

Binomial Theorem

- The coefficients of the expansions of a binomial are arranged in an array. This array is called Pascal's triangle. It can be written as

Index	Coefficient(s)
0	0C_0 (= 1)
1	1C_0 1C_1 (= 1) (= 1)
2	2C_0 2C_1 2C_2 (= 1) (= 2) (= 1)
3	3C_0 3C_1 3C_2 3C_3 (= 1) (= 3) (= 3) (= 1)
4	4C_0 4C_1 4C_2 4C_3 4C_4 (= 1) (= 4) (= 6) (= 4) (= 1)
5	

- General Term:** The $(r + 1)^{\text{th}}$ term (denoted by T_{r+1}) is known as the general term of the expansion $(a + b)^n$ and it is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Example 1: In the expansion of $(5x - 7y)^9$, find the general term?

Solution: $T_{r+1} = {}^9C_r (5x)^{9-r} (-7y)^r = (-1)^r {}^9C_r (5x)^{9-r} (7y)^r$

- Middle term in the expansion of $(a + b)^n$:**
 - If n is even, then the number of terms in the expansion will be $n + 1$. Since n is even, $n + 1$ is odd. Therefore, the middle term is $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term.
 - If n is odd, then $n + 1$ is even. So, there will be two middle terms in the expansion. They are $\left(\frac{n+1}{2}\right)^{\text{th}}$ term and $\left(\frac{n+1}{2} + 1\right)^{\text{th}}$ term.

- In the expansion of $\left(x + \frac{1}{x}\right)^{2n}$, where $x \neq 0$, the middle term is $\left(\frac{2n}{2} + 1\right)^{\text{th}}$, i.e., $(n + 1)^{\text{th}}$ term [since $2n$ is even].

It is given by ${}^{2n}C_n x^n \left(\frac{1}{x}\right)^n = {}^{2n}C_n$ which is a constant.

This term is called the term independent of x or the constant term.

Note: In the expansion of $(a + b)^n$, r^{th} term from the end = $(n - r + 2)^{\text{th}}$ term from the beginning

Example 2: In the expansion of _____, find the middle term and find the term

which is independent of x . $\left(\frac{x^3}{4} - \frac{12}{x}\right)^4$

Solution: As 4 is even, the middle term in the expansion of _____ is the $\left(\frac{4}{2} + 1\right)^{\text{th}}$

term, i.e., 3rd term, which is given by

$$\left(\frac{x^3}{4} - \frac{12}{x}\right)^4$$

$$T_3 = T_{2+1} = {}^4C_2 \left(\frac{x^3}{4}\right)^2 \left(\frac{-12}{x}\right)^2$$

$$= 6 \times \frac{x^6}{16} \times \frac{144}{x^2}$$

$$= 54x^4$$

Now, we will find the term in the expansion which is independent of x . Suppose $(r + 1)^{\text{th}}$ term is independent of x .

The $(r + 1)^{\text{th}}$ term in the expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Hence, the $(r + 1)^{\text{th}}$ term in the expansion of $\left(\frac{x^3}{4} - \frac{12}{x}\right)^4$ is given by

$$\left(\frac{x^3}{4} - \frac{12}{x}\right)^4$$