}{2} - 5 \times \frac{1}{2} + 2 \\ &= \frac{3}{4} - \frac{1}{4} - \frac{5}{2} + 2 \\ &= \frac{1}{2} - \frac{5}{2} + 2 = -2 + 2 = 0 \end{aligned}$$

Hence  $(2x-1)$  is a factor of  $6x^3 - x^2 - 5x + 2$

$(x-3)$  is a factor of  $5x^2 - 21x + 18$ .

**Answer 3.**

$$\Rightarrow x = -2 \dots (i)$$

$$(2x-1) \Rightarrow x = \frac{1}{2} \dots (ii)$$

Putting (i) in polynomial , we get

$$f(-2) = 2x(-2)x(-2)x(-2) + ax(-2)x(-2) + bx(-2) + 10 = 0$$

$$\Rightarrow -16 + 4a - 2b + 10 = 0$$

$$\Rightarrow a = \frac{b}{2} + \frac{3}{2} \dots (iii)$$

Putting (ii) in polynomial , we get

$$\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + a \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + b \times \left(\frac{1}{2}\right) + 10 = 0$$

$$\Rightarrow \frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 10 = 0$$

$$\Rightarrow a = -2b - 41 \dots (iv)$$

Combining (iii) and (iv), we get,

$$\frac{b}{2} + \frac{3}{2} = a = -2b - 41$$

$$\Rightarrow \frac{b+3}{2} = -2b - 41$$

$$\Rightarrow b+3 = -4b - 82$$

$$\Rightarrow 5b = -85$$

$$\Rightarrow b = -17$$

$$\text{and } a = -7$$

$$\Rightarrow a = -7, b = -17$$

**Answer 4.**

$$x-1=0 \Rightarrow x=1 \text{ and remainder is } 2m$$

Substituting this value, we get :

$$f(1) = 1 \times 1 \times 1 + 5 \times 1 \times 1 - m \times 1 + 6 = 2m$$

$$\Rightarrow 3m = 12$$

$$\Rightarrow m = 4$$

**Answer 5.**

$$x-2=0 \Rightarrow x=2 \text{ and remainder is } 0$$

Substituting this value, we get :

$$f(2) = 2x^2x^2 + 3x^2x^2 - mx^2 + 4 = 0$$

$$\Rightarrow 2m=24$$

$$\Rightarrow m=12$$

**Answer 6.**

$$(x-1) \Rightarrow x=1 \dots (i)$$

$$(x-2) \Rightarrow x = 2 \dots (ii)$$

Putting (i) in polynomial , we get

$$f(1) = 1x^1x^1 - px^1x^1 + 14x^1 - q = 0$$

$$\Rightarrow p + q = 15$$

$$\Rightarrow p = 15 - q \dots (iii)$$

Putting (ii) in polynomial , we get

$$f(2) = 2x^2x^2 - px^2x^2 + 14x^2 - q = 0$$

$$4p + q = 36, \Rightarrow q = 36 - 4p \dots (iv)$$

Combining (iii) and (iv), we get,

$$p = 15 - (36 - 4p)$$

$$\Rightarrow p = 15 - 36 + 4p$$

$$\Rightarrow 3p = 21$$

$$q = 36 - 4 \times 7 = 8$$

$$\Rightarrow p = 7, q = 8$$

**Answer 7.**

$$3) \Rightarrow x = -\frac{3}{2} \dots (i)$$

$$(x+2) \Rightarrow x = -2 \dots (ii)$$

Putting (i) in polynomial , we get

$$f\left(-\frac{3}{2}\right) = ax\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right) + 3x\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right) + bx\left(-\frac{3}{2}\right) - 3 \\ = 0$$

$$-27a + 54 - 12b - 24 = 0$$

$$\Rightarrow 27a = -12b + 30 \dots (iii)$$

Putting (ii) in polynomial, and remainder is -3 we get

$$f(-2) = ax(-2)x(-2)x(-2) + 3x(-2)x(-2) + bx(-2) - 3 = -3$$

$$b = 6 - 4a \dots (iv)$$

Combining (iii) and (iv), we get,

$$27a = -12(6-4a) + 30$$

$$\Rightarrow 27a = -72 + 48a + 30,$$

$$\Rightarrow a = 2, b = 6 - 4 \times 2 = -2$$

$$a = 2, b = -2$$

**Answer 8.**

$$(x+2) \Rightarrow x = -2 \dots (i)$$

$$(x+1) \Rightarrow x = -1 \dots (ii)$$

Putting (i) in polynomial, we get

$$f(-2) = (-2)x(-2)x(-2) - 2x(-2)x(-2) + mx(-2) + n = 0$$

$$\Rightarrow -8 - 8 - 2m + n = 0$$

$$\Rightarrow n = 2m + 16 \dots (iii)$$

Putting (ii) in polynomial, and remainder is 9 we get

$$f(-1) = (-1)x(-1)x(-1) - 2x(-1)x(-1) + mx(-1) + n = 9$$

$$\Rightarrow -1 - 2 - m + n = 9,$$

$$\Rightarrow m = n - 12 \dots (iv)$$

Combining (iii) and (iv), we get,

$$n = 2(n - 12) + 16,$$

$$\Rightarrow n = 8.$$

$$\text{Hence, } m = n - 12 = 8 - 12 = -4$$

$$m = -4, n = 8$$

**Answer 9.**

$$(2x+1) \Rightarrow x = -\frac{1}{2}$$

Solving Equation (i), we get

$$f\left(-\frac{1}{2}\right) = 2 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) - 5 \times \left(-\frac{1}{2}\right) + a = 0$$

$$\Rightarrow \frac{1}{2} + \frac{5}{2} + a = 0$$

$$\Rightarrow a = -3$$

$$g\left(-\frac{1}{2}\right) = 2 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) + 5 \times \left(-\frac{1}{2}\right) + b = 0$$

$$\Rightarrow \frac{1}{2} - \frac{5}{2} + b = 0$$

$$\Rightarrow b = 2$$

$$\Rightarrow a = -3, b = 2$$

**Answer 10.**

$$x = -3 \dots (i)$$

$$x = \frac{1}{2} \dots (ii)$$

Putting (i) in polynomial, we get

$$0 = ax(-3)x(-3)x(-3) + bx(-3)x(-3) + (-3) - a = 0$$

$$-27a + 9b - 3 - a = 0$$

$$= \frac{9b}{28} - \frac{3}{28} \dots (iii)$$

Putting (ii) in polynomial, we get

$$0 = ax\left(\frac{1}{2}\right)x\left(\frac{1}{2}\right)x\left(\frac{1}{2}\right) + bx\left(\frac{1}{2}\right)x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - a = 0$$

$$\frac{a}{8} + \frac{b}{4} + \frac{1}{2} - a = 0$$

$$= \frac{7a}{8} - 2 \dots (iv)$$

Adding (iii) and (iv), we get,

$$\frac{a}{8} \times \left(\frac{7a}{8} - 2\right) - \frac{3}{28}$$

$$= 63a - 42$$

$$6$$

$$\frac{7 \times 6}{2} - 2 = 21 - 2 = 19$$

$$b = 19$$

Putting these values in polynomial, we get

$$6x^3 + 19x^2 + x - 6$$

$$\text{equation becomes } (x+3)(2x-1)(3x+2) = 0$$

**Answer 11.**

$$(x-2)=0 \Rightarrow x=2$$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

$$f(2) = 2 \times 2 + 2 + 1 - a = 0$$

$$\Rightarrow a = 7$$

Hence answer = 7

**Answer 12.**

$$(x-1)=0 \Rightarrow x=1$$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be added so that polynomial is exactly subtracted by the factor.

$$f(1) = 2 \times 1 \times 1 \times 1 - 3 \times 1 \times 1 + 7 \times 1 - 8 + a = 0$$

$$\Rightarrow a = 2$$

Hence answer = 2

**Answer 13.**

$$(2x-5)=0 \Rightarrow x=\frac{5}{2}$$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{5}{2}\right) = 2 \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) - 5 \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) + 8 \times \left(\frac{5}{2}\right) - 17 - a = 0$$

$$\Rightarrow \frac{125}{4} - \frac{125}{4} + 20 - 17 - a = 0$$

$$\Rightarrow a = 3$$

- + 20 - 17 - a = 0 Hence answer = 3

**Answer 14.**

$$(2x-1) \Rightarrow x = \frac{1}{2}$$

When we substitute this value in the polynomial, whatever we get as a remainder (say  $a$ ) should be added so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{1}{2}\right) = 12 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + 16 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) - 5 \times \left(\frac{1}{2}\right) - 8 + a = 0$$

$$\Rightarrow \frac{3}{2} + 4 - \frac{5}{2} - 8 + a = 0$$

$$\Rightarrow a = 5$$

$$+ 4 - - 8 + a = 0 \text{ Hence answer } = 5$$

**Answer 15.**

We know that  $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \dots (i)$

And if we put  $a=b \Rightarrow a=b$ , and substitute this to the polynomial, we get:

$$f(x) = 0 + (a-c)^3 + (c-a)^3 = (a-c)^3 - (a-c)^3 = 0$$

Hence,  $(a-b)$  is a factor.  $\Rightarrow a=b \dots (ii)$

Substituting (i) in problem polynomial, we get

$$f(x) = 0 + (b^3 - 3b^2c + 3bc^2 - c^3) + (c^3 - 3c^2a + 3ca^2 - a^3)$$

$$= -3b^2c + 3bc^2 - 3ca^2 + 3ca^2$$

$$= 3(-b^2c + bc^2 - ca^2 + ca^2)$$

If we put  $b=c \Rightarrow b=c$ , and substitute this to the polynomial, we get:

$$f(b=c), 3(-c^2xc + cxc^2 - cxc^2 + cxc^2) = 0$$

Hence, till now factors are  $3x(a-b)x(b-c) \dots (iii)$

Similarly if we had put  $c=a$ , we would have got similar result.

So  $(c-a)$  is also a factor.....(iv)

From (ii), (iii), and (iv), we get

$3(a-b)(b-c)(c-a)$  is a complete factorization of the given polynomial.

**Answer 16.**

If  $p-q$  is assumed to be factor, then  $p=q$ . Substituting this in problem polynomial, we get:

$$\begin{aligned} f(p=q) &= (p-r)^3 + (r-p)^3 \\ &= (p-r)^3 + (-(p-r))^3 \\ &= (p-r)^3 - (p-r)^3 \\ &= 0 \end{aligned}$$

Hence,  $(p-q)$  is a factor.

**Answer 17.**

If  $x-y$  is assumed to be factor, then  $x=y$ . Substituting this in problem polynomial, we get:

$$\begin{aligned} f(x=y) &= yz(y^2-z^2) + zy(z^2-y^2) + yy(y^2-y^2) \\ &= yz(y^2-z^2) + zy(-(y^2-z^2)) + 0 \\ &= yz(y^2-z^2) - yz(y^2-z^2) = 0 \end{aligned}$$

Hence,  $(x-y)$  is a factor.

**Answer 18.**

If  $x-3$  is assumed to be factor, then  $x=3$ . Substituting this in problem polynomial, we get:

$$f(3) = 3 \times 3 \times 3 - 3 \times 3 - 9 \times 3 + 9 = 0$$

Hence its proved that  $x-3$  is a factor of the polynomial.

### Answer 19.

If  $x + 1$  is assumed to be factor, then  $x = -1$ . Substituting this in problem polynomial, we get:

$$f(-1) = (-1)x(-1)x(-1) - 6x(-1)x(-1) + 5x(-1) + 12 = 0$$

Hence  $(x+1)$  is a factor of the polynomial.

Multiplying  $(x+1)$  by  $x^2$ , we get  $x^3 + x^2$ , hence we are left with  $-7x^2 + 5x + 12$  (and 1<sup>st</sup> part of factor as  $x^2$ ).

Multiplying  $(x+1)$  by  $-7x$ , we get  $-7x^2 - 7x$ , hence we are left with  $12x + 12$  (and 2<sup>nd</sup> part of factor as  $-7x$ ).

Multiplying  $(x+1)$  by  $12$ , we get  $12x + 12$ , hence we are left with  $0$  (and 3<sup>rd</sup> part of factor as  $12$ ).

Hence complete factor is  $(x+1)(x^2-7x+12)$ .

Further factorizing  $(x^2-7x+12)$ , we get:

$$x^2 - 3x - 4x + 12 = 0$$

$$\Rightarrow (x-4)(x-3)=0$$

Hence answer is  $(x+1)(x-4)(x-3) = 0$

**Answer 20.**

If  $5x - 4$  is assumed to be factor, then  $x = \frac{4}{5}$ . Substituting this in problem polynomial, we get:

$$\begin{aligned}f\left(\frac{4}{5}\right) &= 5 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 4 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 5 \times \left(\frac{4}{5}\right) + 4 \\&= \frac{64}{25} - \frac{64}{25} - 4 + 4 \\&= 0\end{aligned}$$

Hence  $(5x-4)$  is a factor of the polynomial.

Multiplying  $(5x-4)$  by  $x^2$ , we get  $5x^3 - 4x^2$ , hence we are left with  $-5x + 4$  (and 1<sup>st</sup> part of factor as  $x^2$ ).

Multiplying  $(5x-4)$  by  $-1$ , we get  $-5x + 4$ , hence we are left with 0 (and 2<sup>nd</sup> part of factor as  $-7x$ ).

Hence complete factor is  $(5x-4)(x^2-1)$ .

Further factorizing  $(x^2-1)$ , we get:

$$\Rightarrow (x-1)(x+1) = 0$$

Hence answer is  $(5x-4)(x-1)(x+1) = 0$

**Answer 21.**

$$\text{Given } f(x) = (x-1)(x-2)+(-2x+5)$$

$$=(x^2-3x+2)+(-2x+5)$$

$$f(x)=x^2-5x+7$$

Substituting  $x=1$

$$f(x) = 1-5+7 = 3$$

when  $f(x)$  is divided by  $(x-1)$ , remainder = 3

substituting  $x=2$

$$f(x) = 4-10+7 = 1$$

when  $f(x)$  is divided by  $(x-2)$ , remainder = 1

$$\frac{x^2 - 5x + 7}{x^2 - 3x + 2} = 1 \frac{(-2x + 5)}{(x - 1)(x - 2)}$$

and

when  $f(x)$  is divided by  $(x-1)(x-2)$ , remainder =  $(-2x+5)$ .

**Answer 22.**

When  $x + 1$  is a factor, we can substitute  $x=-1$  to evaluate values. ...(i)

When  $x - 2$  is a factor, we can substitute  $x=2$  to evaluate values. ...(ii)

When  $2x - 1$  is a factor, we can substitute  $x=\frac{1}{2}$  to evaluate values. ...(iii)

Substituting (i), we get

$$f(-1) = a(-1)^4 + (-1)^3 + b(-1)^2 - 4(-1) + c = 0$$

$$\Rightarrow a + b + c = -3,$$

$$\Rightarrow a = -b - c - 3 \dots (iv)$$

Substituting (ii), we get

$$\Rightarrow f(2) = a(2)^4 + (2)^3 + b(2)^2 - 4(2) + c = 0$$

$$\Rightarrow 16a + 4b + c = 0 \dots (v)$$

Substituting (iii), we get

$$\Rightarrow f\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + c = 0$$

$$\Rightarrow \frac{a}{16} + \frac{b}{4} + c = 2 - \frac{1}{8}$$

$$\Rightarrow a + 4b + 16c = 30 \dots (vi)$$

Putting (iv) in (v) and (vi), we get:

$$5a + 4b + c = 0; \Rightarrow 16x(-b - c - 3) + 4b + c = 0$$

$$\Rightarrow -12b - 15c - 48 = 0 \Rightarrow 4b + 5c = -16$$

$$\Rightarrow b = -4 - \frac{5c}{4} \dots (vii)$$

$$+ 4b + 16c = 30 \Rightarrow (-b - c - 3) + 4b + 16c = 30$$

$$\Rightarrow 3b + 15c = 33 \dots (viii)$$

Putting (vii) in (viii), we get,

$$\Rightarrow 3\left(-4 - \frac{5c}{4}\right) + 15c = 33,$$

$\Rightarrow$  Solving this, we get

$$\Rightarrow c = 4$$

Putting this value of c in (viii), we get: