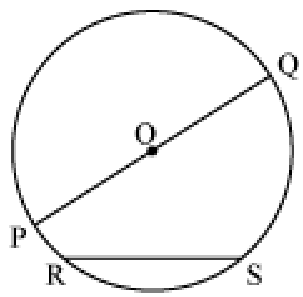
Circles

- **Circle:** Circle is a simple closed curve.



1. The fixed point O is the centre of the circle.

2. The fixed distance OP = OQ is the **radius** of the circle.

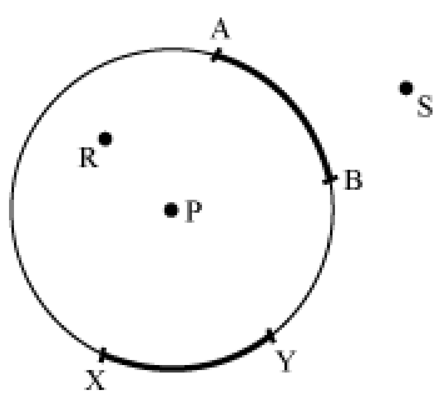
3. The distance around the circle is its **circumference**.

4. A line joining any two points on a circle is known as **chord**. In the given figure, RS and PQ are the chords.

5. The chord passing through the centre of a circle is called **diameter**. The diameter of a circle divides it into two semicircles.

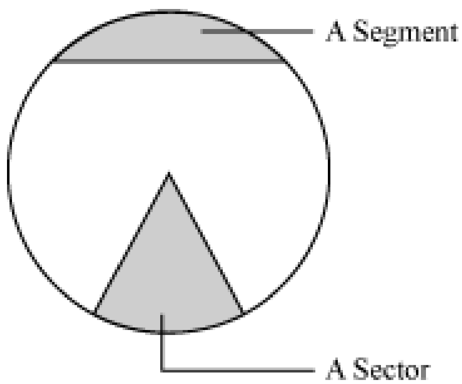
6. The diameter of a circle is the longest chord of the circle and it is twice the radius.

7. The portions on a circle are known as arcs. In the figure, XY and AB are arcs.

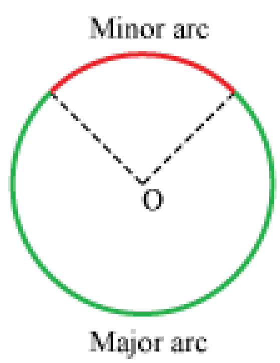


8. The region in the interior of a circle enclosed by a chord and an arc is known as **segment.**

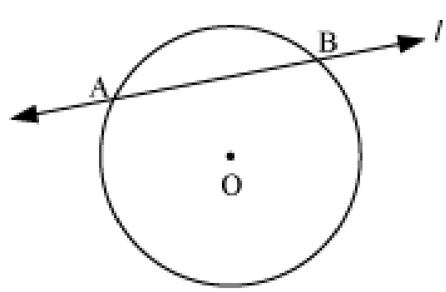
9. The region in the interior of a circle enclosed by an arc on one side and a pair of radii on the other side is called **sector.**



- An arc less than one-half of the entire arc of a circle is called the **minor arc** of the circle, while an arc greater than one-half of the entire arc of a circle is called the **major arc** of the circle.

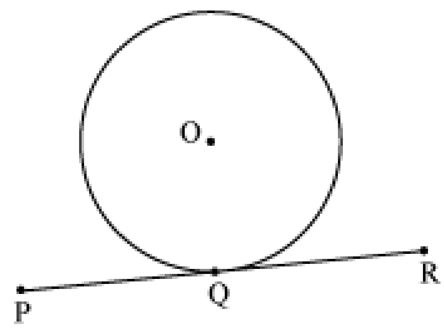


- A line that meets a circle at two points is called the secant of the circle.



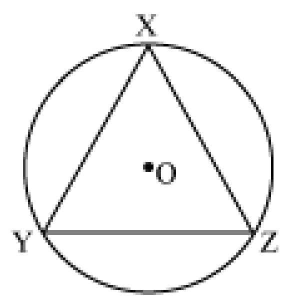
In the figure, a line l is the **secant** to the circle.

- A line that meets a circle at one and only one point is called a tangent to the circle. The point where the tangent touches the circle is called the point of contact.



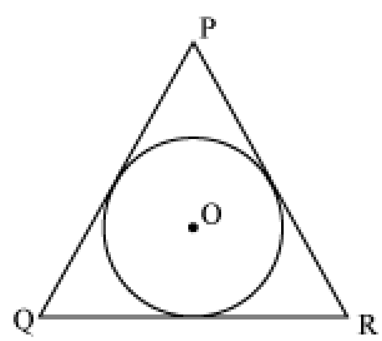
In the above figure, line PR is the tangent to the circle.

- A circle which passes through all the three vertices of a triangle is called the **circumcircle** of the triangle.



In the above figure, circumcircle of ΔXYZ is drawn.

- A circle (drawn inside a triangle) which touches all the three sides of the triangle is called the **incircle** of the triangle.



In the above figure, incircle of ΔPQR is drawn.

- Circles with the same centre and different radii are called concentric circles.

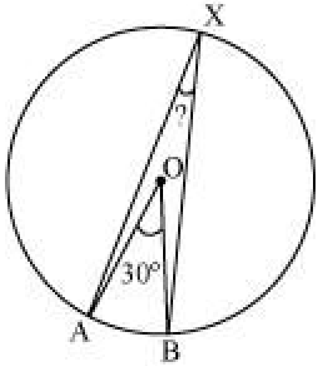
- Circles with the same radius are called congruent circles.

• A straight line cutting a circle at two different points is called a secant.

• A line that meets a circle at one and only one point is called a tangent to the circle. The point where the tangent touches the circle is called the point of contact.

- The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc at the remaining part of the circle.

In the given figure, ∠AOB and ∠AXB are the angles subtended by arc AB at the centre and at remaining part of the circle.

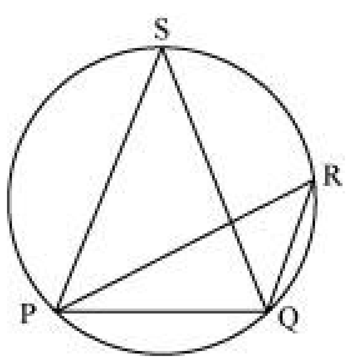


$\therefore 2 \times \angle AXB = \angle AOB$

$\Rightarrow \angle AXB = \frac{1}{2} \times 30° = 15°$

- The angle lying in the major segment is an acute angle and the angle lying in the minor segment is an obtuse angle. This statement is true for all major and minor segments in a circle.
- Angles in the same segment of a circle are equal.

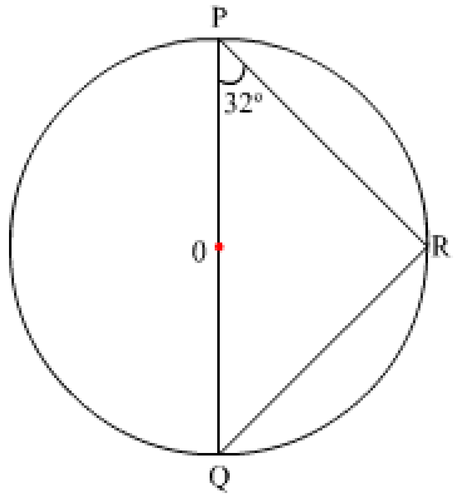
In the given figure, ∠PRQ and ∠PSQ lie in the same segment of the circle.
$\therefore \angle PRQ = \angle PSQ$



- Angle in a semi circle is a **right angle**.

Example:

Find the value of $\angle PQR$ in the given figure.



Solution:

In ΔPQR, $\angle PQR + \angle QRP + \angle RPQ = 180°$

$\Rightarrow \angle PQR + 90° + 32° = 180°$

$\Rightarrow \angle PQR = 180° - 90° - 32° = 58°$

- A quadrilateral whose vertices lie on a circle is known as a **cyclic quadrilateral**.

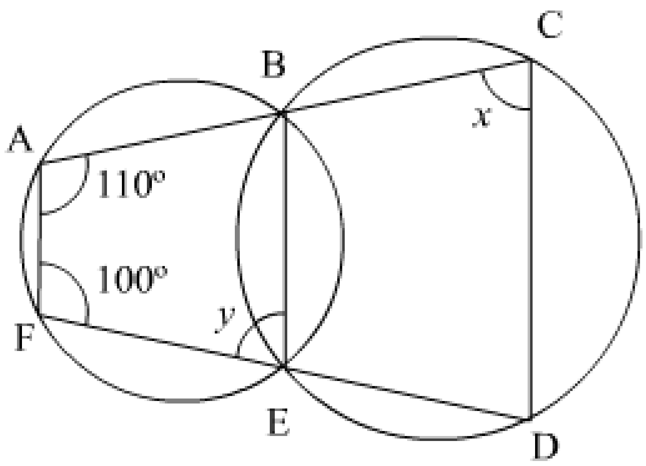
Properties of cyclic quadrilateral:

  - The sum of each pair of opposite angles of a cyclic quadrilateral is 180°.

    Converse of the property also holds true, which states that if the sum of a pair of opposite angles of a quadrilateral is 180° then the quadrilateral is cyclic.
  - Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Example:

In the given figure, find the value of x and y.



Solution:

We know that in a cyclic quadrilateral opposite angles are supplementary.

In cyclic quadrilateral ABEF, ∠A + ∠BEF = 180°

⇒∠BEF = y = 70°

Also,∠BEF + BED = 180° (Linear pair)

⇒∠BED = 110°

In cyclic quadrilateral BCDE, ∠BED + ∠C = 180°

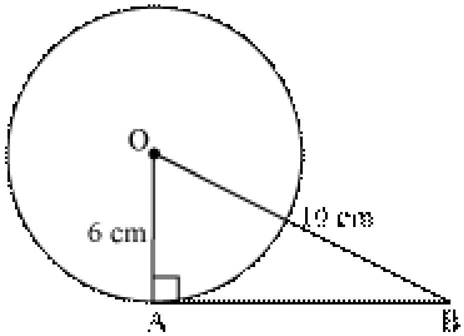
⇒∠C = x =70°

- **Concept of tangent at any point of the circle**

  Theorem: The tangent at any point on a circle is perpendicular to the radius through the point of contact.

  Example:
  A tangent AB at a point A of a circle of radius 6 cm meets a line through the centre O at the point B, such that OB = 10 cm. Find the length of AB.

  Solution:



It is known that the tangent at any point on a circle is perpendicular to the radius through the point of contact.
OA ⊥ AB

By applying Pythagoras theorem in right triangle OAB, we obtain

$OA^2 + AB^2 = OB^2$
$\Rightarrow 6^2 + AB^2 = 10^2$
$\Rightarrow AB^2 = (100 - 36)\ cm^2$
$\Rightarrow AB^2 = 64\ cm^2$
$\Rightarrow AB = \sqrt{64\ cm^2} = 8\ cm$

No tangent can be drawn to a circle passing through a point lying inside the circle.

One and only one tangent can be drawn to a circle passing through a point lying on the circle.

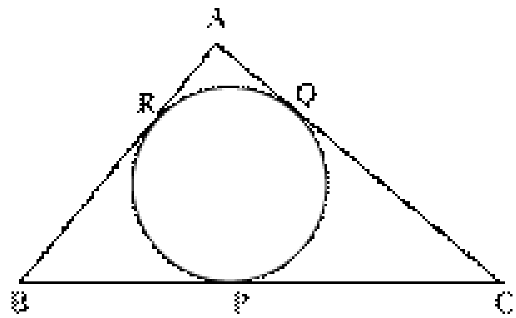
Exactly two tangents can be drawn to a circle through a point lying outside the circle.

- **Tangent drawn from an external point to a circle**

Length of the tangent: The length of the segment of the tangent from an external point P to the point of contact with the circle is called the length of the tangent from the point P to the circle.

Theorem: The lengths of tangents drawn from an external point to a circle are equal.

Example:
In the given figure, a circle is inscribed in ΔABC touching the points, P, Q, and R.



If AB = 7 cm, BC = 9 cm, CA = 8 cm, then find the measures of AR, AQ, BR, BP, CP, and CQ.

Solution:
It is known that the lengths of tangents drawn from an external point to a circle are equal.
AR = AQ = a (say)
BR = BP = b (say)
CP = CQ = c (say)

AB + BC + CA = (7 + 9 + 8) cm = 24 cm

$\Rightarrow (a + b) + (b + c) + (c + a) = 24$ cm
$\Rightarrow 2(a + b + c) = 24$ cm
$\Rightarrow a + b + c = 12$ cm

AB = 7 cm
$\Rightarrow a + b = 7$ cm
$\therefore c + 7$ cm = 12 cm
$\Rightarrow c = (12 - 7)$ cm = 5 cm

BC = 9 cm
$\Rightarrow b + c = 9$ cm
$\Rightarrow b = 9 - c = (9 - 5)$ cm = 4 cm

$a + b + c = 12$ cm
$\therefore$ 9 cm + a = 12 cm
$\Rightarrow a = (12 - 9)$ cm = 3 cm

Hence, AR = AQ = 3 cm,
BR = BP = 4 cm,
CP = CQ = 5 cm.

Results: If two tangents are drawn to a circle from an external point, then

1. they subtend equal angles at the centre.
2. they are equally inclined to the segment, joining the centre to that point.

- If two circles touch then the point of contact lies on the straight line joining the centres.

- If r_1 and r_2 be the radii of bigger and smaller circles respectively and d be the distance between their centres then

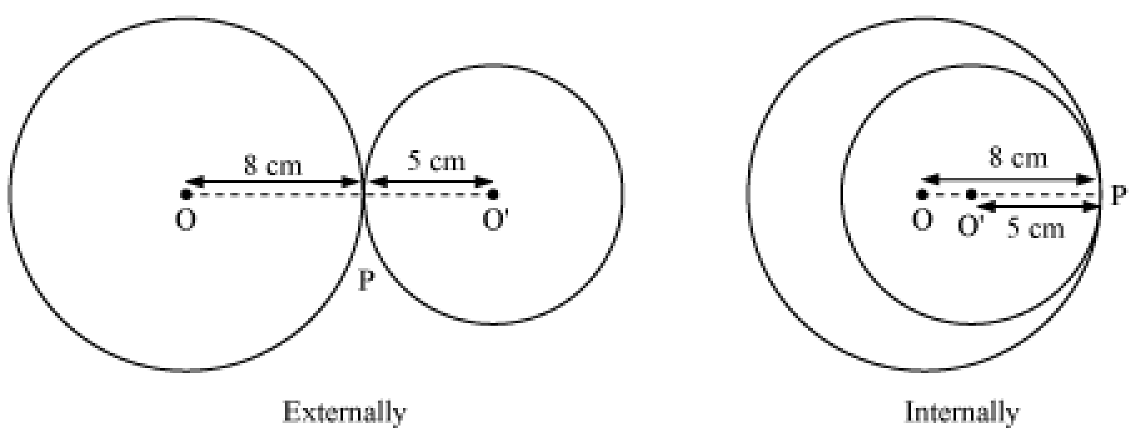
  (i) $d = r_1 + r_2$, when they touch externally

  (ii) $d = r_1 - r_2$, when they touch internally

  For example, if two circles with centres O and O' and of radius 8 cm and 5 cm respectively touch each other at point P then P lies on $O'O$ i.e., O, P, and O' lie on a straight line.

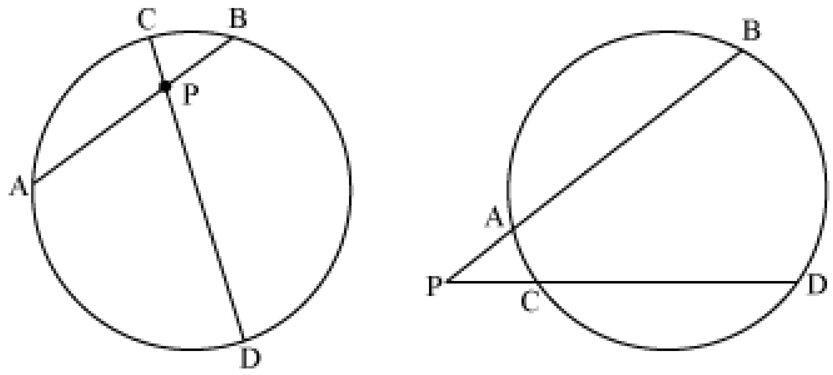
  Distance between the centres of the circles when they touch externally = $O'O$ = OP + $O'P$ = 8 cm + 5 cm = 13 cm

  And, distance between the centres of the circles when they touch internally = $O'O$ = OP − $O'P$ = 8 cm − 5 cm = 3 cm



- If two chords of a circle intersect internally or externally, then the product of the lengths of segments are equal.

For two chords AB and CD to intersect internally or externally at P, we have PA.PB = PC.PD

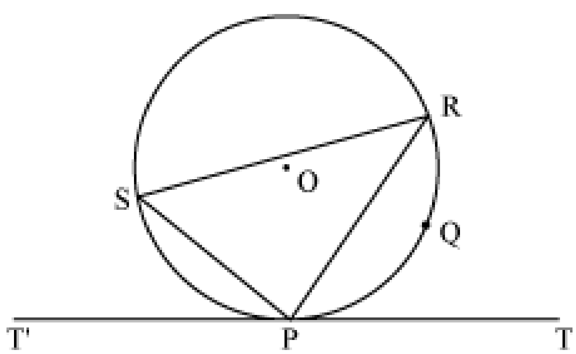


Two chords intersect internally

Two chords intersect externally

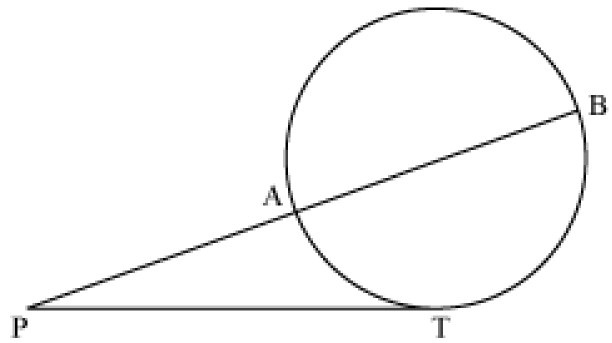
- **Alternate Segment theorem** states that "if a line touches a circle and a chord is drawn from the point of contact, then the angle between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments."

For example, in the given figure ∠RPT = ∠RSP



- If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

For chord AB and the tangent through a point T to the circle intersecting each other at point P, we have $PT^2 = PA \times PB$.


B
A
P
T