Chapter 21. Trigonometric Identities

Ex 21.1

Answer 5.

(i) $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

$(\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} \left(\because 1-\sin^2\theta = \cos^2\theta\right)$

$= \frac{1-\sin\theta}{1+\sin\theta}$

(ii) $\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{2\sin A}{1-2\cos^2 A}$

$\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A}$

$= \frac{\sin A - \cos A + \sin A + \cos A}{\sin^2 A - \cos^2 A}$

$= \frac{2\sin A}{1-\cos^2 A - \cos^2 A} = \frac{2\sin A}{1-2\cos^2 A}$

(iii) $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$

$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A + \cos A)(\sin A - \cos A)}$

$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin\cos A}{\sin^2 A - \cos^2 A}$

$= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$

$= \frac{2}{\sin^2 A - \cos^2 A}$ $[\sin^2 A + \cos^2 A = 1]$

$= \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - (1-\sin^2 A)}$

$\Rightarrow \frac{2}{2\sin^2 A - 1}$

(iv) $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

$\text{L.H.S} = \tan^2 A - \tan^2 B$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\cos^2 B}$

$= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A\cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A\cos^2 B - \cos^2 A + \cos^2 A\cos^2 B}{\cos^2 A\cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B}$

$= \frac{(1-\sin^2 B) - (1-\sin^2 A)}{\cos^2 A\cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

Hence $\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

(v) $\frac{\cos A}{1-\tan A}+\frac{\sin^2 A}{\sin A-\cos A}=\cos A+\sin A$

$$\frac{\cos A}{1-\tan A}+\frac{\sin^2 A}{\sin A-\cos A}$$
$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin^2 A}{\sin A-\cos A}$$
$$=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin^2 A}{\sin A-\cos A}$$
$$=\frac{\cos^2 A}{(\cos A-\sin A)}-\frac{\sin^2 A}{(\cos A-\sin A)}$$
$$=\frac{(\cos^2 A-\sin^2 A)}{(\cos A-\sin A)}=\frac{(\cos A+\sin A)(\cos A-\sin A)}{(\cos A-\sin A)}$$
$$=(\cos A+\sin A)$$

(vi) $(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$

$$(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)$$
$$=\left(1+\frac{\sin^2 A}{\cos^2 A}\right)+\left(1+\frac{1}{\frac{\sin^2 A}{\cos^2 A}}\right)$$
$$=\left(\frac{\cos^2 A+\sin^2 A}{\cos^2 A}\right)+\left(\frac{\cos^2 A+\sin^2 A}{\sin^2 A}\right)$$
$$=\frac{1}{1-\sin^2 A}+\frac{1}{\sin^2 A}(\because \cos^2 A+\sin^2 A=1)$$
$$=\frac{\sin^2 A+1-\sin^2 A}{\sin^2 A(1-\sin^2 A)}=\frac{1}{\sin^2 A-\sin^4 A}$$

(vii) $\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}=2$

$$\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$$
$$=\frac{(\cos^3 A+\sin^3 A)(\cos A-\sin A)+(\cos^3 A-\sin^3 A)(\cos A+\sin A)}{\cos^2 A-\sin^2 A}$$
$$=\frac{\cos^4 A-\cos^3 A\sin A+\sin^3 A\cos A-\sin^4 A+\cos^4 A+\cos^3 A\sin A-\sin^3 A\cos A=\sin^4 A}{\cos^2 A-\sin^2 A}$$
$$=\frac{2(\cos^4 A-\sin^4 A)}{\cos^2 A-\sin^2 A}=\frac{2(\cos^2 A+\sin^2 A)(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)}=2(\cos^2 A+\sin^2 A)$$
$$=2\ (\because \cos^2 A+\sin^2 A=1)$$

OR

$$\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$$
$$=\frac{(\cos A+\sin A)(\cos^2 A+\sin^2 A-\cos A\sin A)}{(\cos A+\sin A)}+\frac{(\cos A-\sin A)(\cos^2 A+\sin^2 A+\cos A\sin A)}{(\cos A-\sin A)}$$
$$(\because a^3\pm b^3=(a\pm b)(a^2+b^2\mp ab))$$
$$=(\cos^2 A+\sin^2 A-\cos A\sin A)+(\cos^2 A+\sin^2 A+\cos A\sin A)$$
$$=1-\cos A\sin A+1+\cos A\sin A(\because \cos^2 A+\sin^2 A=1)$$
$$=2$$

(viii) $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

$$\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2$$
$$=\left(\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)^2+\left(\frac{\sin\theta}{\cos\theta}-\frac{1}{\cos\theta}\right)^2$$
$$=\left(\frac{\sin\theta+1}{\cos\theta}\right)^2+\left(\frac{\sin\theta-1}{\cos\theta}\right)^2$$
$$=\frac{(\sin\theta+1)^2}{\cos^2\theta}+\frac{(\sin\theta-1)^2}{\cos^2\theta}$$
$$=\frac{(\sin\theta+1)^2+(\sin\theta-1)^2}{\cos^2\theta}$$
$$=\frac{\sin^2\theta+1+2\sin\theta+\sin^2\theta+1-2\sin\theta}{1-\sin^2\theta}$$
$$=\frac{2(1+\sin^2\theta)}{1-\sin^2\theta}$$

(ix) $\dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B} = 0$

$$\text{L.H.S} = \frac{\sin A + \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A - \sin B}$$
$$= \frac{(\sin A + \sin B)(\sin A - \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A - \sin B)}$$
$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A - \sin B)}$$
$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A - \sin B)}$$
$$= \frac{1-1}{(\cos A + \cos B)(\sin A - \sin B)}$$
$$= \frac{0}{(\cos A + \cos B)(\sin A - \sin B)}$$
$$= 0$$

$$\frac{\sin A + \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A - \sin B} = 0$$

Hence proved.

(x) $\dfrac{1}{\cos A + \sin A - 1} + \dfrac{1}{\cos A + \sin A + 1} = \operatorname{cosec} A + \sec A$

$$\frac{1}{(\cos A + \sin A) - 1} + \frac{1}{(\cos A + \sin A) + 1}$$
$$= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1}$$
$$= \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1}$$
$$= \frac{2(\cos A + \sin A)}{1 + 2\cos A \sin A - 1} = \frac{\cos A + \sin A}{\cos A \sin A}$$
$$= \frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A}$$
$$= \frac{1}{\sin A} + \frac{1}{\cos A}$$
$$= \operatorname{cosec} A + \sec A$$

(xi) $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{\cos A + 1}{\sin A}$

$$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$
$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \qquad [\operatorname{cosec}^2 A - \cot^2 A = 1]$$
$$= \frac{\cot A + \operatorname{cosec} A - [(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$
$$= \frac{\cot A + \operatorname{cosec} A[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1}$$
$$= \cot A + \operatorname{cosec} A$$
$$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$
$$= \frac{1 + \cos A}{\sin A}$$

(xii) $\dfrac{\sec A - 1}{\sec A + 1} = \dfrac{\sin^2 A}{(1 + \cos A)^2}$

$$\frac{\sec A - 1}{\sec A + 1}$$
$$= \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} = \frac{1 - \cos A}{1 + \cos A}$$
$$= \frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}$$
$$= \frac{1 - \cos^2 A}{(1 + \cos A)^2}$$
$$= \frac{\sin^2 A}{(1 + \cos A)^2} \quad (\because 1 - \cos^2 A = \sin^2 A)$$

Answer 6.

(i) $(1+\cot A)^2+(1-\cot A)^2=2\operatorname{cosec}^2 A$

$(1+\cot A)^2+(1-\cot A)^2$

$=1+\cot^2 A+2\cot A+1+\cot^2 A-2\cot A$

$=2+2\cot^2 A=2\left(1+\cot^2 A\right)$

$=2\operatorname{cosec}^2 A$

(ii) $\dfrac{\operatorname{cosec}\theta}{\tan\theta+\cot\theta}=\cos\theta$

$\dfrac{\operatorname{cosec}\theta}{\tan\theta+\cot\theta}$

$=\dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$

$=\dfrac{\frac{1}{\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}=\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta\sin\theta}}$

$=\dfrac{1}{\sin\theta}\times\dfrac{\cos\theta\sin\theta}{1}=\cos\theta$

(iii) $\left(1+\tan^2\theta\right)\sin\theta\cos\theta=\tan\theta$

$\left(1+\tan^2\theta\right)\sin\theta\cos\theta$

$=\left(1+\dfrac{\sin^2\theta}{\cos^2\theta}\right)\sin\theta\cos\theta$

$=\left(\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)\sin\theta\cos\theta$

$=\dfrac{1}{\cos^2\theta}\times\sin\theta\cos\theta\ \left(\because \cos^2\theta+\sin^2\theta=1\right)$

$=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

(iv) $\dfrac{1+\sin\theta}{\operatorname{cosec}\theta-\cot\theta}-\dfrac{1-\sin\theta}{\operatorname{cosec}\theta+\cot\theta}=2\left(1+\cot\theta\right)$

$\dfrac{1+\sin\theta}{\operatorname{cosec}\theta-\cot\theta}-\dfrac{1-\sin\theta}{\operatorname{cosec}\theta+\cot\theta}$

$=\dfrac{(1+\sin\theta)(\operatorname{cosec}\theta+\cot\theta)-(1-\sin\theta)(\operatorname{cosec}\theta-\cot\theta)}{\operatorname{cosec}^2\theta-\cot^2\theta}$

$=\dfrac{\operatorname{cosec}\theta+\cot\theta+1+\cos\theta-\operatorname{cosec}\theta+\cot\theta+1-\cos\theta}{1+\cot^2\theta-\cot^2\theta}\quad \left(\because \operatorname{cosec}^2\theta=1+\cot^2\theta\right)$

$=2+2\cot\theta=2(1+\cot\theta)$

(v) $(1+\cot A+\tan A)(\sin A-\cos A)=\dfrac{\sec A}{\operatorname{cosec}^2 A}-\dfrac{\operatorname{cosec} A}{\sec^2 A}$

$(1+\cot A+\tan A)(\sin A-\cos A)$

$=\left(1+\dfrac{\cos A}{\sin A}+\dfrac{\sin A}{\cos A}\right)(\sin A-\cos A)$

$=\left(\dfrac{\sin A\cos A+\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)$

$=\dfrac{\left(\sin^3 A-\cos^3 A\right)}{\sin A\cos A}\ \left(\because \left(\sin^3 A-\cos^3 A\right)=(\sin A-\cos A)\left(\sin A\cos A+\cos^2 A+\sin^2 A\right)\right)$

$=\dfrac{\sin^3 A}{\sin A\cos A}-\dfrac{\cos^3 A}{\sin A\cos A}$

$=\dfrac{\sin^2 A}{\cos A}-\dfrac{\cos^2 A}{\sin A}=\dfrac{1}{\cos A}\times\sin^2 A-\dfrac{1}{\sin A}\times\cos^2 A$

$=\sec A\sin^2 A-\operatorname{cosec} A\cos^2 A$

$=\dfrac{\sec A}{\operatorname{cosec}^2 A}-\dfrac{\operatorname{cosec} A}{\sec^2 A}$

$(vi)\ 2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1=0$

$$2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1$$
$$=2\left[\left(\sin^2\theta\right)^3+\left(\cos^2\theta\right)^3\right]-3\left(\sin^4\theta+\cos^4\theta\right)+1$$
$$=2\left[\left(\sin^2\theta+\cos^2\theta\right)\left\{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2-\sin^2\theta\cos^2\theta\right\}\right]-3\left(\sin^4\theta+\cos^4\theta\right)+1$$
$$=2\left\{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2-\sin^2\theta\cos^2\theta\right\}-3\left(\sin^4\theta+\cos^4\theta\right)+1$$
$$=2\sin^4\theta+2\cos^4\theta-2\sin^2\theta\cos^2\theta-3\sin^4\theta-3\cos^4\theta+1$$
$$=-\sin^4\theta-\cos^4\theta-2\sin^2\theta\cos^2\theta+1$$
$$=-\left(\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta\right)+1$$
$$=-\left(\sin^2\theta+\cos^2\theta\right)^2+1=-1+1=0$$

$(vii)\ \sin^8\theta-\cos^8\theta=\left(\sin^2\theta-\cos^2\theta\right)\left(1-2\sin^2\theta\cos^2\theta\right)$

$$\sin^8\theta-\cos^8\theta$$
$$=\left(\sin^4\theta\right)^2-\left(\cos^4\theta\right)^2$$
$$=\left(\sin^4\theta-\cos^4\theta\right)\left(\sin^4\theta+\cos^4\theta\right)$$
$$=\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)\left(\sin^4\theta+\cos^4\theta\right)$$
$$=\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^4\theta+\cos^4\theta\right)$$
$$=\left(\sin^2\theta-\cos^2\theta\right)\left(\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta\right)$$
$$=\left(\sin^2\theta-\cos^2\theta\right)\left(\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta\right)$$
$$=\left(\sin^2\theta-\cos^2\theta\right)\left(1-2\sin^2\theta\cos^2\theta\right)$$

$(viii)\ \sec^4 A-\sec^2 A=\dfrac{\sin^2 A}{\cos^4 A}$

$$\sec^4 A-\sec^2 A=\frac{1}{\cos^4 A}-\frac{1}{\cos^2 A}$$
$$=\frac{1-\cos^2 A}{\cos^4 A}$$
$$=\frac{\sin^2 A}{\cos^4 A}\ \left[\because \sin^2 A=1-\cos^2 A\right]$$

$(ix)\ \dfrac{\tan^2\theta}{\tan^2\theta-1}+\dfrac{\operatorname{cosec}^2\theta}{\sec^2\theta-\operatorname{cosec}^2\theta}=\dfrac{1}{\sin^2\theta-\cos^2\theta}$

$$\frac{\tan^2\theta}{\tan^2\theta-1}+\frac{\operatorname{cosec}^2\theta}{\sec^2\theta-\operatorname{cosec}^2\theta}=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}-1}+\frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}-\frac{1}{\sin^2\theta}}$$
$$=\frac{\sin^2\theta}{\sin^2\theta-\cos^2\theta}+\frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta-\cos^2\theta}{\cos^2\theta\sin^2\theta}}$$
$$=\frac{\sin^2\theta}{\sin^2\theta-\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta-\cos^2\theta}$$
$$=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=\frac{1}{\sin^2\theta-\cos^2\theta}\ \left(\because \sin^2\theta+\cos^2\theta=1\right)$$

$(x)\ \dfrac{\sec^2\theta-\sin^2\theta}{\tan^2\theta}=\operatorname{cosec}^2\theta-\cos^2\theta$

$$\frac{\sec^2\theta-\sin^2\theta}{\tan^2\theta}=\frac{\frac{1}{\cos^2\theta}-\sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$=\frac{\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$=\frac{1-\sin^2\theta\cos^2\theta}{\sin^2\theta}$$
$$=\frac{1}{\sin^2\theta}-\frac{\sin^2\theta\cos^2\theta}{\sin^2\theta}$$
$$=\operatorname{cosec}^2\theta-\cos^2\theta$$

(xi) $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=2$

$$\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos^3\theta+\sin^3\theta)(\cos\theta-\sin\theta)+(\cos^3\theta-\sin^3\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$$

$$=\frac{\cos^4\theta-\cos^3\theta\sin\theta+\sin^3\theta\cos\theta-\sin^4\theta+\cos^4\theta+\cos^3\theta\sin\theta-\sin^3\theta\cos\theta-\sin^4\theta}{\cos^2\theta-\sin^2\theta}$$

$$=\frac{2\cos^4\theta-2\sin^4\theta}{\cos^2\theta-\sin^2\theta}=\frac{2(\cos^4\theta-\sin^4\theta)}{\cos^2\theta-\sin^2\theta}$$

$$=\frac{2(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)}{(\cos^2\theta-\sin^2\theta)}=2(\cos^2\theta+\sin^2\theta)$$

$$=2\ \left(\because (\cos^2\theta+\sin^2\theta)=1\right)$$

OR

$$\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)}{(\cos\theta+\sin\theta)}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{(\cos\theta-\sin\theta)}$$

$(\because a^3\pm b^3=(a\pm b)(a^2+b^2\mp ab))$

$$=(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)+(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)$$

$$=1-\cos\theta\sin\theta+1+\cos\theta\sin\theta\ (\because \cos^2\theta+\sin^2\theta=1)$$

$$=2$$

(xii) $\dfrac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta}=\dfrac{\sec\theta+1}{\sec\theta-1}$

$$\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}=\frac{\sin\theta+\sin\theta\cos\theta}{\sin\theta+\sin\theta\cos\theta}$$

$$=\frac{\sin\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}=\frac{1+\cos\theta}{1-\cos\theta}$$

$$=\frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}=\frac{\frac{\sec\theta+1}{\sec\theta}}{\frac{\sec\theta-1}{\sec\theta}}=\frac{\sec\theta+1}{\sec\theta-1}$$

(xiii) $\left[\dfrac{1}{(\sec^2\theta-\cos^2\theta)}+\dfrac{1}{(\operatorname{cosec}^2\theta-\sin^2\theta)}\right](\sin^2\theta\cos^2\theta)=\dfrac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

$$\left[\frac{1}{(\sec^2\theta-\cos^2\theta)}+\frac{1}{(\operatorname{cosec}^2\theta-\sin^2\theta)}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{1}{\left(\frac{1}{\cos^2\theta}-\cos^2\theta\right)}+\frac{1}{\left(\frac{1}{\sin^2\theta}-\sin^2\theta\right)}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{1}{\left(\frac{1-\cos^4\theta}{\cos^2\theta}\right)}+\frac{1}{\left(\frac{1-\sin^4\theta}{\sin^2\theta}\right)}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{\cos^2\theta-\cos^2\theta\sin^2\theta+\sin^2\theta-\sin^2\theta\cos^4\theta}{(1-\cos^4\theta)(1-\sin^4\theta)}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{\cos^2\theta+\sin^2\theta-\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}\right](\sin^2\theta\cos^2\theta)$$

$$=\left[\frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta(1+\cos^2\theta)\cos^2\theta(1+\sin^2\theta)}\right](\sin^2\theta\cos^2\theta)$$

$(\because \cos^2\theta+\sin^2\theta=1, (1-\cos^2\theta)=\sin^2\theta, (1-\sin^2\theta)=\cos^2\theta)$

$$=\frac{1-\cos^2\theta\sin^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}=\frac{1-\cos^2\theta\sin^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$

$$=\frac{1-\cos^2\theta\sin^2\theta}{1+1+\sin^2\theta\cos^2\theta}=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

(xiv) $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}=\sec^2\theta\left(\frac{1-\sin\theta}{1+\sec\theta}\right)$

$$\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}$$

$$=\frac{\cot^2\theta(\sec\theta-1)(1-\sin\theta)(\sec\theta+1)}{(1+\sin\theta)(1-\sin\theta)(\sec\theta+1)}$$

$$=\frac{\cot^2\theta(\sec\theta-1)(\sec\theta+1)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)(\sec\theta+1)}$$

$$=\frac{\cot^2\theta(\sec^2\theta-1)(1-\sin\theta)}{(1-\sin^2\theta)(1+\sec\theta)}$$

$$=\frac{\cot^2\theta(\tan^2\theta)(1-\sin\theta)}{(\cos^2\theta)(1+\sec\theta)}\ (\because \tan^2\theta=\sec^2\theta-1, 1-\sin^2\theta=\cos^2\theta)$$

$$=\frac{(\cot\theta\tan\theta)^2(1-\sin\theta)}{(\cos^2\theta)(1+\sec\theta)}$$

$$=\frac{1(1-\sin\theta)}{(\cos^2\theta)(1+\sec\theta)}\ (\because \cot\theta\tan\theta=1)$$

$$=\sec^2\theta\left(\frac{1-\sin\theta}{1+\sec\theta}\right)$$