

Chapter 11. Matrices

Ex 11.1

Answer 2.

$$\begin{bmatrix} 2a+3b & a-b \end{bmatrix} = \begin{bmatrix} 19 & 2 \end{bmatrix}$$

$\begin{bmatrix} 2a+3b & a-b \end{bmatrix}$ is 1×2 matrix and $\begin{bmatrix} 19 & 2 \end{bmatrix}$ is 1×2 matrix

$$2a+3b = 19 \quad \text{--- (1)}$$

$$a-b = 2 \quad \text{--- (2)}$$

$$\Rightarrow a = 2+b$$

Putting the value of a in equation (1)

$$4+2b+3b = 19 \dots (1)$$

$$\Rightarrow 5b = 15$$

$$\Rightarrow b = 3$$

From (2)

$$a = 2+3 = 5$$

Answer 3.

$$\begin{bmatrix} 3x-y \\ 5 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 7 \\ x+y \end{bmatrix}_{2 \times 1}$$

$$3x-y=7 \quad \text{--- (1)}$$

$$x+y=5 \quad \text{--- (2)}$$

$$\Rightarrow x = 5-y$$

putting the value of x in (1)

$$3(5-y)-y=7$$

$$\Rightarrow 15-3y-y=7$$

$$\Rightarrow -4y=-8$$

$$\Rightarrow y=2$$

from (2)

$$x+2=5$$

$$\Rightarrow x=3$$

Answer 6.

$$A = \begin{bmatrix} 4 & 7 \end{bmatrix}_{1 \times 2} \quad B = \begin{bmatrix} 3 & 1 \end{bmatrix}_{1 \times 2}$$

(i) $A + 2B$

$$\begin{aligned} A + 2B &= \begin{bmatrix} 4 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 4+6 & 7+2 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 9 \end{bmatrix}_{1 \times 2} \end{aligned}$$

(ii) $A - B$

$$\begin{aligned} A - B &= \begin{bmatrix} 4-3 & 7-1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 6 \end{bmatrix}_{1 \times 2} \end{aligned}$$

(iii) $2A - 3B$

$$\begin{aligned} 2A - 3B &= \begin{bmatrix} 8 & 14 \end{bmatrix} - \begin{bmatrix} 9 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 11 \end{bmatrix}_{1 \times 2} \end{aligned}$$

Answer 7.

$$P = \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix}_{2 \times 2}, \quad Q = \begin{bmatrix} 7 & 3 \\ 4 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} \text{(i)} \quad 2P + 3Q &= \begin{bmatrix} 4 & 18 \\ 10 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 9 \\ 12 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 4+21 & 18+9 \\ 10+12 & 14+3 \end{bmatrix} = \begin{bmatrix} 25 & 27 \\ 22 & 17 \end{bmatrix}_{2 \times 2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 2Q - P &= \begin{bmatrix} 14 & 6 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 14-2 & 6-9 \\ 8-5 & 2-7 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 3 & -5 \end{bmatrix}_{2 \times 2} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 3P - 2Q &= \begin{bmatrix} 6 & 27 \\ 15 & 21 \end{bmatrix} - \begin{bmatrix} 14 & 6 \\ 8 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 6-14 & 27-6 \\ 15-8 & 21-2 \end{bmatrix} = \begin{bmatrix} -8 & 21 \\ 7 & 19 \end{bmatrix}_{2 \times 2} \end{aligned}$$

Answer 8.

$$A = \begin{bmatrix} 17 & 5 & 19 \\ 11 & 8 & 13 \end{bmatrix}_{2 \times 3}, B = \begin{bmatrix} 9 & 3 & 7 \\ 1 & 6 & 5 \end{bmatrix}_{2 \times 3}$$

$$\begin{aligned} 2A - 3B &= \begin{bmatrix} 34 & 10 & 38 \\ 22 & 16 & 26 \end{bmatrix} - \begin{bmatrix} 27 & 9 & 21 \\ 3 & 18 & 15 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 1 & 17 \\ 19 & -2 & 11 \end{bmatrix}_{2 \times 3} \end{aligned}$$

Answer 9.

$$M = \begin{bmatrix} 8 & 3 \\ 9 & 7 \\ 4 & 3 \end{bmatrix}_{3 \times 2}, N = \begin{bmatrix} 4 & 7 \\ 5 & 3 \\ 10 & 1 \end{bmatrix}_{3 \times 2}$$

$$(i) \quad M + N = \begin{bmatrix} 8+4 & 3+7 \\ 9+5 & 7+3 \\ 4+10 & 3+1 \end{bmatrix} = \begin{bmatrix} 12 & 10 \\ 14 & 10 \\ 14 & 4 \end{bmatrix}_{3 \times 2}$$

$$(ii) \quad M - N = \begin{bmatrix} 8-4 & 3-7 \\ 9-5 & 7-3 \\ 4-10 & 3-1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & 4 \\ -6 & 2 \end{bmatrix}$$

Answer 10.

$$A = \begin{bmatrix} 1 & 9 & 4 \\ 5 & 0 & 3 \end{bmatrix}_{2 \times 3}$$

$$(i) \quad \text{Negative } A = \begin{bmatrix} -1 & -9 & -4 \\ -5 & 0 & -3 \end{bmatrix}_{2 \times 3}$$

$$(ii) \quad A^t = \begin{bmatrix} 1 & 5 \\ 9 & 0 \\ 4 & 3 \end{bmatrix}_{3 \times 2}$$

Answer 11.

$$P = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix}_{2 \times 2}$$

$$(i) \quad P^t = \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}_{2 \times 2}$$

$$(ii) \quad P + P^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix} \\ = \begin{bmatrix} 16 & 12 \\ 12 & 4 \end{bmatrix}_{2 \times 2}$$

$$(iii) \quad P - P^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix} \\ = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}_{2 \times 2}$$

Answer 12.

$$B = \begin{bmatrix} 15 & 13 \\ 11 & 12 \\ 10 & 17 \end{bmatrix}_{3 \times 2}$$

$$B^t = \begin{bmatrix} 15 & 11 & 10 \\ 13 & 12 & 17 \end{bmatrix}_{2 \times 3}$$

To add two matrixes their order i.e. their corresponding number of rows and number of columns should be same, whereas in this case order of B and B^t are not same hence, we cannot add them.

Answer 13.

$$A = \begin{bmatrix} 5 & r \\ p & 7 \end{bmatrix}, B = \begin{bmatrix} q & 4 \\ 3 & s \end{bmatrix}$$

$$A + B = \begin{bmatrix} s & r \\ p & 7 \end{bmatrix} + \begin{bmatrix} q & 4 \\ 3 & s \end{bmatrix}$$

$$A + B = \begin{bmatrix} s+q & r+4 \\ p+3 & 7+s \end{bmatrix}_{2 \times 2} \quad -(1)$$

But, given

$$A + B = \begin{bmatrix} 9 & 7 \\ 5 & 8 \end{bmatrix}_{2 \times 2} \quad -(2)$$

from (1) & (2)

$$\begin{bmatrix} s+q & r+4 \\ p+3 & 7+s \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 5 & 8 \end{bmatrix}$$

$$s+q=9$$

$$\Rightarrow q=4$$

$$r+4=7$$

$$\Rightarrow r=3$$

$$p+3=5$$

$$\Rightarrow p=2$$

$$7+s=8$$

$$\Rightarrow s=1$$

Answer 14.

$$A = \begin{bmatrix} p & q \\ 8 & 5 \end{bmatrix}_{2 \times 2}, \quad B = \begin{bmatrix} 3p & 5q \\ 2q & 7 \end{bmatrix}_{2 \times 2}$$

$$A + B = \begin{bmatrix} p + 3q & q + 5q \\ 8 + 2q & 5 + 7 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4p & 6q \\ 8 + 2q & 12 \end{bmatrix}_{2 \times 2} \quad - \quad (1)$$

$$\text{But given, } A + B = \begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}_{2 \times 2} \quad - (2)$$

from (1) and (2)

$$\begin{bmatrix} 4p & 6q \\ 8 + 2q & 12 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}$$

$$4p = 12$$

$$\Rightarrow p = 3$$

$$6q = 6$$

$$\Rightarrow q = 1$$

$$8 + 2q = 2r$$

$$8 + 2 = 2r$$

$$\Rightarrow r = 5$$

$$12 = 3s$$

$$\Rightarrow s = 4$$

Answer 15.

Given,

$$\begin{bmatrix} 2a+b & c \\ d & 3a-b \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 4 & 3a \\ 7 & 6 \end{bmatrix}_{2 \times 2}$$

$$2a+b=4 \quad \text{---(1)}$$

$$3a-b=6 \quad \text{---(2)}$$

Adding (1) and (2), we get

$$5a=10$$

$$\Rightarrow a=2$$

from (1)

$$2(2)+b=4$$

$$\Rightarrow b=0$$

$$c=3a$$

$$\Rightarrow c=3 \times 2$$

$$\Rightarrow c=6$$

$$\Rightarrow d=7$$

Answer 17.

$$A = \begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix}_{2 \times 2}, B = \begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}_{2 \times 2}$$

$$(i) \quad A + X = B$$

$$X = B - A$$

$$\therefore X = \begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix} - \begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 14 & 3 \end{bmatrix}_{2 \times 2}$$

$$(ii) \quad 2A - X = B$$

$$X = 2A - B$$

$$\begin{aligned} X &= \begin{bmatrix} 30 & 14 \\ 26 & 16 \end{bmatrix} - \begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 2 \\ -1 & 5 \end{bmatrix}_{2 \times 2} \end{aligned}$$

Answer 18.

$$P = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix}_{2 \times 2}, \quad Q = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}_{2 \times 2}$$

$$P - M = 3Q$$

$$M = P - 3Q$$

$$M = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 9 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 14 - 6 & 17 - 3 \\ 13 - 9 & 1 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 14 \\ 4 & 10 \end{bmatrix}_{2 \times 2}$$

Ex 11.2**Answer 2.**

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}_{2 \times 2}, B = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} \text{(a)} \quad AB &= \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -2-12 & 3+3 \\ -6-8 & 9+2 \end{bmatrix} = \begin{bmatrix} -14 & 6 \\ -14 & 11 \end{bmatrix}_{2 \times 2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad BA &= \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2+9 & -6+6 \\ -4+3 & -12+2 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 0 \\ -1 & -10 \end{bmatrix}_{2 \times 2} \end{aligned}$$

Answer 3.

$$\text{Sol: Let } A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} AI &= \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} p+0 & 0+q \\ r+0 & 0+s \end{bmatrix} \\ &= \begin{bmatrix} p & q \\ r & s \end{bmatrix} = A \end{aligned}$$

$$\begin{aligned} IA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} \\ &= \begin{bmatrix} p+0 & q+0 \\ 0+r & 0+s \end{bmatrix} \\ &= \begin{bmatrix} p & q \\ r & s \end{bmatrix} = A \end{aligned}$$

Hence proved $AI = IA = A$.

Answer 4.

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2 \times 2}, \quad Q = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} QP &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}_{2 \times 2} \end{aligned}$$

$$\begin{aligned} P(QP) &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}_{2 \times 2} \end{aligned}$$