Similarity

- **Similar and Congruent Figures**
  - Two geometric figures having the same shape and size are said to be congruent figures.
  - Two geometric figures having the same shape, but not necessarily the same size, are called similar figures.

  Example:
  (1) All circles are similar.
  (2) All equilateral triangles are similar.
  (3) All congruent figures are similar. However, the converse is not true.

- **Similarity of Polygons**

  Two polygons with the same number of sides are similar, if

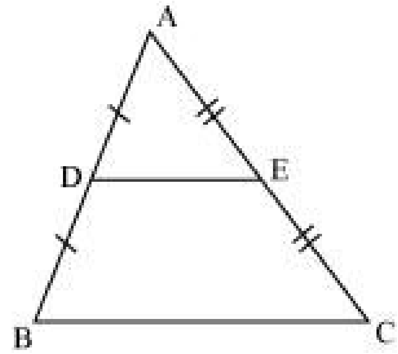
  - their corresponding angles are equal
  - their corresponding sides are in the same ratio (or proportion)

- Two lines segments are congruent, if they are equal in length.
- Two angles are congruent, if they have the same measure.
- **CPCT:**

CPCT stands for Corresponding Parts of Congruent Triangles.

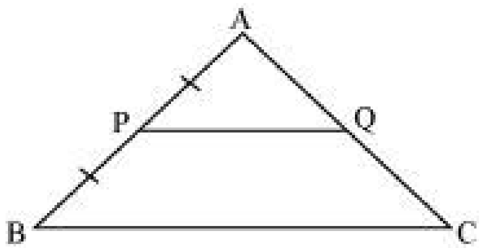
If $\Delta ABC \cong \Delta PQR$, then corresponding sides are equal i.e., AB = PQ, BC = QR, and CA = RP and corresponding angles are equal i.e., $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

- **Mid-point theorem** states that the line segment joining the mid-point of any two sides of a triangle is parallel to the third side and is half of it.



In ΔABC, if D and E are the mid-points of sides AB and AC respectively then by mid-point theorem DE ∥ BC and DE = $\frac{BC}{2}$

Converse of the mid-point theorem is also true, which states that a line through the mid-point of one side of a triangle and parallel to the other side bisects the third side.



In ΔABC, if AP = PB and PQ ∥ BC then PQ bisects AC i.e., Q is the mid-point of AC.

• When a line intersects two distinct lines at different points, then this line is known as transversal and the portion of the **transversal** lying between these two distinct lines is known as **intercept**.

In the given figure, MN is the intercept made by lines *a* and *b* on transversal *x*.

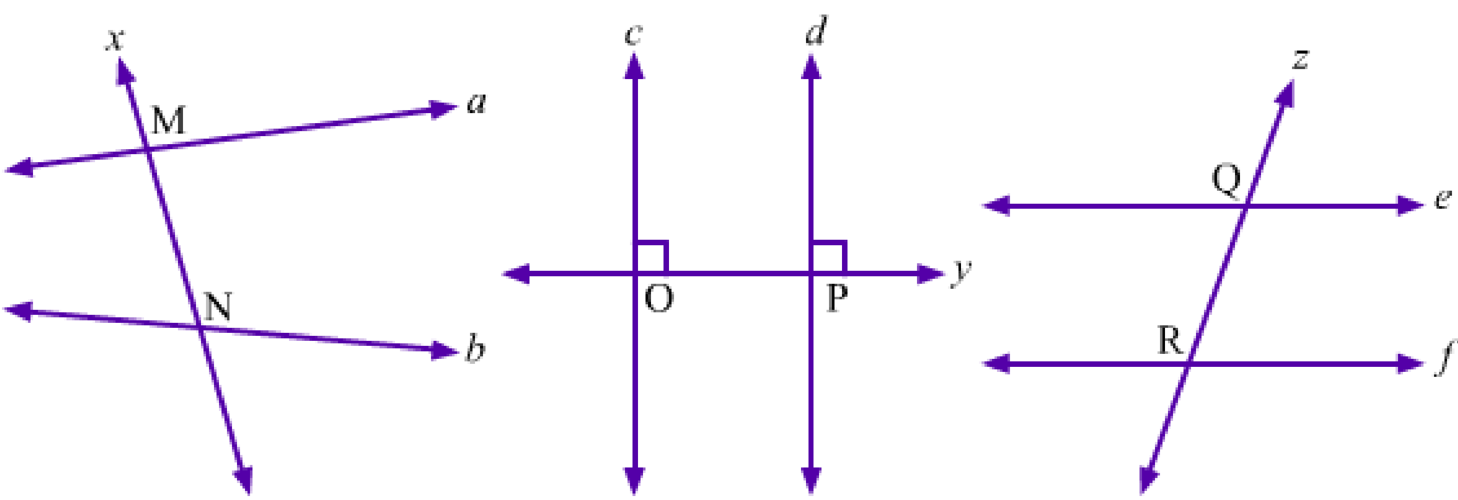
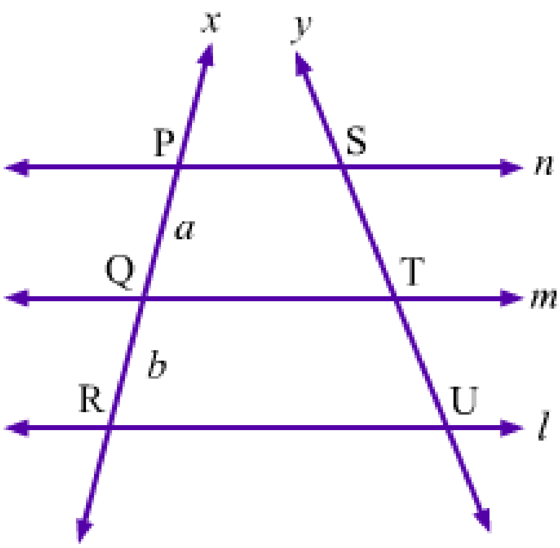


Fig. (1) Fig. (2) Fig. (3)

• The lengths of the intercepts made by three parallel lines on one transversal are in the same ratio as the lengths of the corresponding intercepts made by the same lines on any other transversal.

In the given figure, $\frac{PQ}{QR} = \frac{ST}{TU} = \frac{a}{b}$

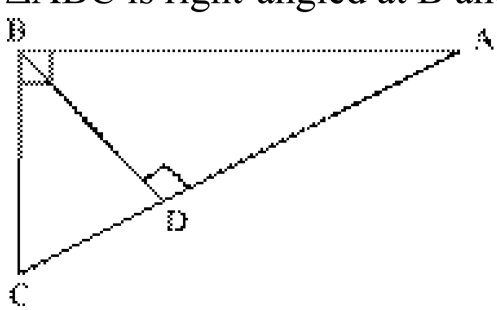


• If three parallel lines form congruent intercepts on one transversal, then the intercepts formed by them on the other transversals are also congruent.

- **Pythagoras theorem:**

  In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

  Example:
  ΔABC is right-angled at B and BD⊥CA.



  Prove that $BD^2 = CD \times DA$.
  Solution:
  By applying Pythagoras theorem in ΔBDC, ΔBDA, and ΔABC, we obtain
  $BC^2 = CD^2 + BD^2$ … (1)

$BA^2 = BD^2 + DA^2$ … (2)

$CA^2 = BC^2 + BA^2$ … (3)

Adding equations (1) and (2), we obtain

$BC^2 + BA^2 = 2BD^2 + CD^2 + DA^2$

$\Rightarrow CA^2 = 2BD^2 + CD^2 + DA^2$ ... [Using (3)]

$\Rightarrow (CD + DA)^2 = 2BD^2 + CD^2 + DA^2$

$\Rightarrow CD^2 + DA^2 + 2 \times CD \times DA = 2BD^2 + CD^2 + DA^2$

$\Rightarrow CD \times DA = BD^2$

- **Converse of Pythagoras theorem:**

  In a triangle, if the square of one side is equal to the sum of the squares of other two sides, then the angle opposite to the first side is a right angle.