

Simultaneous Linear Equations

- **Substitution Method of Solving Pairs of Linear Equations**

In this method, we have **substituted** the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why this method is known as the **substitution method**.

Example:

Solve the following system of equations by substitution method.

$$x - 4y + 7 = 0$$

$$3x + 2y = 0$$

Solution:

The given equations are

$$x - 4y + 7 = 0 \quad \dots (1)$$

$$3x + 2y = 0 \quad \dots (2)$$

From equation (2),

$$3x = -2$$

$$\Rightarrow x = -\frac{2}{3}y$$

Put $x = -\frac{2}{3}y$ in equation (1)

$$-\frac{2}{3}y - 4y + 7 = 0$$

$$\Rightarrow \frac{-2y - 12y}{3} = -7$$

$$\Rightarrow -14y = -21$$

$$\Rightarrow y = \frac{-21}{-14} = \frac{3}{2}$$

$$\therefore x = -\frac{2}{3}\left(\frac{3}{2}\right) = -1$$

Therefore, the required solution is $\left(-1, \frac{3}{2}\right)$.

- **Elimination Method to Solve a Pair of Linear Equations**

Example:

Solve the following pair of linear equations by elimination method.

$$7x - 2y = 10$$

$$5x + 3y = 6$$

Solution:

$$7x - 2y = 10 \quad \dots (1)$$

$$5x + 3y = 6 \quad \dots (2)$$

Multiplying equation (1) by 5 and equation (2) by 7, we get

$$35x - 10y = 50 \quad \dots (3)$$

$$35x + 21y = 42 \quad \dots (4)$$

Subtracting equation (4) from (3), we get

$$-31y = 8 \Rightarrow y = -\frac{8}{31}$$

Now, using equation (1):

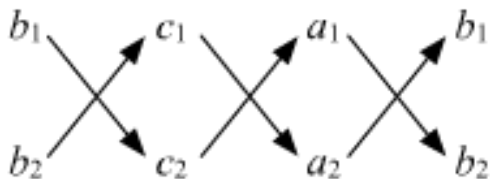
$$7x = 10 + 2y$$

$$\Rightarrow x = \frac{1}{7} \left\{ 10 + 2 \times \frac{-8}{31} \right\} = \frac{42}{31}$$

Required solution is $\left(\frac{42}{31}, -\frac{8}{31} \right)$.

- **Cross-Multiplication Method of Solving Pairs of Linear Equations**

The solution of the system of linear equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ can be determined by the following diagram.



That is,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad (a_1b_2 - a_2b_1 \neq 0)$$

Example:

Solve the following pair of linear equations by the cross-multiplication method

$$x - 5y = 14, 4x + 3y = 10$$

Solution:

$$x - 5y - 14 = 0$$

$$4x + 3y - 10 = 0$$

$$\frac{x}{(-5) \times (-10) - 3 \times (-14)} = \frac{y}{(-14) \times 4 - (-10) \times 1} = \frac{1}{1 \times 3 - 4 \times (-5)}$$

$$\Rightarrow \frac{x}{50 + 42} = \frac{y}{-56 + 10} = \frac{1}{3 + 20}$$

$$\Rightarrow \frac{x}{92} = \frac{y}{-46} = \frac{1}{23}$$

$$\Rightarrow x = \frac{92}{23} = 4, y = -\frac{46}{23} = -2$$

$\therefore$ Required solution is $(4, -2)$.

Equations reducible to a pair of linear equations in two variables

Some pair of equations which are not linear can be reduced to linear form by suitable substitutions.

Example: Solve the following system of equations

$$\frac{2}{x-2} - \frac{1}{y-1} = 1$$

$$\frac{5}{x-2} - \frac{6}{y-1} = 20$$

Solution:

Let $\frac{1}{x-2} = u$ and $\frac{1}{y-1} = v$. Then, the given system of equations reduces to

$$2u - v = 1 \quad \dots (1)$$

$$5u - 6v = 20 \quad \dots (2)$$

Multiplying equation (1) by 6 and then subtracting from (2), we get

$$5u - 6v = 20$$

$$12u - 6v = 6$$

$$\begin{array}{r} - \quad + \quad - \\ \hline -7u = 14 \end{array}$$

$$\Rightarrow u = \frac{14}{-7} = -2$$

$$\begin{aligned} \text{Equation (1)} \Rightarrow v &= 2u - 1 \\ &= 2(-2) - 1 \\ &= -4 - 1 = -5 \end{aligned}$$

$$\therefore \frac{1}{x-2} = -2, \frac{1}{y-1} = -5$$

$$\Rightarrow x-2 = -\frac{1}{2} \quad \Rightarrow y-1 = -\frac{1}{5}$$

$$\Rightarrow x = 2 - \frac{1}{2} = \frac{3}{2} \quad \Rightarrow y = 1 - \frac{1}{5} = \frac{4}{5}$$

$\therefore \left(\frac{3}{2}, \frac{4}{5}\right)$ is the solution of the given system of equations.