Similarity

- **Similar and Congruent Figures**
    - Two geometric figures having the same shape and size are said to be congruent figures.
    - Two geometric figures having the same shape, but not necessarily the same size, are called similar figures.

  Example:
  (1) All circles are similar.
  (2) All equilateral triangles are similar.
  (3) All congruent figures are similar. However, the converse is not true.

- **Similarity of Polygons**

  Two polygons with the same number of sides are similar, if

    - their corresponding angles are equal
    - their corresponding sides are in the same ratio (or proportion)

- Two lines segments are congruent, if they are equal in length.
- Two angles are congruent, if they have the same measure.
- **CPCT:**

CPCT stands for Corresponding Parts of Congruent Triangles.

If $\Delta ABC \cong \Delta PQR$, then corresponding sides are equal i.e., AB = PQ, BC = QR, and CA = RP and corresponding angles are equal i.e., $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

- **Theorem: (AAA similarity criterion)**

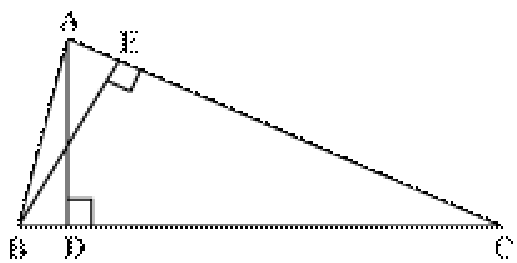
  If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence, the two triangles are similar.

- **Theorem: (AA similarity criterion)**

  If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar.

Example:
In ΔABC, $\angle C$ is acute, D and E are points on sides BC and AC respectively, such that $AD \perp BC$ and $BE \perp AC$. Show that $BC \times CD = AC \times CE$.

Solution:



In ΔADC and ΔBEC,
$\angle ADC = \angle BEC = 90°$
$\angle DCA = \angle ECB$ [Common]
$\therefore$ By AA similarity criterion, $\Delta ADC \sim \Delta BEC$

$$\therefore \frac{CD}{CE} = \frac{AC}{BC}$$
$$\Rightarrow BC \times CD = AC \times CE$$

Hence, the result is proved.

- **Theorem: (SSS similarity criterion)**

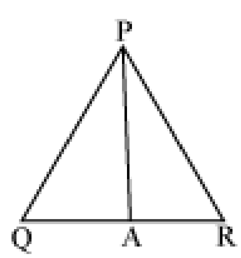
If in two triangles, sides of one triangle are proportional to the sides of the other triangle then the two triangles are similar by SSS similarity criterion.

Example:

If PQR is an isosceles triangle with PQ = PR and A is the mid-point of side QR then prove that ΔPAQ is similar to ΔPAR.

Solution:

It is given that ΔPQR is an isosceles triangle and PQ = PR.



In triangles PAQ and PAR,

PQ = PR

Also, A is the mid-point of QR, therefore QA = AR

And, PA = PA (Common to both triangles)

Therefore, we can say that

$$\frac{PQ}{PR} = \frac{QA}{AR} = \frac{PA}{PA}$$

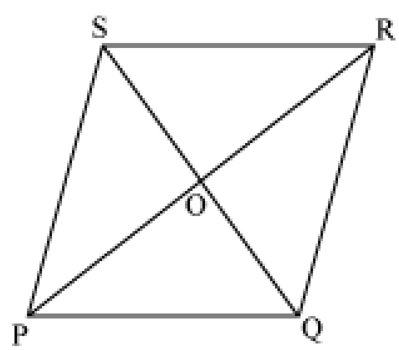
∴ Using SSS similarity criterion, we obtain $\Delta PAQ \sim \Delta PAR$

- **Theorem: (SAS similarity criterion)**

  If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar by SAS similarity criterion.

  Example:

  If PQRS is a parallelogram, then prove that ΔSOR is similar to ΔPOQ.



  Solution:

  Consider ΔSOR and ΔPOQ.

  Since PQRS is a parallelogram, the diagonals bisect each other.

  ∴ SO = OQ and PO = OR

  and ∠POQ = ∠SOR (Vertically opposite angles)

  By SAS similarity criterion, we obtain

$\Delta SOR \sim \Delta QOP$

- **Basic proportionality theorem:**

  If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

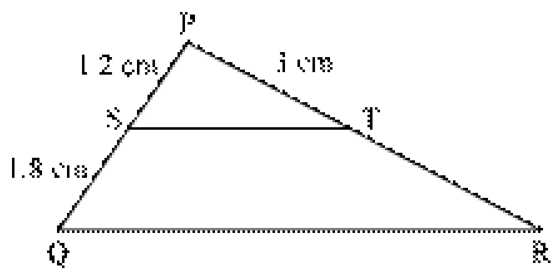
  Corollary: If D and E are points on the sides, AB and AC, respectively of ΔABC such that DE || BC, then

  $$\frac{AB}{AD} = \frac{AC}{AE}$$

  $$\frac{AB}{DB} = \frac{AC}{EC}$$

  Example:

  In the given figure, S and T are points on PQ and PR respectively of ΔPQR such that ST || QR.



  Determine the length of PR.

  Solution:
  Since ST || QR, by basic proportionality theorem, we have

  $$\frac{PS}{SQ} = \frac{PT}{TR}$$

  $$\Rightarrow \frac{1.2}{1.8} = \frac{3}{TR}$$

  $$\Rightarrow TR = \frac{3 \times 1.8}{1.2} = 4.5 \text{ cm}$$

  $$\therefore PR = PT + TR = (3 + 4.5) \text{ cm} = 7.5 \text{ cm}$$

- **Converse of basic proportionality theorem:**

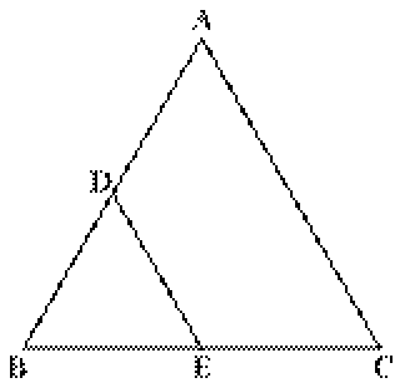
If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

- **Areas of similar triangles**

Theorem: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Example:
In ΔABC, D and E are the respective mid-points of sides AB and BC. Find the ratio of the areas of ΔDBE and ΔABC.

Solution:



In ΔABC, D and E are the respective mid-points of the sides, AB and BC.
By the converse of BPT, DE||AC
In ΔDBE and ΔABC,
∠DBE = ∠ABC [Common]
∠BED = ∠BCA [Corresponding angles]
∠BDE = ∠BAC [Corresponding angles]
∴ By AAA similarity criterion, ΔDBE~ ΔABC

$$\Rightarrow \frac{\text{Area}(\Delta DBE)}{\text{Area}(\Delta ABC)} = \left(\frac{BE}{BC}\right)^2$$

$$= \left(\frac{BE}{2BE}\right)^2 \quad [\text{E is mid-point of BC}]$$

$$= \frac{1}{4}$$

$$= 1:4$$

Result: Using the above theorem, the result " the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians or altitudes or angle bisector” can be proved.

- The **scale of a map** can be defined as the ratio of the distance between two points on the map to the actual distance of these two points on the ground. This ratio is known as the **scale factor**, and is denoted by the letter *k*.

- In the case of models:

$$k = \frac{\text{Length of the model}}{\text{Length of the object}} = \frac{\text{Height of the model}}{\text{Height of the object}}$$

Scale factor	**Transformation**	**Size**
$k = 1$	Identify transformation	Size of the model = Size of the object
$k > 1$	Enlargement	Size of the model > Size of the object
$k < 1$	Reduction	Size of the model < Size of the object