Expansions

- An identity is an equality which is true for all values of the variables in it. It helps us in shortening our calculations.
- Identities for "Square of Sum or Difference of Two Terms" are:
  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$

  Example:

  Evaluate $(5x + 2y)^2 - (3x - y)^2$.

  Solution:

  Using identities (i) and (ii), we obtain

  $(5x + 2y)^2 = (5x)^2 + 2\,(5x)\,(2y) + (2y)^2$

  $= 25x^2 + 20xy + 4y^2$

  $(3x - y)^2 = (3x)^2 - 2\,(3x)\,(y) + (y)^2$

  $= 9x^2 - 6xy + y^2$

  $\therefore (5x + 2y)^2 - (3x - y)^2 = 25x^2 + 20xy + 4y^2 - 9x^2 + 6xy - y^2 = 16x^2 + 26xy + 3y^2$

- **Identities:** $(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$ and $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$

Other ways to represent these identities are:

  - $x^3 + y^3 = (x + y)^3 - 3xy\,(x + y)$
  - $x^3 + y^3 = (x + y)\,(x^2 - xy + y^2)$
  - $x^3 - y^3 = (x - y)^3 + 3xy\,(x - y)$
  - $x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$

Example:

Expand $(3x + 2y)^3 - (3x - 2y)^3$

Solution:

$(3x + 2y)^3 = (3x)^3 + (2y)^3 + 3\,(3x)\,(2y)\,(3x + 2y)$

$= 27x^3 + 8y^3 + 54x^2y + 36xy^2$ ... (1)

$(3x - 2y)^3 = (3x)^3 - (2y)^3 - 3\,(3x)\,(2y)\,(3x - 2y)$

$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$ ... (2)

From equations (1) and (2) in given expression, we get

$(3x + 2y)^3 - (3x - 2y)^3 = (27x^3 + 8y^3 + 54x^2y + 36xy^2) - (27x^3 - 8y^3 - 54x^2y + 36xy^2)$

$= 27x^3 + 8y^3 + 54x^2y + 36xy^2 - 27x^3 + 8y^3 + 54x^2y - 36xy^2$

$= 16y^3 + 108x^2y$

- $(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$

Example:

Find 206 × 198.

Solution:

We have,

$206 \times 198 = (200 + 6)\,(200 - 2)$

$= (200)^2 + (6 + (-2)) \times 200 + (6)\,(-2)$ [Using identity $(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$]

$= 40000 + 800 - 12$

$= 40800 - 12$

$= 40788$

- **Identity:** $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can use this identity to factorize and expand the polynomials.

For example, the given expression can be factorized as follows:

$$2x^2 + 27y^2 + 25z^2 + 6\sqrt{6}xy - 30\sqrt{3}yz - 10\sqrt{2}xz$$
$$= \left(\sqrt{2}x\right)^2 + \left(3\sqrt{3}y\right)^2 + \left(-5z\right)^2 + 2\cdot\left(\sqrt{2}x\right)\left(3\sqrt{3}y\right) + 2\left(3\sqrt{3}y\right)(-5z) + 2\left(\sqrt{2}x\right)(-5z)$$

On comparing the expression with $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$, we get

$$2x^2 + 27y^2 + 25z^2 + 6\sqrt{6}xy - 30\sqrt{3}yz - 10\sqrt{2}xz$$
$$= \left(\sqrt{2}x + 3\sqrt{3}y - 5z\right)^2$$

• Some well-known identities are as follows:

$(a + b)^2 \equiv a^2 + 2ab + b^2$

$(a - b)^2 \equiv a^2 - 2ab + b^2$

$(a - b)^3 \equiv a^3 - b^3 - 3ab\,(a - b)$

• If an equation is true for all values of the variables involved in it under a certain condition, then the equation is known as conditional identity.

For example, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Here, $a^3 + b^3 + c^3 = 3abc$ is true only when $a + b + c = 0$, so it is a conditional identity.

- Identities for sum and difference of two cubes are:
    - $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
    - $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

For example, $x^6 - 729y^6$ can be factorized as:

$x^6 - 729y^6$

$= (x^3)^2 - (27y^3)^2$

$= (x^3 + 27y^3)\ (x^3 - 27y^3)$ [Using $a^2 - b^2 = (a + b)\ (a - b)$]

$= [(x)^3 + (3y)^3]\ [(x)^3 - (3y)^3]$

$= (x + 3y)\ (x^2 + 9y^2 - 3xy)\ (x - 3y)(x^2 + 9y^2 + 3xy)$