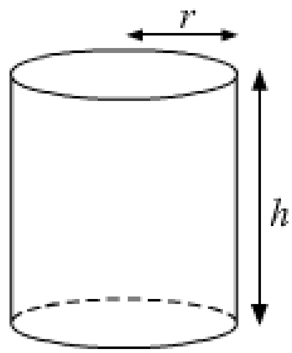
Surface Area and Volume of Solids

- **Surface areas of solid cylinder**
  - Curved surface area = $2\pi rh$, where r and h are the radius and height
  - Total surface area = $2\pi r\,(r + h)$, where r and h are the radius and height



Example :

What is the curved surface area of a cylinder of radius 2 cm and height 14 cm?

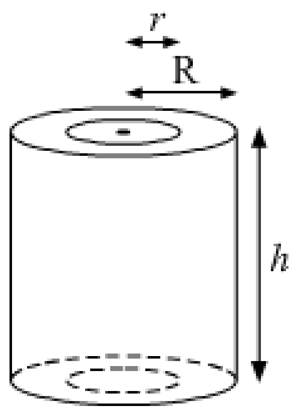
Solution:

Curved surface area of cylinder = $2\pi rh$

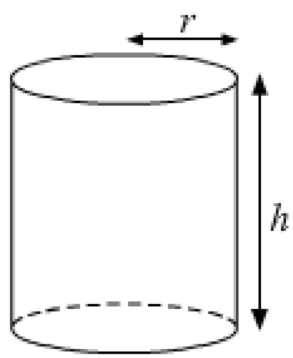
$$= 2 \times \frac{22}{7} \times 2 \times 14\ \text{cm}^2$$
$$= 176\ \text{cm}^2$$

- **Surface areas of hollow cylinder**
  - Curved surface area = $2\pi h\,(r + R)$, where r, R and h are the inner radius, outer radius and height
  - Total surface area = CSA of outer cylinder + CSA of inner cylinder + 2 × Area of base

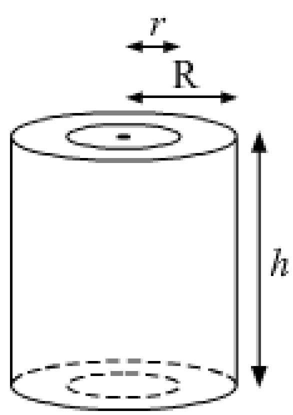
    = $2\pi\,(r + R)\,(h + R - r)$, where r, R and h are the inner radius, outer radius and height



- **Volume of the solid cylinder and hollow cylinder**
  - Volume of solid cylinder = $\pi r^2 h$, where r and h are the radius and height of the solid cylinder



  - Volume of the hollow cylinder = $\pi (R^2 - r^2) h$, where r, R and h are the inner radius, outer radius and height of hollow cylinder



Example:

Find the volume of the pillar of radius 70 cm and height 10 m.

Solution:

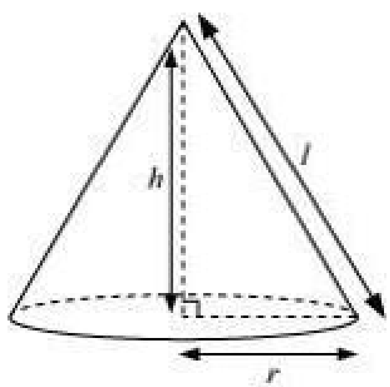
Radius of the pillar (r) = 70 cm = $\frac{70\text{ m}}{100}$ = 0.7 m

Height of the pillar (h) = 10 m

$$\text{Volume of the pillar} = \pi r^2 h$$
$$= \frac{22}{7} \times (0.7)^2 \times 10\text{ m}^3$$
$$= 15.4\text{ m}^3$$

- **Surface areas of cone**
  - Curved surface area = πrl, where r and l are the radius and slant height
  - Total surface area = $\pi r(l + r)$, where r and l are the radius and slant height

Here, $l = \sqrt{h^2 + r^2}$, where h is the height.



Example:
Calculate the curved surface area of a cone of base radius 3 cm and height 4 cm.
Solution:
Here, r = 3 cm and h = 4 cm
$\therefore l = \sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2}\text{ m} = 5\text{ cm}$
Curved surface area = $\pi rl = \pi \times 3 \times 5\text{ cm}^2 = 15\pi\text{ cm}^2$

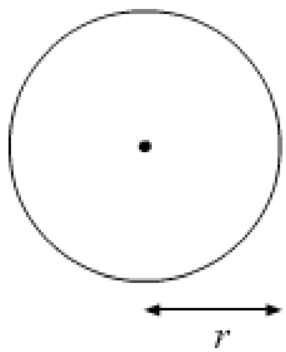
- **Volume of a cone** = $\frac{1}{3}\pi r^2 h$, where r and h are the radius of base and height of the cone.

Example:

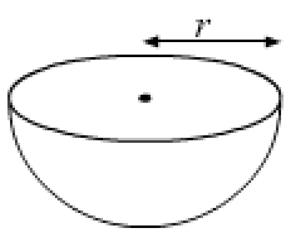
Calculate the volume of a cone of base radius 3 cm and height 4 cm.
Solution:
Here, r = 3 cm and h = 4 cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times 3\text{ cm} \times 3\text{ cm} \times 4\text{ cm}$$
$$= 12\pi\text{ cm}^3$$

- **Surface areas of sphere and hemisphere**
  - Surface area of sphere = $4\pi r^2$, where r is the radius



  - Curved surface area of hemisphere = $2\pi r^2$, where r is the radius
  - Total surface area of hemisphere = $3\pi r^2$, where r is the radius



Example:

What is the radius of a balloon whose surface area is 5544 cm^2?

Solution:

Let radius of the balloon be r.

Surface area of the balloon = $4\pi r^2 = 5544\text{ cm}^2$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544\text{ cm}^2$

$\Rightarrow r^2 = \frac{5544 \times 7}{88}$ cm^2

$\Rightarrow r^2 = 441$ cm^2

$\Rightarrow r = \sqrt{441} = 21$ cm

Thus, the radius of the balloon is 21 cm.

- **Volume of sphere and hemisphere**
  - Volume of sphere $= \frac{4}{3}\pi r^3$
  - Volume of hemisphere $= \frac{2}{3}\pi r^3$

Example 1:

Calculate the volume of a sphere whose surface area is 9π cm^2.

Solution:

Surface area = 9π cm^2

$\Rightarrow 4\pi r^2 = 9\pi$

$\Rightarrow r^2 = \frac{9}{4}$

$\Rightarrow r = \frac{3}{2}$ cm

$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{3}{2}\text{ cm}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right)\text{ cm}^3 = 4.5\pi\text{ cm}^3$

Example 2:

The inner radius of a hemispherical bowl is 4.2 cm. What is the capacity of the bowl?

Solution:

Here, $r = 4.2$ cm

Volume of the bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4.2\text{ cm})^3 = 155.232\text{ cm}^3$

1 ml = 1 cm^3

Thus, the capacity of the bowl is 155.232 ml.

- **Conversion of a solid from one shape into another**

    When a solid is converted into another solid of a different shape, the volume of the solid does not change.

Example:

A metallic block of dimensions 8.4 cm × 6 cm × 4.4 cm is melted and recast into the shape of a cone of radius 4.2 cm. Find the height of the cone.

Solution:

Volume of the cuboidal block = Length × Breadth × Height

$= 8.4 \times 6 \times 4.4\ \text{cm}^3$

Let h be the height of the cone.

Radius of the cone = 4.2 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 4.2^2 \times h$

Since the cuboidal block is converted into the shape of a cone

Volume of the cuboid = Volume of the cone

$$\Rightarrow (8.4 \times 6 \times 4.4)\ \text{cm}^3 = \frac{1}{3}\pi \times (4.2\ \text{cm})^2 \times h$$

$$\Rightarrow h = \frac{8.4 \times 6 \times 4.4 \times 3 \times 7}{4.2 \times 4.2 \times 22}\ \text{cm} = 12\ \text{cm}$$

Thus, the height of the cone is 12 cm.

- **Volume of combination of solids**

    1. Volume of a cuboid = $l \times b \times h$, where l, b, h are respectively length, breadth and height of the cuboid.
    2. Volume of a cube = a^3, where a is the edge of the cube.
    3. Volume of a cylinder = $\pi r^2 h$, where r is the radius and h is the height of the cylinder.
    4. Volume of a cone $= \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone.

5. Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$.

6. Volume of a hemisphere of radius $r = \frac{2}{3}\pi r^3$.

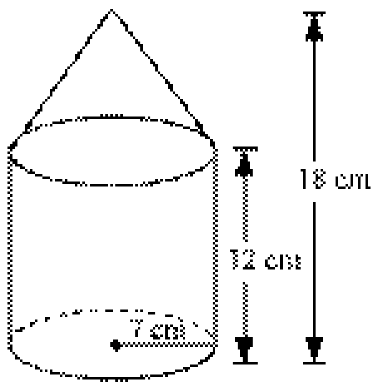
Note: Volume of the combination of solids is the sum of the volumes of the individual solids.

Example:

A solid is in the shape of a cylinder surmounted by a cone. The diameter of the base of the solid is 14 cm and the height of the solid is 18 cm. If the length of the cylindrical part in 12 cm, then find the volume of the solid.

Solution:

Volume of the solid = Volume of the cylindrical part + Volume of the conical part



Diameter of the base = 14 cm

Radius of the base $= \frac{14}{2}\ \text{cm} = 7\ \text{cm}$

Radius of the conical part = Radius of the cylindrical part = 7 cm
Height of the solid = 18 cm
Height of the cylindrical part = 12 cm
Height of the conical part = (18 – 12) cm = 6 cm
Volume of the cylindrical part

$$= \pi \times (\text{Radius})^2 \times \text{Height} = \frac{22}{7} \times 7^2 \times 12\ \text{cm}^3 = 1848\ \text{cm}^3$$

Volume of the conical part

$$= \frac{1}{3}\pi \times (\text{Radius})^2 \times \text{Height} = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 6\ \text{cm}^3 = 308\ \text{cm}^3$$

Thus, volume of the solid = (1848 + 308) cm^3 = 2156 cm^3.