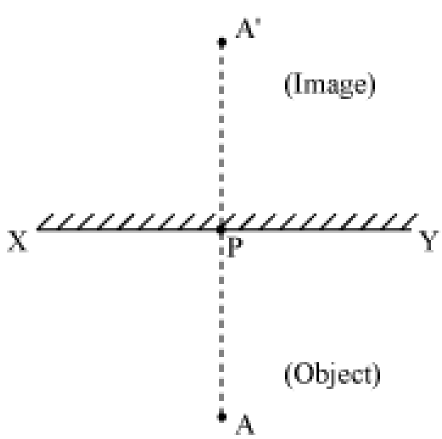
Co-ordinate Geometry

- Some figures look exactly similar after drawing a dotted line i.e., the left half of the figure is exactly same as the right half of the figure. These figures are known as **symmetrical figures** and the dotted line through which the figure is divided is called **line of symmetry.**
- The line of symmetry is closely related to mirror reflection. When we deal with mirror reflection, we have to take into account that the object and image are symmetrical with reference to the mirror line. There is no change in the length and angle of the object and the corresponding length and angle of the image, with respect to the mirror line; only the left-and-right alignment changes.

- **Axis of reflection** or **mediator**

  Let XY be a mirror. Let A be a point (object) placed in front of it. We obtain its image A′ as shown below:



  We can notice that:

  1. The distance of the image (A′) behind the mirror is same as the distance of the object (A) from it i.e., PA = PA′
  2. The mirror line XY is perpendicular to the line joining the object and the image i.e., XY $\perp$ AA′

  Here, XY (the mirror line) is called the **axis of reflection** or **mediator**.

  If the point A lies on XY, then its image will be this point itself. In such case, A is called an **invariant point** with respect to mirror line XY.

• **Section formula:**

The co-ordinates of the point P (x,y), which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m{:}n$, are given by:

$P\ (x,\ y) = \left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$ P x, y=mx2+nx1m+n, my2+ny1m+n

Example: In what ratio does the point (–4, 7) divide the line segment joining the points P (–1, 1) and Q (–6, 11).

Solution: Let the point (–4, 7) divide the line segment joining the points P (–1, 1) and Q(–6, 11) in the ratio λ : 1.
Thus, by section formula, we have:

$\left(\frac{-6\lambda+(-1)}{\lambda+1},\ \frac{11\lambda+1}{\lambda+1}\right) = (-4,\ 7)$ -6λ+-1λ+1,11λ+1λ+1=-4, 7⇒-6λ-1λ+1=-4, 11λ+1λ+1=7

$\Rightarrow \frac{-6\lambda-1}{\lambda+1} = -4,\ \frac{11\lambda+1}{\lambda+1} = 7$

$\Rightarrow -6\lambda - 1 = -4\lambda - 4$

$\Rightarrow 2\lambda = 3$

$\Rightarrow \lambda = \frac{3}{2}$

⇒-6λ-1=-4λ-4⇒2λ=3⇒λ=32

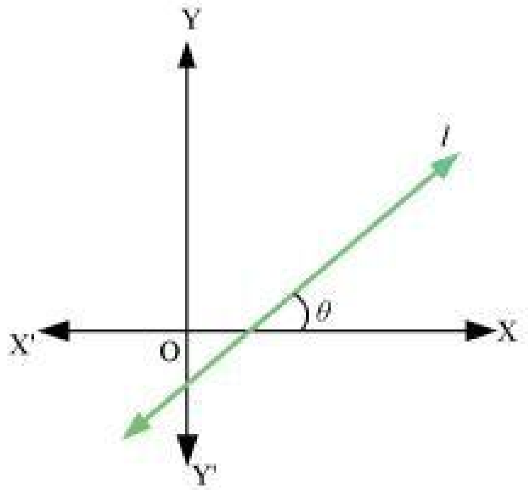
Therefore, the required ratio is 3:2.

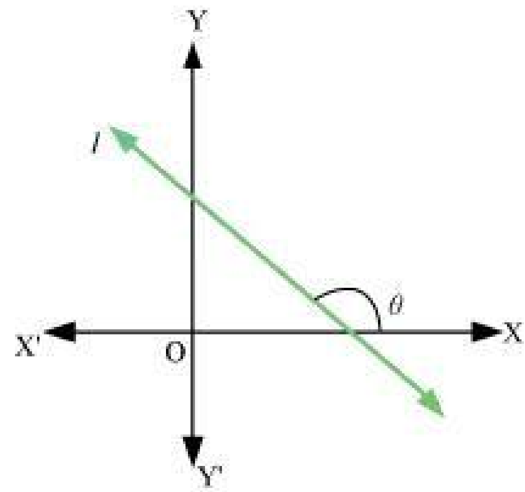
• The **mid-point** of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) is $\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$ x1+x22, y1+y22 .

[Note: Here, $m = n = 1$]

• If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC, then the coordinates of its **centroid** are given by the point $\left(\frac{x_1+x_2+x_3}{3},\ \frac{y_1+y_2+y_3}{3}\right)$ x1+x2+x33, y1+y2+y33 .

- **Slope of a line:** If θ is the inclination of a line l (the angle between positive x-axis and line l), then $m = \tan\theta$ is called the slope or gradient of line l.





- The slope of a line whose inclination is 90º is not defined. Hence, the slope of the vertical line, y-axis is undefined.
- The slope of the horizontal line, x-axis is zero.

For example, the slope of a line making an angle of 135° with the positive direction of x-axis is $m = \tan 135° = \tan (180° – 45°) = – \tan 45° = –1$

- **Slope of line passing through two given points:**

  The slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

  For example, the slope of the line joining the points (–1, 3) and (4, –2) is given by,
  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 3}{4 - (-1)} = -\frac{5}{5} = -1$

- **Conditions for parallelism and perpendicularity of lines:**

  Suppose l_1 and l_2 are non-vertical lines having slopes m_1 and m_2 respectively.

  - l_1 is parallel to l_2 if and only if $m_1 = m_2$ i.e., their slopes are equal.
  - l_1 is perpendicular to l_2 if and only if $m_1 m_2 = –1$ i.e., the product of their slopes is –1.

Example:
Find the slope of the line which makes an angle of 45° with a line of slope 3.
Solution:
Let m be the slope of the required line.

$$\therefore \tan 45° = \left|\frac{m-3}{1+3m}\right|$$

$$\Rightarrow \left|\frac{m-3}{1+3m}\right| = 1$$

$$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \pm 1$$

$$\Rightarrow \frac{m-3}{1+3m} = 1 \quad \text{or} \quad \frac{m-3}{1+3m} = -1$$

$$\Rightarrow m-3 = 1+3m \quad \text{or} \quad m-3 = -1-3m$$

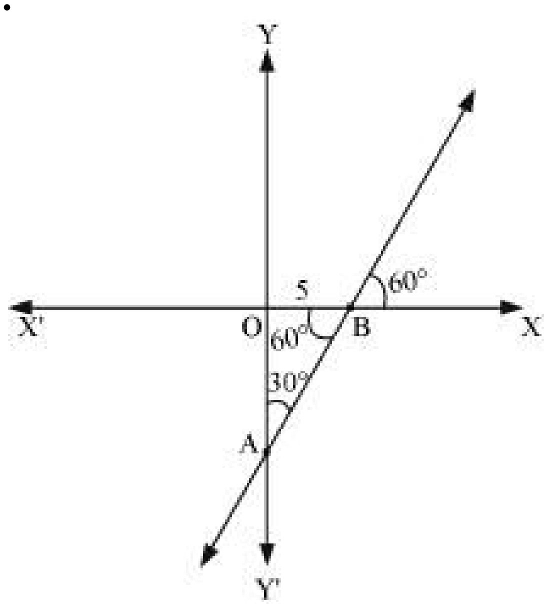
$$\Rightarrow -2m = 4 \quad \text{or} \quad 4m = 2$$

$$\Rightarrow m = -2 \quad \text{or} \quad m = \frac{1}{2}$$

- **Collinearity of three points:** Three points A, B and C are collinear if and only if slope of AB = slope of BC

- **Slope-intercept form of a line**
  - The equation of the line, with slope m, which makes y-intercept c is given by $y = mx + c$.
  - The equation of the line, with slope m, which makes x-intercept d is given by $y = m(x - d)$.

Example:
Find the equation of the line which cuts off an intercept 5 on the x-axis and makes an angle of 30° with the y-axis.
Solution:



Slope of the line, $m = \tan 60° = \sqrt{3}$
OB = 5
Intercept on the x-axis, $c = –OB = –5$ and $\tan 60° = -5\sqrt{3}$
Equation of the required line is $y = \sqrt{3}x + \left(-5\sqrt{3}\right)$.

- **General equation of line**

Any equation of the form $Ax + By + C = 0$, where A and B are not zero simultaneously is called the general linear equation or general equation of line.

Slope of the line $= -\dfrac{\text{Coefficient of}\,x}{\text{Coefficient of}\,y} = -\dfrac{A}{B}$
y- intercept $= -\dfrac{C}{B}$

Example:
Find the slope and the y-intercept of the line $2x - 3y = -16$.
Solution:
The equation of the given line can be rewritten as $2x - 3y + 16 = 0$.
Here, $A = 2$, $B = -3$ and $C = 16$.
Slope of the line $= -\dfrac{A}{B} = -\dfrac{2}{(-3)} = \dfrac{2}{3}$
Intercept on the y-axis $= -\dfrac{C}{B} = -\dfrac{16}{(-3)} = \dfrac{16}{3}$

- **Point-slope form of the equation of a line**

The point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) if and only if its coordinates satisfy the equation. This means $y - y_0 = m(x - x_0)$.

Example: Find the equation of the line passing through (4, 5) and making an angle of 120° with the positive direction of x-axis?
Solution: Slope of the line,
$m = \tan 120° = \tan\left(180° - 60°\right) = -\tan 60° = -\sqrt{3}$
Equation of the required line is,
$$y - 5 = -\sqrt{3}(x - 4)$$
$$\Rightarrow y - 5 = -\sqrt{3}x + 4\sqrt{3}$$
$$\Rightarrow \sqrt{3}x + y - \left(5 + 4\sqrt{3}\right) = 0$$

- **Two-point form of the equation of a line**

The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Example: Find the equation of the line passing through the points (–5, 2) and (1, 6).
Solution: Equation of the line passing through points (–5, 2) and (1, 6) is

$$y - 2 = \frac{6-2}{1-(-5)}(x - (-5))$$

$$\Rightarrow y - 2 = \frac{4}{6}(x + 5)$$

$$\Rightarrow y - 2 = \frac{2}{3}(x + 5)$$

$$\Rightarrow 3y - 6 = 2x + 10$$

$$\Rightarrow 2x - 3y + 16 = 0$$