Factorisation

• **Factorization** is the decomposition of an algebraic expression into product of factors. Factors of an algebraic term can be numbers or algebraic variables or algebraic expressions.

For example, the factors of $2a^2b$ are 2, a, a, b, since $2a^2b = 2 \times a \times a \times b$

The factors, 2, a, a, b, are said to be irreducible factors of $2a^2b$ since they cannot be expressed further as a product of factors.

Also,
$$2a^2b = 1 \times 2 \times a \times a \times b$$

Therefore, 1 is also a factor of $2a^2b$. In fact, 1 is a factor of every term. However, we do not represent 1 as a separate factor of any term unless it is specially required.

For example, the expression, $2x^2(x+1)$, can be factorized as $2 \times x \times x \times (x+1)$.

Here, the algebraic expression (x + 1) is a factor of $2x^2(x + 1)$.

• Factorization of expressions by the method of common factors

This method involves the following steps.

Step 1: Write each term of the expression as a product of irreducible factors.

Step 2: Observe the factors, which are common to the terms and separate them.

Step 3: Combine the remaining factors of each term by making use of distributive law.

Example: Factorize $12p^2q + 8pq^2 + 18pq$.

Solution: We have,

$$12p^2q = 2 \times 2 \times 3 \times p \times p \times q$$

$$8pq^2 = 2 \times 2 \times 2 \times p \times q \times q$$

$$18pq = 2 \times 3 \times 3 \times p \times q$$

The common factors are 2, p, and q.

$$\therefore 12p^{2}q + 8pq^{2} + 18pq$$

$$= 2 \times p \times q \left[(2 \times 3 \times p) + (2 \times 2 \times q) + (3 \times 3) \right]$$

$$= 2pq (6p + 4q + 9)$$

• Factorization by regrouping terms

Sometimes, all terms in a given expression do not have a common factor. However, the terms can be grouped by trial and error method in such a way that all the terms in each group have a common factor. Then, there happens to occur a common factor amongst each group, which leads to the required factorization.

Example:

Factorize
$$2a^2 - b + 2a - ab$$
.

Solution:

$$2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$$

The terms, $2a^2$ and 2a, have common factors, 2 and a.

The terms, -b and -ab have common factors, -1 and b.

Therefore,

$$2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$$

$$=2a(a+1)-b(1+a)$$

$$= (a + 1) (2a - b)$$
 (As the factor, $(1 + a)$, is common to both the terms)

Thus, the factors of the given expression are (a + 1) and (2a - b).

• Factorisation of quadratic polynomials of the form $ax^2 + bx + c$ can be done using Factor theorem and splitting the middle term.

Example 1:

Factorize $x^2 - 7x + 10$ using the factor theorem.

Solution:

Let
$$p(x) = x^2 - 7x + 10$$

The constant term is 10 and its factors are ± 1 , ± 2 , ± 5 and ± 10 .

Let us check the value of the polynomial for each of these factors of 10.

$$p(1) = 1^2 - 7 \cdot 1 + 10 = 1 - 7 + 10 = 4 \neq 0$$

Hence, x - 1 is not a factor of p(x).

$$p(2) = 2^2 - 7 \cdot 2 + 10 = 4 - 14 + 10 = 0$$

Hence, x - 2 is a factor of p(x).

$$p(5) = 5^2 - 7 \cdot 5 + 10 = 25 - 35 + 10 = 0$$

Hence, x - 5 is a factor of p(x).

We know that a quadratic polynomial can have a maximum of two factors. We have obtained the two factors of the given polynomial, which are x - 2 and x - 5.

Thus, we can write the given polynomial as:

$$p(x) = x^2 - 7x + 10 = (x - 2)(x - 5)$$

Example 2:

Factorize $2x^2 - 11x + 15$ by splitting the middle term.

Solution:

The given polynomial is $2x^2 - 11x + 15$.

Here, $a = 2 \times 15 = 30$. The middle term is -11. Therefore, we have to split -11 into two numbers such that their product is 30 and their sum is -11. These numbers are -5 and -6 [As (-5) + (-6) = -11 and $(-5) \times (-6) = 30$].

Thus, we have:

$$2x^{2} - 11x + 15 = 2x^{2} - 5x - 6x + 15$$
$$= x (2x - 5) - 3 (2x - 5)$$
$$= (2x - 5) (x - 3)$$

Note: A quadratic polynomial can have a maximum of two factors.

•
$$(a+b)(a-b) = a^2 - b^2$$

Example:

Evaluate 95×105 .

Solution:

We have,
$$95 \times 105 = (100 - 5) \times (100 + 5)$$

= $(100)^2 - (5)^2$ [Using identity $(a + b) (a - b) = a^2 - b^2$]
= $10000 - 25$
= 9975

• Identities for sum and difference of two cubes are:

o
$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

o $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

For example, $x^6 - 729y^6$ can be factorized as:

$$x^{6} - 729y^{6}$$

$$= (x^{3})^{2} - (27y^{3})^{2}$$

$$= (x^{3} + 27y^{3}) (x^{3} - 27y^{3}) [Using a^{2} - b^{2} = (a+b) (a-b)]$$

$$= [(x)^{3} + (3y)^{3}] [(x)^{3} - (3y)^{3}]$$

$$= (x + 3y) (x^{2} + 9y^{2} - 3xy) (x - 3y)(x^{2} + 9y^{2} + 3xy)$$