

Ex 2.1

Q1

By the definition of equality of ordered pairs

$$\begin{aligned} & \left(\frac{a}{3} + 1, b - \frac{2}{3} \right) = \left(\frac{5}{3}, \frac{1}{3} \right) \\ \Rightarrow & \frac{a}{3} + 1 = \frac{5}{3} \quad \text{and} \quad b - \frac{2}{3} = \frac{1}{3} \\ \Rightarrow & \frac{a}{3} = \frac{5}{3} - 1 \quad \text{and} \quad b = \frac{1}{3} + \frac{2}{3} \\ \Rightarrow & \frac{a}{3} = \frac{5-3}{3} \quad \text{and} \quad b = \frac{1+2}{3} \\ \Rightarrow & \frac{a}{3} = \frac{2}{3} \quad \text{and} \quad b = \frac{3}{3} \\ \Rightarrow & a = 2 \quad \text{and} \quad b = 1 \end{aligned}$$

By the definition of equality of ordered pairs

$$\begin{aligned} & (x+1, 1) = (3, y-2) \\ \Rightarrow & x+1 = 3 \quad \text{and} \quad 1 = y-2 \\ \Rightarrow & x = 3-1 \quad \text{and} \quad 1+2 = y \\ \Rightarrow & x = 2 \quad \text{and} \quad 3 = y \\ \Rightarrow & x = 2 \quad \text{and} \quad y = 3 \end{aligned}$$

Q2

We have,

$$\begin{aligned} & (x, -1) \in \{(a, b) : b = 2a - 3\} \\ \text{and, } & (5, y) \in \{(a, b) : b = 2a - 3\} \\ \Rightarrow & -1 = 2 \times x - 3 \quad \text{and} \quad y = 2 \times 5 - 3 \\ \Rightarrow & -1 = 2x - 3 \quad \text{and} \quad y = 10 - 3 \\ \Rightarrow & 3 - 1 = 2x \quad \text{and} \quad y = 7 \\ \Rightarrow & 2 = 2x \quad \text{and} \quad y = 7 \\ \Rightarrow & x = 1 \quad \text{and} \quad y = 7 \end{aligned}$$

Q3

We have,

$$a + b = 5$$

$$\Rightarrow a = 5 - b$$

$$\therefore b = 0 \Rightarrow a = 5 - 0 = 5,$$

$$b = 3 \Rightarrow a = 5 - 3 = 2,$$

$$b = 6 \Rightarrow a = 5 - 6 = -1,$$

Hence, the required set of ordered pairs (a, b) is $\{(-1, 6), (2, 3), (5, 0)\}$

Q4

We have,

$$a \in \{2, 4, 6, 9\}$$

$$\text{and, } b \in \{4, 6, 18, 27\}$$

Now, a/b stands for ' a divides b '. For the elements of the given sets, we find that $2/4$, $2/6$, $2/18$, $6/18$, $9/18$ and $9/27$

$\therefore \{(2, 4), (2, 6), (2, 18), (6, 18), (9, 18), (9, 27)\}$ are the required set of ordered pairs (a, b) .

Q5

We have,

$$A = \{1, 2\} \text{ and } B = \{1, 3\}$$

$$\text{Now, } A \times B = \{1, 2\} \times \{1, 3\}$$

$$= \{(1, 1), (1, 3), (2, 1), (2, 3)\}$$

$$\text{and, } B \times A = \{1, 3\} \times \{1, 2\}$$

$$= \{(1, 1), (1, 2), (3, 1), (3, 2)\}$$

Q6

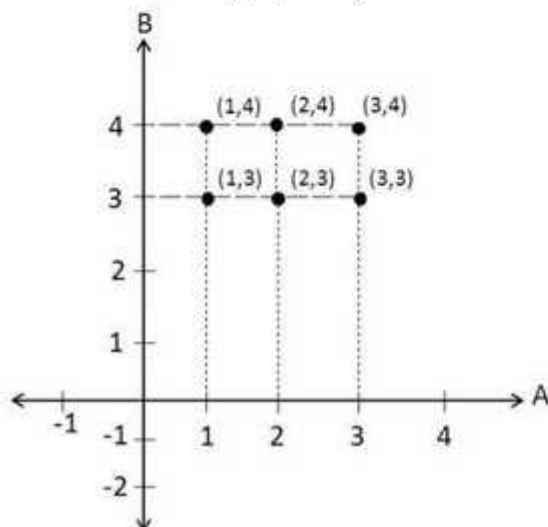
We have,

$$A = \{1, 2, 3\} \text{ and } B = \{3, 4\}$$

$$\begin{aligned} \therefore A \times B &= \{1, 2, 3\} \times \{3, 4\} \\ &= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\} \end{aligned}$$

In order to represent $A \times B$ graphically, we follow the following steps:

- Draw two mutually perpendicular line one horizontal and other vertical.
- On the horizontal line represent the element of set A and on the vertical line represent the elements of set B .
- Draw vertical dotted lines through points representing elements of A on horizontal line and horizontal lines through points representing elements of B on the vertical line points of intersection of these lines will represent $A \times B$ graphically.



Q7

We have,

$$A = \{1, 2, 3\} \text{ and } B = \{2, 4\}$$

$$A \times B = \{1, 2, 3\} \times \{2, 4\}$$

$$= \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\},$$

$$B \times A = \{2, 4\} \times \{1, 2, 3\}$$

$$= \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}.$$

$$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$$

$$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\},$$

$$B \times B = \{2, 4\} \times \{2, 4\}$$

$$= \{(2, 2), (2, 4), (4, 2), (4, 4)\},$$

$$\text{and, } (A \times B) \cap (B \times A)$$

$$= \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\} \cap \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}$$

$$= \{(2, 2)\}$$

$$\Rightarrow (A \times B) \cap (B \times A) = \{(2, 2)\}.$$

Q8

We have,

$$n(A) = 5 \text{ and } n(B) = 4$$

We know that, if A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$

$$\therefore n(A \times B) = 5 \times 4 = 20$$

Now,

$$n[(A \times B) \cap (B \times A)] = 3 \times 3 = 9$$

$$[\because A \text{ and } B \text{ have 3 elements in common}]$$

Q9

Let (a, b) be an arbitrary element of $(A \times B) \cap (B \times A)$. Then,

$$\begin{aligned} & (a, b) \in (A \times B) \cap (B \times A) \\ \Leftrightarrow & (a, b) \in A \times B \quad \text{and} \quad (a, b) \in B \times A \\ \Leftrightarrow & (a \in A \text{ and } b \in B) \quad \text{and} \quad (a \in B \text{ and } b \in A) \\ \Leftrightarrow & (a \in A \text{ and } a \in B) \quad \text{and} \quad (b \in A \text{ and } b \in B) \\ \Leftrightarrow & a \in A \cap B \quad \text{and} \quad b \in A \cap B \end{aligned}$$

Hence, the sets $A \times B$ and $B \times A$ have an element in common iff the sets A and B have an element in common.

Q10

Since $(x, 1)$, $(y, 2)$, $(z, 1)$ are elements of $A \times B$. Therefore, $x, y, z \in A$ and $1, 2 \in B$

It is given that $n(A) = 3$ and $n(B) = 2$

$$\therefore x, y, z \in A \text{ and } n(A) = 3$$

$$\Rightarrow A = \{x, y, z\}$$

$$1, 2 \in B \text{ and } n(B) = 2$$

$$\Rightarrow B = \{1, 2\}.$$

Q11

We have,

$$A = \{1, 2, 3, 4\}$$

$$\text{and, } R = \{(a, b) = a \in A, b \in A, a \text{ divides } b\}$$

Now,

a/b stands for ' a divides b '. For the elements of the given sets, we find that $1/1$, $1/2$, $1/3$, $1/4$, $2/2$, $3/3$ and $4/4$

$$\therefore R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}$$

Q12

We have,

$$A = \{-1, 1\}$$

$$\begin{aligned}\therefore A \times A &= \{-1, 1\} \times \{-1, 1\} \\ &= \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}\end{aligned}$$

$$\begin{aligned}\therefore A \times A \times A &= \{-1, 1\} \times \{(-1, -1), (-1, 1), (1, -1), (1, 1)\} \\ &= \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}\end{aligned}$$

Q13

(i) False,

$$\text{If } P = \{m, n\} \text{ and } Q = \{n, m\},$$

Then,

$$P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$$

(ii) False,

If A and B are non-empty sets, then AB is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

(iii) True

Q14

We have,

$$A = \{1, 2\}$$

$$\begin{aligned}\therefore A \times A &= \{1, 2\} \times \{1, 2\} \\ &= \{(1, 1), (1, 2), (2, 1), (2, 2)\}\end{aligned}$$

$$\begin{aligned}\therefore A \times A \times A &= \{1, 2\} \times \{(1, 1), (1, 2), (2, 1), (2, 2)\} \\ &= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}\end{aligned}$$

Q15

We have,

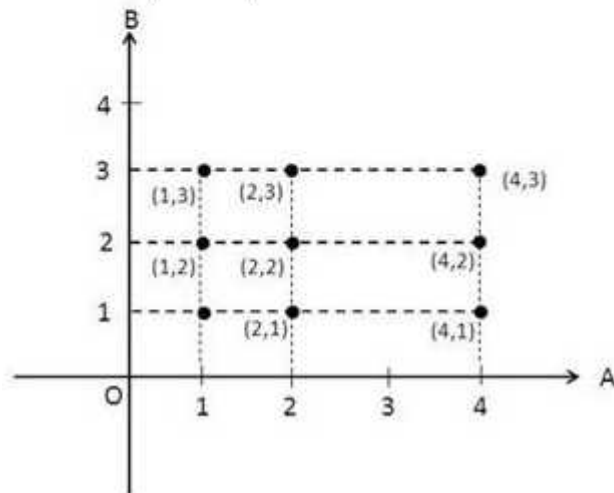
$$A = \{1, 2, 4\} \text{ and } B = \{1, 2, 3\}$$

$$\therefore A \times B = \{1, 2, 4\} \times \{1, 2, 3\}$$

$$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}$$

Hence, we represent A on the horizontal line and B on vertical line.

Graphical representation of $A \times B$ is as shown below:



We have,

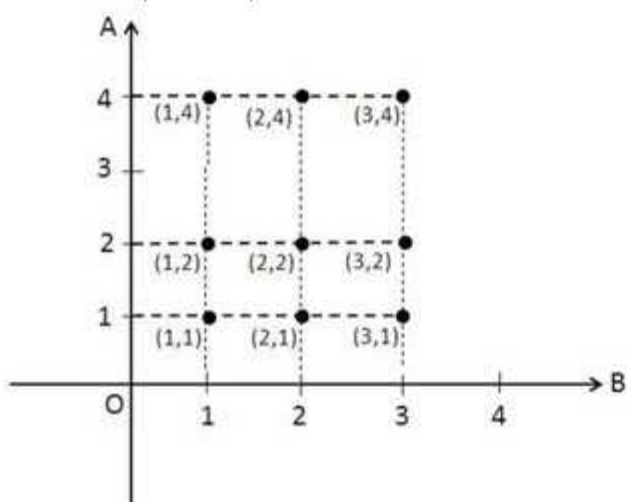
$$A = \{1, 2, 4\} \text{ and } B = \{1, 2, 3\}$$

$$\therefore B \times A = \{1, 2, 3\} \times \{1, 2, 4\}$$

$$= \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2), (3, 4)\}$$

Hence, we represent B on the horizontal line and A on vertical line.

Graphical representation of $B \times A$ is as shown below:



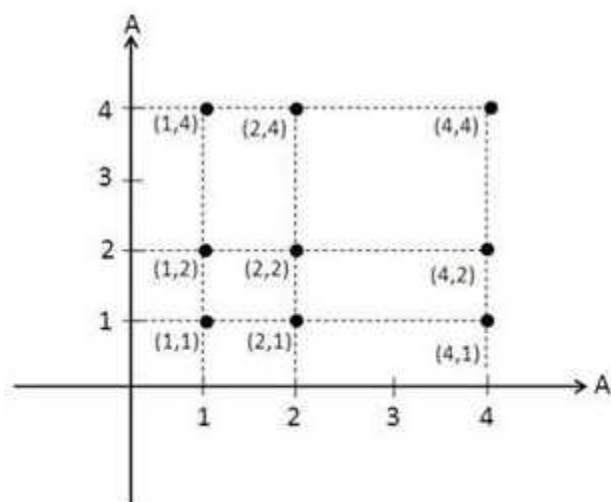
We have,

$$A = \{1, 2, 4\}$$

$$\therefore A \times A = \{1, 2, 4\} \times \{1, 2, 4\}$$

$$= \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (4, 1), (4, 2), (4, 4)\}$$

Graphical representation of $A \times A$ is shown below:



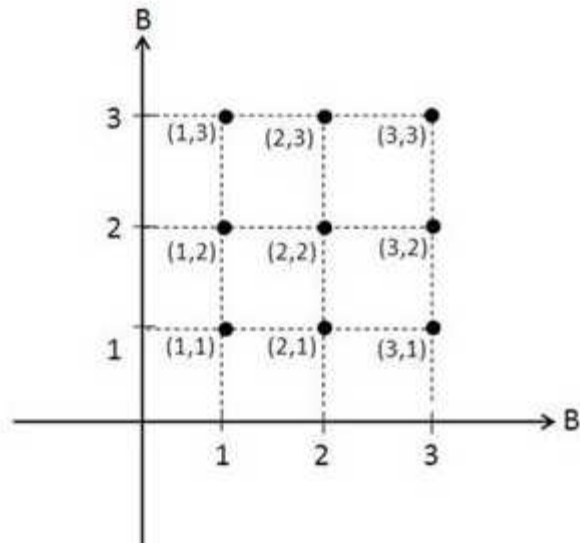
We have,

$$B = \{1, 2, 3\}$$

$$\therefore B \times B = \{1, 2, 3\} \times \{1, 2, 3\}$$

$$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Graphical representation of $B \times B$ is shown below:



Ex 2.2

Q1

We have,

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\}$$

$$\begin{aligned}\therefore A \times B &= \{1, 2, 3\} \times \{3, 4\} \\ &= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}\end{aligned}$$

$$\begin{aligned}\text{and, } B \times C &= \{3, 4\} \times \{4, 5, 6\} \\ &= \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}\end{aligned}$$

$$\therefore (A \times B) \cap (B \times C) = \{3, 4\}.$$

Q2

We have,

$$A = \{2, 3\}, B = \{4, 5\} \text{ and } C = \{5, 6\}$$

$$\begin{aligned}\therefore B \cup C &= \{4, 5\} \cup \{5, 6\} \\ &= \{4, 5, 6\}\end{aligned}$$

$$\begin{aligned}\therefore A \times (B \cup C) &= \{2, 3\} \times \{4, 5, 6\} \\ &= \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}\end{aligned}$$

Now,

$$B \cap C = \{4, 5\} \cap \{5, 6\} = \{5\}$$

$$\begin{aligned}\therefore A \times (B \cap C) &= \{2, 3\} \times \{5\} \\ &= \{(2, 5), (3, 5)\}\end{aligned}$$

Now,

$$\begin{aligned}A \times B &= \{2, 3\} \times \{4, 5\} \\ &= \{(2, 4), (2, 5), (3, 4), (3, 5)\}\end{aligned}$$

$$\begin{aligned}\text{and, } A \times C &= \{2, 3\} \times \{5, 6\} \\ &= \{(2, 5), (2, 6), (3, 5), (3, 6)\}\end{aligned}$$

$$\therefore (A \times B) \cup (A \times C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

Q3

We have,

$$A = \{1, 2, 3\}, B = \{4\} \text{ and } C = \{5\}$$

$$\therefore B \cup C = \{4\} \cup \{5\} = \{4, 5\}$$

$$\therefore A \times (B \cup C) = \{1, 2, 3\} \times \{4, 5\}$$

$$\Rightarrow A \times (B \cup C) = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\} \quad \text{--- (i)}$$

Now,

$$A \times B = \{1, 2, 3\} \times \{4\}$$

$$= \{(1, 4), (2, 4), (3, 4)\}$$

$$\text{and, } A \times C = \{1, 2, 3\} \times \{5\}$$

$$= \{(1, 5), (2, 5), (3, 5)\}$$

$$\therefore (A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4)\} \cup \{(1, 5), (2, 5), (3, 5)\}$$

$$\Rightarrow (A \times B) \cup (A \times C) = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\} \quad \text{--- (ii)}$$

From equation(i) and(ii), we get

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Hence verified.

We have,

$$A = \{1, 2, 3\}, B = \{4\} \text{ and } C = \{5\}$$

$$\therefore B \cap C = \{4\} \cap \{5\} = \emptyset$$

$$\therefore A \times (B \cap C) = \{1, 2, 3\} \times \emptyset$$

$$\Rightarrow A \times (B \cap C) = \emptyset \quad \text{--- (i)}$$

Now,

$$A \times B = \{1, 2, 3\} \times \{4\}$$

$$= \{(1, 4), (2, 4), (3, 4)\}$$

$$\text{and, } A \times C = \{1, 2, 3\} \times \{5\}$$

$$= \{(1, 5), (2, 5), (3, 5)\}$$

$$\therefore (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\} \cap \{(1, 5), (2, 5), (3, 5)\}$$

$$\Rightarrow (A \times B) \cap (A \times C) = \emptyset \quad \text{--- (ii)}$$

From equation(i) and equation(ii), we get

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Hence verified.

We have,

$$A = \{1, 2, 3\}, B = \{4\} \text{ and } C = \{5\}$$

$$\therefore B - C = \{4\}$$

$$\therefore A \times (B - C) = \{1, 2, 3\} \times \{4\}$$

$$\Rightarrow A \times (B - C) = \{(1, 4), (2, 4), (3, 4)\} \quad \text{--- (i)}$$

Now,

$$A \times B = \{1, 2, 3\} \times \{4\}$$

$$= \{(1, 4), (2, 4), (3, 4)\}$$

$$\text{and, } A \times C = \{1, 2, 3\} \times \{5\}$$

$$= \{(1, 5), (2, 5), (3, 5)\}$$

$$\therefore (A \times B) - (A \times C) = \{(1, 4), (2, 4), (3, 4)\} \quad \text{--- (ii)}$$

From equation(i) and equation(ii), we get

$$A \times (B - C) = (A \times B) - (A \times C)$$

Hence verified.

Q4

We have,

$$A = \{1, 2\}, B = \{1, 2, 3, 4\}, C = \{5, 6\} \text{ and } D = \{5, 6, 7, 8\}$$

$$\therefore B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$$

$$= \left[\begin{array}{l} (1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), \\ (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8) \end{array} \right] \quad \text{--- (i)}$$

$$\text{and, } A \times C = \{1, 2\} \times \{5, 6\}$$

$$= \{(1, 5), (1, 6), (2, 5), (2, 6)\} \quad \text{--- (ii)}$$

Clearly from equation(i) and equation(ii), we get

$$A \times C \subset B \times D$$

Hence verified.

We have,

$$A = \{1, 2\}, B = \{1, 2, 3, 4\}, C = \{5, 6\} \text{ and } D = \{5, 6, 7, 8\}$$

$$\therefore B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \emptyset$$

$$A \times (B \cap C) = \{1, 2\} \times \emptyset = \emptyset \quad \text{--- (i)}$$

Now,

$$\begin{aligned} A \times B &= \{1, 2\} \times \{1, 2, 3, 4\} \\ &= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\} \end{aligned}$$

$$\begin{aligned} \text{and, } A \times C &= \{1, 2\} \times \{5, 6\} \\ &= \{(1, 5), (1, 6), (2, 5), (2, 6)\} \end{aligned}$$

$$\therefore (A \times B) \cap (A \times C) = \emptyset \quad \text{--- (ii)}$$

From equation (i) and equation (ii), we get

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Hence verified.

Q5

Given:

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\}$$

(i) $A \times (B \cap C)$

Now,

$$(B \cap C) = \{4\}$$

$$\therefore A \times (B \cap C) = \{(1, 4), (2, 4), (3, 4)\}$$

(ii) $(A \times B) \cap (A \times C)$

Now,

$$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

And,

$$(A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$\therefore (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$$

(iii) $A \times (B \cup C)$

Now,

$$(B \cup C) = \{3, 4, 5, 6\}$$

$$\therefore A \times (B \cup C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

(iv) $(A \times B) \cup (A \times C)$

Now,

$$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

And,

$$(A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$\therefore (A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

Q6

Let (a, b) be an arbitrary element of $(A \cup B) \times C$. Then,

$$\begin{aligned} & (a, b) \in (A \cup B) \times C \\ \Rightarrow & a \in A \cup B \text{ and } b \in C && [\text{By definition}] \\ \Rightarrow & (a \in A \text{ or } a \in B) \text{ and } b \in C && [\text{By definition}] \\ \Rightarrow & (a \in A \text{ and } b \in C) \text{ or } (a \in B \text{ and } b \in C) \\ \Rightarrow & (a, b) \in A \times C \text{ or } (a, b) \in B \times C \\ \Rightarrow & (a, b) \in (A \times C) \cup (B \times C) \\ \Rightarrow & (a, b) \in (A \cup B) \times C \\ \Rightarrow & (a, b) \in (A \times C) \cup (B \times C) \\ \Rightarrow & (A \cup B) \times C \subseteq (A \times C) \cup (B \times C) && \text{---(i)} \end{aligned}$$

Again, let (x, y) be an arbitrary element of $(A \times C) \cup (B \times C)$. Then,

$$\begin{aligned} & (x, y) \in (A \times C) \cup (B \times C) \\ \Rightarrow & (x, y) \in A \times C \quad \text{or} \quad (x, y) \in B \times C \\ \Rightarrow & x \in A \text{ and } y \in C \quad \text{or} \quad x \in B \text{ and } y \in C \\ \Rightarrow & (x \in A \text{ or } x \in B) \quad \text{and} \quad y \in C \\ \Rightarrow & x \in A \cup B \quad \text{and} \quad y \in C \\ \Rightarrow & (x, y) \in (A \cup B) \times C \\ \Rightarrow & (x, y) \in (A \times C) \cup (B \times C) \\ \Rightarrow & (x, y) \in (A \cup B) \times C \\ \Rightarrow & (A \times C) \cup (B \times C) \subseteq (A \cup B) \times C && \text{---(ii)} \end{aligned}$$

Using equation (i) and equation (ii), we get

$$(A \cup B) \times C = (A \times C) \cup (B \times C)$$

Hence proved.

Let (a, b) be an arbitrary element of $(A \cap B) \times C$. Then,

$$\begin{aligned}
 & (a, b) \in (A \cap B) \times C \\
 \Rightarrow & a \in A \cap B \text{ and } b \in C \\
 \Rightarrow & (a \in A \text{ and } a \in B) \text{ and } b \in C & [\text{By definition}] \\
 \Rightarrow & (a \in A \text{ and } b \in C) \text{ and } (a \in B \text{ and } b \in C) \\
 \Rightarrow & (a, b) \in A \times C \text{ and } (a, b) \in B \times C \\
 \Rightarrow & (a, b) \in (A \times C) \cap (B \times C) \\
 \Rightarrow & (a, b) \in (A \cap B) \times C \\
 \Rightarrow & (a, b) \in (A \times C) \cap (B \times C) \\
 \Rightarrow & (A \cap B) \times C \subseteq (A \times C) \cap (B \times C) & \text{---(i)}
 \end{aligned}$$

Let (x, y) be an arbitrary element of $(A \times C) \cap (B \times C)$. Then,

$$\begin{aligned}
 & (x, y) \in (A \times C) \cap (B \times C) \\
 \Rightarrow & (x, y) \in A \times C \text{ and } (x, y) \in B \times C & [\text{By definition}] \\
 \Rightarrow & (x \in A \text{ and } y \in C) \text{ and } (x \in B \text{ and } y \in C) \\
 \Rightarrow & (x \in A \text{ and } x \in B) \text{ and } y \in C \\
 \Rightarrow & x \in A \cap B \text{ and } y \in C \\
 \Rightarrow & (x, y) \in (A \cap B) \times C \\
 \Rightarrow & (x, y) \in (A \times C) \cap (B \times C) \\
 \Rightarrow & (x, y) \in (A \cap B) \times C \\
 \Rightarrow & (A \times C) \cap (B \times C) \subseteq (A \cap B) \times C & \text{---(ii)}
 \end{aligned}$$

Using equation (i) and equation (ii), we get

$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$

Q7

Let (a, b) be an arbitrary element of $A \times B$. then,

$$\begin{aligned} & (a, b) \in A \times B \\ \Rightarrow & a \in A \text{ and } b \in B \end{aligned} \quad \text{---(i)}$$

Now,

$$\begin{aligned} & (a, b) \in A \times B \\ \Rightarrow & (a, b) \in C \times D & [\because A \times B \subseteq C \times D] \\ \Rightarrow & a \in C \text{ and } b \in D & \text{---(ii)} \\ \therefore & a \in A \Rightarrow a \in C & [\text{Using (i) and (ii)}] \\ \Rightarrow & A \subseteq C \\ \text{and,} & \\ & b \in B \Rightarrow b \in D \\ \Rightarrow & B \subseteq D \end{aligned}$$

Hence, proved

Ex 2.3

Q1

(i) We have,

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$\{(1, 6), (3, 4), (5, 2)\}$ is not a relation from A to B as it is not a subset of $A \times B$.

(ii) We have,

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$\{(1, 5), (2, 6), (3, 4), (3, 6)\}$ is a subset of $A \times B$, so it is a relation from A to B .

(iii) We have,

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$\{(4, 2), (4, 3), (5, 1)\}$ is not a relation from A to B as it is not a subset of $A \times B$.

(iv) We have,

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$A \times B$ is a relation from A to B .

Q2

We have,

$$A = \{2, 3, 4, 5\} \text{ and } B = \{3, 6, 7, 10\}$$

It is given that $(x, y) \in R \Leftrightarrow x$ is relatively prime to y

$$\therefore \{2, 3\} \in R, \{2, 7\} \in R, \{3, 7\} \in R, \{3, 10\} \in R, \{4, 3\} \in R, \{4, 7\} \in R, \{5, 3\} \in R, \text{ and } \{5, 7\} \in R.$$

Thus,

$$R = \{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 7)\}$$

Clearly, $\text{Domain}(R) = \{2, 3, 4, 5\}$ and $\text{Range} = \{3, 7, 10\}$.

Q3

We have,

$$A = \{1, 2, 3, 4, 5\}$$

∴ A is the set of first five natural number

It is given that R be a relation on A defined as $(x, y) \in R \Leftrightarrow x \leq y$

For the elements of the given sets A and A, we find that

$$1 = 1, 1 < 2, 1 < 3, 1 < 4, 1 < 5, 2 = 2, 2 < 3, 2 < 4, 2 < 5, 3 = 3, 3 < 4, 3 < 5, 4 = 4, 4 < 5, \text{ and } 5 = 5$$

$$\therefore (1, 1) \in R, (1, 2) \in R, (1, 3) \in R, (1, 4) \in R, (1, 5) \in R, (2, 2) \in R, (2, 3) \in R, (2, 4) \in R, (2, 5) \in R, \\ (3, 3) \in R, (3, 4) \in R, (3, 5) \in R, (4, 4) \in R, (4, 5) \in R \text{ and } (5, 5) \in R$$

Thus,

$$R = \left\{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5) \right\}$$

As so,

$$R^{-1} = \left\{ (1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4), (5, 5) \right\}$$

$$(i) \text{ Domain}(R^{-1}) = \{1, 2, 3, 4, 5\}$$

$$(ii) \text{ Range}(R) = \{1, 2, 3, 4, 5\}$$

Q4

(i) We have,

$$R = \{(1,2), (1,3), (2,3), (3,2), (5,6)\}$$

$$\Rightarrow R^{-1} = \{(2,1), (3,1), (3,2), (2,3), (6,5)\}$$

(ii) We have,

$$R = \{(x,y) : x, y \in N, x + 2y = 8\}$$

Now,

$$x + 2y = 8$$

$$\Rightarrow x = 8 - 2y$$

Putting $y = 1, 2, 3$ we get $x = 6, 4, 2$ respectively.

For $y = 4$, we get $x = 0 \notin N$. Also for $y > 4$, $x \notin N$.

$$\therefore R = \{(6,1), (4,2), (2,3)\}$$

Thus,

$$R^{-1} = \{(1,6), (2,4), (3,2)\}$$

$$\Rightarrow R^{-1} = \{(3,2), (2,4), (1,6)\}$$

(iii) We have,

$$R \text{ is a relation from } \{11, 12, 13\} \text{ to } \{8, 10, 12\} \text{ defined by } y = x - 3$$

Now,

$$y = x - 3$$

Putting $x = 11, 12, 13$ we get $y = 8, 9, 10$ respectively

$$\Rightarrow (11,8) \in R, (12,9) \notin R \text{ and } (13,10) \in R$$

Thus,

$$R = \{(11,8), (13,10)\}$$

$$\Rightarrow R^{-1} = \{(8,11), (10,13)\}$$

Q5

(i) We have,

$$x = 2y$$

Putting $y = 1, 2, 3$ we get $x = 2, 4, 6$ respectively.

$$\therefore R = \{(2, 1), (4, 2), (6, 3)\}$$

(ii) We have,

It is given that relation R on the set $\{1, 2, 3, 4, 5, 6, 7\}$ defined by $(x, y) \in R \Leftrightarrow x$ is relatively prime to y .

$$\therefore \begin{aligned} (2, 3) \in R, (2, 5) \in R, (2, 7) \in R, (3, 2) \in R, (3, 4) \in R, (3, 5) \in R, (3, 7) \in R, (4, 3) \in R, (4, 5) \in R, \\ (4, 7) \in R, (5, 2) \in R, (5, 3) \in R, (5, 4) \in R, (5, 6) \in R, (5, 7) \in R, (6, 5) \in R, (6, 7) \in R, (7, 2) \in R, \\ (7, 3) \in R, (7, 4) \in R, (7, 5) \in R \text{ and } (7, 6) \in R. \end{aligned}$$

Thus,

$$R = \left\{ \begin{aligned} &(2, 3), (2, 5), (2, 7), (3, 2), (3, 4), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 2), \\ &(5, 3), (5, 4), (5, 6), (5, 7), (6, 5), (6, 7), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6) \end{aligned} \right\}$$

(iii) We have,

$$2x + 3y = 12$$

$$\Rightarrow 2x = 12 - 3y$$

$$\Rightarrow x = \frac{12 - 3y}{2}$$

Putting $y = 0, 2, 4$ we get $x = 6, 3, 0$ respectively.

For $y = 1, 3, 5, 6, 7, 8, 9, 10$, $x \notin$ given set

$$\begin{aligned} \therefore R &= \{(6, 0), (3, 2), (0, 4)\} \\ &= \{(0, 4), (3, 2), (6, 0)\} \end{aligned}$$

(iv) We have,

$$A = \{5, 6, 7, 8\} \text{ and } B = \{10, 12, 15, 16, 18\}$$

Now,

a/b stands for 'a divides b'. For the elements of the given set A and B , we find that $5/10$, $5/15$, $6/12$, $6/18$ and $8/16$

$$\therefore (5, 10) \in R, (5, 15) \in R, (6, 12) \in R, (6, 18) \in R, \text{ and } (8, 16) \in R$$

Thus,

$$R = \{(5, 10), (5, 15), (6, 12), (6, 18), (8, 16)\}$$

Q6

We have,

$$\{x, y\} \in R \Leftrightarrow x + 2y = 8$$

Now,

$$x + 2y = 8$$

$$\Rightarrow x = 8 - 2y$$

Putting $y = 1, 2, 3$, we get $x = 6, 4, 2$ respectively

For $y = 4$, we get $x = 0 \notin N$

Also, for $y > 4$, $x \notin N$

$$\therefore R = \{(6, 1), (4, 2), (2, 3)\}$$

Thus,

$$R^{-1} = \{(1, 6), (2, 4), (3, 2)\}$$

$$\Rightarrow R^{-1} = \{(3, 2), (2, 4), (1, 6)\}$$

Q7

We have,

$$A = \{3, 5\}, \quad B = \{7, 11\}$$

and, $R = \{(a, b) : a \in A, b \in B, a - b \text{ is odd}\}$

For the elements of the given sets A and B , we find that

$$3 - 7 = -4, \quad 3 - 11 = -8, \quad 5 - 7 = -2 \text{ and } 5 - 11 = -6$$

$$\therefore (3, 7) \notin R, (3, 11) \notin R, (5, 7) \notin R \text{ and } (5, 11) \notin R,$$

Thus, R is an empty relation from A into B .

Q8

We have,

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

$$\therefore n(A) = 2 \text{ and } n(B) = 2$$

$$\Rightarrow n(A) \times n(B) = 2 \times 2 = 4$$

$$\Rightarrow n(A \times B) = 4$$

$$[\because n(A \times B) = n(A) \times n(B)]$$

So, there are $2^4 = 16$ relations from A to B .

$$\left[\begin{array}{l} \because n(x) = a, n(y) = b \\ \Rightarrow \text{Total number of relations} = 2^{ab} \end{array} \right]$$

Q9

(i) We have,

$$R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$$

For the elements of the given sets, we find that

$$R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$$

Clearly, $\text{Domain}(R) = \{0, 1, 2, 3, 4, 5\}$ and $\text{Range}(R) = \{5, 6, 7, 8, 9, 10\}$

(ii) We have,

$$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$$

For the elements of the given sets, we find that

$$x = 2, 3, 5, 7$$

$$\therefore (2, 8) \in R, (3, 27) \in R, (5, 125) \in R \text{ and } (7, 343) \in R$$

$$\Rightarrow R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$$

Clearly, $\text{Domain}(R) = \{2, 3, 5, 7\}$ and $\text{Range}(R) = \{8, 27, 125, 343\}$

Q10

(i) We have,

$$R = \{(a, b) : a \in N, a < 5, b = 4\}$$

$$\Rightarrow a = 1, 2, 3, 4 \text{ and } b = 4$$

$$\text{Thus, } R = \{(1, 4), (2, 4), (3, 4), (4, 4)\}$$

$$\text{Clearly, Domain}(R) = \{1, 2, 3, 4\} \text{ and Range}(R) = \{4\}$$

(ii) We have,

$$S = \{(a, b) : b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$$

$$\Rightarrow a = -3, -2, -1, 0, 1, 2, 3$$

For $a = -3, -2, -1, 0, 1, 2, 3$ we get
 $b = 4, 3, 2, 1, 0, 1, 2$ respectively

$$\text{Thus, } S = \{(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)\}$$

$$\text{Domain}(S) = \{-3, -2, -1, 0, 1, 2, 3\} \text{ and}$$

$$\text{Range}(S) = \{0, 1, 2, 3, 4\}$$

Q11

Here, $A = \{a, b\}$

we know that,

$$\begin{aligned} \text{Number of relations} &= 2^{m \cdot n} \\ &= 2^{2 \cdot 2} \\ &= 2^4 \\ &= 16 \end{aligned}$$

Number of relations on $A = 16$

Relations on A are given by

$$\begin{aligned} R = & \{a, a\}, \{a, b\}, \{b, a\}, \{b, b\} \\ & \{(a, a), (a, b)\}, \{(a, a), (b, a)\}, \{(a, a), (b, b)\}, \\ & \{(a, b), (b, a)\}, \{(a, b), (b, b)\}, \{(b, a), (b, b)\}, \\ & \{(a, a), (a, b), (b, a)\}, \{(a, b), (b, a), (b, b)\}, \\ & \{(b, a), (b, b), (a, a)\}, \{(b, b), (a, a), (a, b)\}, \\ & \{(a, a), (b, a), (b, b)\}, \{(a, a), (b, a), (b, b)\} \end{aligned}$$

Q12

We have,

$$A = \{x, y, z\} \text{ and } B = \{a, b\}$$

$$\Rightarrow n(A) = 3 \text{ and } n(B) = 2$$

$$\Rightarrow n(A) \times n(B) = 3 \times 2 = 6$$

$$\Rightarrow n(A \times B) = 6$$

$$[\because n(A \times B) = n(A) \times n(B)]$$

So, there are $2^6 = 64$ relations from A to B .

$$\left[\begin{array}{l} \because n(x) = a, n(y) = b \\ \Rightarrow \text{Total number of relations} = 2^{a \cdot b} \end{array} \right]$$

Q13

We have,

$$R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a = b^2\}$$

(i) This statement is not true because $(5, 5) \notin R$.

(ii) This statement is not true because $(25, 5) \in R$ but $(5, 25) \notin R$.

(iii) This statement is not true because $(36, 6) \in R$ and $(25, 5) \in R$ but $(36, 5) \notin R$.

Q14

We have,

$$3x - y = 0$$

$$\Rightarrow 3x = y$$

$$\Rightarrow y = 3x$$

Putting $x = 1, 2, 3, 4$ we get, $y = 3, 6, 9, 12$ respectively

For $x > 4$, we get $y > 14$ which does not belong to set A .

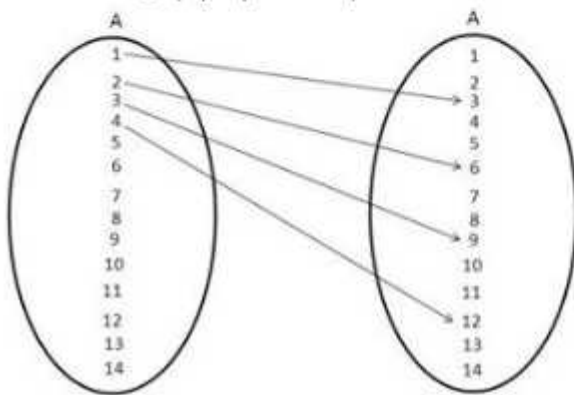
$$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

The arrow diagram representing R is as follows:

Clearly, $\text{Domain}(R) = \{1, 2, 3, 4\}$,

$\text{Co-domain}(R) = \{1, 2, 3, 4, \dots, 14\}$ and

$\text{Range}(R) = \{3, 6, 9, 12\}$



Q15

We have,

$$R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbb{N}\}$$

(i) Putting $x = 1, 2, 3$ we get, $y = 6, 7, 8$ respectively

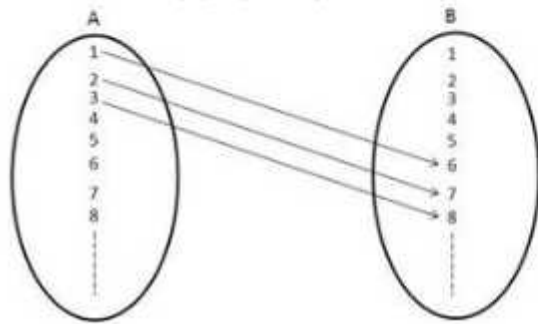
$\therefore$ Relation R in roster form is

$$R = \{(1, 6), (2, 7), (3, 8)\}$$

(ii) The arrow diagram representing R is as follows:

Clearly, $\text{Domain}(R) = \{1, 2, 3\}$ and

$$\text{Range}(R) = \{6, 7, 8\}$$



Q16

We have,

$$A = \{1, 2, 3, 5\} \text{ and } B = \{4, 6, 9\}$$

It is given that,

$$R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd, } x \in A, y \in B\}$$

For the elements of the given sets A and B , we find that

$$(1, 4) \in R, (1, 6) \in R, (2, 9) \in R, (3, 4) \in R, (3, 6) \in R, (5, 4) \in R \text{ and } (5, 6) \in R$$

$$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

Hence, relation R in roster form is $\{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

Q17

We have,

$$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$$

For the elements of the given sets, we find that

$$x = 2, 3, 5, 7$$

$$\therefore (2, 8) \in R, (3, 27) \in R, (5, 125) \in R \text{ and } (7, 343) \in R$$

$$\therefore \text{Relation } R \text{ in roster form is } = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$$

Q18

We have,

$$A = \{1, 2, 3, 4, 5, 6\}$$

and, $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

(i) Now, a/b stands for ' a divides b '. For the elements of the given sets A and A , we find that

$$1/1, 1/2, 1/3, 1/4, 1/5, 1/6, 2/2, 2/4, 2/6, 3/3, 3/6, 4/4, 5/5, 6/6$$

$\therefore$ Relation R in roster form is

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$$

$$(i) \text{ Domain}(R) = \{1, 2, 3, 4, 5, 6\}$$

$$(ii) \text{ Range}(R) = \{1, 2, 3, 4, 5, 6\}$$

Q19

(i) Set builder form of the relation from P to Q is

$$R = \{(x, y) : y = x - 2, x \in P, y \in Q\}$$

(ii) Roster form of the relation from P to Q is

$$R = \{(5, 3), (6, 4), (7, 5)\}$$

$$\text{Domain}(R) = \{5, 6, 7\}$$

$$\text{Range}(R) = \{3, 4, 5\}$$

Q20

We have,

$$R = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is an integer}\}$$

Clearly, $\text{Domain}(R) = \mathbb{Z}$,

$$\text{Range}(R) = \mathbb{Z}.$$

Q21

$$\text{Let } \left(1, \frac{-1}{2}\right) \in R_1 \text{ and } \left(\frac{-1}{2}, -4\right) \in R_1$$

$$\Rightarrow 1 + 1 \times \frac{-1}{2} > 0 \text{ and } 1 + \left(\frac{-1}{2}\right) - 4 > 0$$

$$\begin{aligned} \text{But, } 1 + 1 \times (-4) &= 1 - 4 \\ &= -3 < 0 \end{aligned}$$

$$\text{So, } (1, -4) \notin R_1$$

Q22

We have,

$$(a, b)R(c, d) \Leftrightarrow a + d = b + c \text{ for all } (a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$$

(i) We have,

$$a + b = b + a \text{ for all } a, b \in \mathbb{N}$$

$$\therefore (a, b)R(a, b) \text{ for all } a, b \in \mathbb{N}$$

(ii) Now,

$$(a, b)R(c, d)$$

$$\Rightarrow a + d = b + c$$

$$\Rightarrow c + b = d + a$$

$$\Rightarrow (c, d)R(a, b)$$

(iii) Now,

$$(a, b)R(c, d) \text{ and } (c, d)R(e, f)$$

$$\Rightarrow a + d = b + c \text{ and } c + f = d + e$$

$$\Rightarrow a + d + c + f = b + c + d + e$$

[Adding]

$$\Rightarrow a + f = b + e$$

$$\Rightarrow (a, b)R(e, f)$$