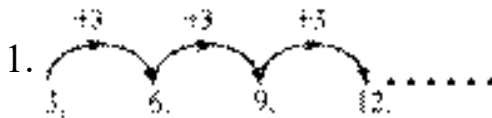


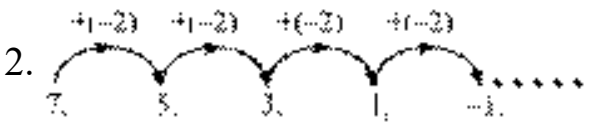
Progressions

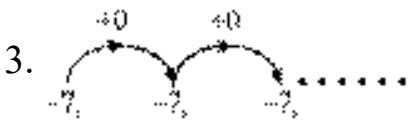
- **The Concept of Arithmetic Progression**

- An arithmetic progression is a list of numbers in which the difference between any two consecutive terms is equal.
- In an AP, each term, except the first term, is obtained by adding a fixed number called common difference to the preceding term.
- The common difference of an AP can be positive, negative or zero.

Example 1:

1.  is an AP whose first term and common difference are 3 and 3 respectively.

2.  is an AP whose first term and common difference are 7 and -2 respectively.

3.  is an AP whose first term and common difference are -7 and 0 respectively.

- The general form of an AP can be written as $a, a + d, a + 2d, a + 3d \dots$, where a is the first term and d is the common difference.
- A given list of numbers i.e., $a_1, a_2, a_3 \dots$ forms an AP if $a_{k+1} - a_k$ is the same for all values of k .

Example 2:

Which of the following lists of numbers forms an AP? If it forms an AP, then write its next three terms.

(a) $-4, 0, 4, 8, \dots$

(b) $2, 4, 8, 16, \dots$

Solution:

(a) $-4, 0, 4, 8, \dots$

$$a_2 - a_1 = 0 - (-4) = 4$$

$$a_3 - a_2 = 4 - 0 = 4$$

$$a_4 - a_3 = 8 - 4 = 4$$

$$a_{n+1} - a_n = 4; \text{ for all values of } n$$

Therefore, the given list of numbers forms an AP with 4 being its common difference.

The next three terms of the AP are $8 + 4 = 12$, $12 + 4 = 16$, $16 + 4 = 20$

Hence, AP: $-4, 0, 4, 8, 12, 16, 20 \dots$

(b) $2, 4, 8, 16, \dots$

$$a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$a_3 - a_2 \neq a_2 - a_1$$

Therefore, the given list of numbers does not form an AP.

- **The terminology related to arithmetic progression**

- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- The fixed number is called the common difference (d) of the A.P. The common difference can be either positive or negative or zero.

- **The general form of an A.P.**

- $a, (a + d), (a + 2d), (a + 3d), \dots, [a + (n - 1)d], \dots$ where a is the first term and d is common difference

- **Type of AP**

- Finite AP: The APs have finite number of terms.
- Infinite AP: The APs have not finite number of terms.

- **n^{th} term of an AP**

The n^{th} term (a_n) of an AP with first term a and common difference d is given by $a_n = a + (n - 1)d$.

Here, a_n is called the general term of the AP.

- **n^{th} term from the end of an AP**

The n^{th} term from the end of an AP with last term l and common difference d is given by $l - (n - 1) d$.

Example:

Find the 12th term of the AP 5, 9, 13 ...

Solution:

Here, $a = 5$, $d = 9 - 5 = 4$, $n = 12$

$$\begin{aligned} a_{12} &= a + (n - 1) d \\ &= 5 + (12 - 1) 4 \\ &= 5 + 11 \times 4 \\ &= 5 + 44 \\ &= 49 \end{aligned}$$

• **Sum of n terms of an AP**

- The sum of the first n terms of an AP is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d] \quad S_n = n[2a + (n - 1)d], \text{ where } a \text{ is the first term and } d \text{ is the common difference.}$$

•

- If there are only n terms in an AP, then $S_n = \frac{n}{2} [a + l]$ $S_n = n[2a + l]$, where $l = a_n$ is the last term.

Example :

Find the value of $2 + 10 + 18 + \dots + 802$.

Solution:

2, 10, 18... 802 is an AP where $a = 2$, $d = 8$, and $l = 802$.

Let there be n terms in the series. Then,

$$a_n = 802$$

$$\Rightarrow a + (n - 1) d = 802$$

$$\Rightarrow 2 + (n - 1) 8 = 802$$

$$\Rightarrow 8(n - 1) = 800$$

$$\Rightarrow n - 1 = 100$$

$$\Rightarrow n = 101$$

Thus, required sum =

$$\frac{n}{2} (a + 1) = \frac{101}{2} (2 + 802) = 40602 \quad n2a+1 = 10122+802 = 40602$$

- **Properties of an Arithmetic progression**

- If a constant is added or subtracted or multiplied to each term of an A.P. then the resulting sequence is also an A.P.
- If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

- **Arithmetic mean**

- For any two numbers a and b , we can insert a number A between them such that a, A, b is an A.P. Such a number i.e., A is called the arithmetic mean (A.M) of numbers a and b and it is given by $A = \frac{a+b}{2}$.
- For any two given numbers a and b , we can insert as many numbers between them as we want such that the resulting sequence becomes an A.P.

Let $A_1, A_2 \dots A_n$ be n numbers between a and b such that $a, A_1, A_2 \dots A_n, b$ is an A.P.

Here, common difference (d) is given by $\frac{b-a}{n+1}$.

Example:

Insert three numbers between -2 and 18 such that the resulting sequence is an A.P.

Solution:

Let A_1, A_2 , and A_3 be three numbers between -2 and 18 such that $-2, A_1, A_2, A_3, 18$ are in an A.P.

Here, $a = -2, b = 18, n = 5$

$$\therefore 18 = -2 + (5 - 1) d$$

$$\Rightarrow 20 = 4 d$$

$$\Rightarrow d = 5$$

$$\text{Thus, } A_1 = a + d = -2 + 5 = 3$$

$$A_2 = a + 2d = -2 + 10 = 8$$

$$A_3 = a + 3d = -2 + 15 = 13$$

Hence, the required three numbers between -2 and 18 are $3, 8$, and 13 .

- **Geometric Progression:** A sequence is said to be a geometric progression (G.P.) if the ratio of any term to its preceding term is the same throughout. This constant factor is called the common ratio and it is denoted by r .
- In standard form, the G.P. is written as $a, ar, ar^2 \dots$ where, a is the first term and r is the common ratio.
- **General Term of a G.P.:** The n^{th} term (or general term) of a G.P. is given by $a_n = ar^{n-1}$

Example: Find the number of terms in G.P. 5, 20, 80 ... 5120.

Solution: Let the number of terms be n .

Here $a = 5$, $r = 4$ and $t_n = 5120$

n^{th} term of G.P. $= ar^{n-1}$

$$\therefore 5(4)^{n-1} = 5120$$

$$\Rightarrow 4^{n-1} = \frac{5120}{5} = 1024$$

$$\Rightarrow (2)^{2n-2} = (2)^{10}$$

$$\Rightarrow 2n - 2 = 10$$

$$\Rightarrow 2n = 12$$

$$\therefore n = 6$$

- **Sum of n Term of a G.P.:** The sum of n terms (S_n) of a G.P. is given by

$$S_n = \begin{cases} \frac{a(1-r^n)}{1-r}, & \text{if } r < 1 \\ \frac{a(r^n-1)}{r-1}, & \text{if } r > 1 \\ na, & \text{if } r = 1 \end{cases}$$

Example: Find the sum of the series $1 + 3 + 9 + 27 + \dots$ to 10 terms.

Solution: The sequence 1, 3, 9, 27, ... is a G.P.

Here, $a = 1, r = 3$.

$$\text{Sum of } n \text{ terms of G.P.} = \frac{a(r^n - 1)}{r - 1} \quad [r > 1]$$

$$S_{10} = 1 + 3 + 9 + 27 + \dots \text{ to 10 terms}$$

$$= \frac{1 \times [(3)^{10} - 1]}{(3 - 1)}$$

$$= \frac{59049 - 1}{2}$$

$$= \frac{59048}{2}$$

$$= 29524$$

- Three consecutive terms can be taken as $\frac{a}{r}, a, ar$. Here, common ratio is r .
- Four consecutive terms can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$. Here, common ratio is r^2 .

- **Geometric Mean:** For any two positive numbers a and b , we can insert a number G between them such that a, G, b is a G.P. Such a number i.e., G is called a geometric mean (G.M.) and is given by $G = \sqrt{ab}$

In general, if $G_1, G_2, \dots, G_n$ be n numbers between positive numbers a and b such that $a, G_1, G_2, \dots, G_n, b$ is a G.P., then $G_1, G_2, \dots, G_n$ are given by

$$G_1 = ar, G_2 = ar^2, \dots, G_n = ar^n$$

Where, r is calculated from the relation $b = ar^{n+1}$, that is $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$.

Example: Insert three geometric means between 2 and 162.

Solution:

Let G_1, G_2, G_3 be 3 G.M.'s between 2 and 162.

Therefore 2, $G_1, G_2, G_3, 162$ are in G.P.

Let r be the common ratio of G.P.

Here, $a = 2$, $b = 162$ and $n = 3$

$$r = \left(\frac{162}{2}\right)^{\frac{1}{3+1}} = (81)^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3$$

$$G_1 = ar = 2 \times 3 = 6$$

$$G_2 = ar^2 = 2 \times (3)^2 = 2 \times 9 = 18$$

$$G_3 = ar^3 = 2 \times (3)^3 = 2 \times 27 = 54$$

Thus, the required three geometric means between 2 and 162 are 6, 18, and 54.

- **Relation between A.M. and G.M.:** Let A and G be the respective A.M. and G.M. of two given positive real numbers a and b . Accordingly,

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}.$$

Then, we will always have the following relationship between the A.M. and G.M.: $A \geq G$