Indices and Logarithms

- **Laws of rational exponents of real numbers:**

Let *a* and *b* be two real numbers and *m* and *n* be two rational numbers then

  - $a^p \cdot a^q = a^{p+q}$
  - $(a^p)^q = a^{pq}$
  - $\frac{a^p}{a^q} = a^{p-q}$
  - $a^p b^p = (ab)^p$
  - $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
  - $a^{-p} = \frac{1}{a^p}$

Example:

$\sqrt[3]{(512)^{-2}}$

$= \left[(512)^{-2}\right]^{\frac{1}{3}}$

$= (512)^{\frac{-2}{3}} \quad [(a^m)^n = a^{mn}]$

$= (8^3)^{\frac{-2}{3}}$

$= (8)^{3\times\frac{-2}{3}} \quad [(a^m)^n = a^{mn}]$

$= (8)^{-2}$

$= \frac{1}{8^2} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{64}$

- If a is any positive real number (except 1), n is any rational number and $a^n = b$, then n is called the **logarithm** of b to the base a, and is written as $\log_a b$.

Thus, $a^n = b$ if and only if $\log_a b = n$.

$a^n = b$ is called the exponential form and $\log_a b = n$ is called the logarithmic form.

- **Properties of logarithm:**
  - Logarithms are only defined for positive real numbers.
  - $\log_a 1 = 0$ and $\log_a a = 1$ where, a is any positive real number except 1.
  - $\log_a x = \log_a y = n(\text{say}) \Rightarrow x = y$
  - Logarithms to the base 10 are called common logarithms.
  - If no base is given, the base is always taken as 10. For example, log 5 = $\log_{10} 5$

- **Laws of logarithm:**
  - **Product Law:**

    $$\log_a mn = \log_a m + \log_a n$$

    In general, $\log_a (mnp...) = \log_a m + \log_a n + \log_a p + ...$

  - **Quotient Law:**

    $$\log_a \frac{m}{n} = \log_a m - \log_a n$$

  - **Power Law:**

    $$\log_a m^n = n \log_a m$$

Example:

Find the value of x if $\log_7 343 = 5x - 4$.

Solution:

$$\log_7 343 = 5x - 4$$

$$\Rightarrow 7^{(5x-4)} = 343$$

$$\Rightarrow 7^{(5x-4)} = 7^3$$

$$\Rightarrow 5x - 4 = 3$$

$$\Rightarrow 5x = 7$$

$$\Rightarrow x = \frac{7}{5}$$