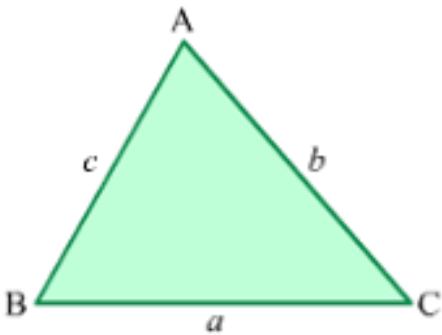


Area and Perimeter of Plane Figure

- **Perimeter** is the length of the boundary of a closed figure.
- The perimeter of a polygon is the sum of the lengths of all its sides.

In case of a triangle ABC, with sides of lengths a , b and c units:



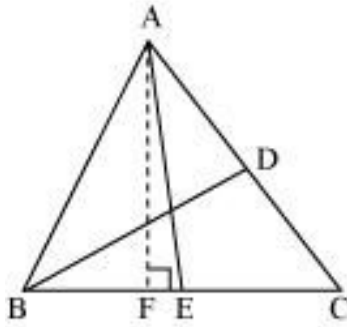
$$\text{Perimeter of ABC} = AB + BC + AC = a + b + c$$

- The **semi-perimeter** of a triangle is half the perimeter of the triangle.

The semi-perimeter (s) of a triangle with sides a , b and c is $\frac{a+b+c}{2}$.

- The semi-perimeter of a triangle is used for calculating its area when the length of altitude is not known.
- Area of a triangle:
 - Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$
 - All the congruent triangles are equal in area, but the triangles having equal areas may or may not be congruent.

Example: $\triangle ABC$ is isosceles with $AC = BC = 6$ cm. AE and BD are the medians and $AF = 4$ cm. What is the area of $\triangle ABD$?



Solution: In $\triangle ABE$ and $\triangle BAD$, we have

$$BE = AD \quad \left[AC = BC \Rightarrow \frac{1}{2}AC = \frac{1}{2}BC \right]$$

$$\angle ABE = \angle BAD \quad [\text{Angles opposite to equal sides}]$$

$$AB = AB \quad [\text{Common}]$$

$$\Rightarrow \triangle ABE \cong \triangle BAD \quad [\text{By SAS congruency criterion}]$$

$$\text{Area}(\triangle ABE) = \text{Area}(\triangle BAD)$$

$$\text{Now, Area } \triangle ABE = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times BE \times AF$$

$$= \frac{1}{2} \times \left(\frac{6 \text{ cm}}{2} \right) \times 4 \text{ cm}$$

$$= 6 \text{ cm}^2$$

$$\Rightarrow \text{Area } \triangle ABD = 6 \text{ cm}^2$$

- **Area of triangle using Heron's formula:**

When all the three sides of a triangle are given, its area can be calculated using Heron's formula, which is given by:

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Here, s is the semi-perimeter of the triangle and is given by, $s = \frac{a+b+c}{2}$

Example:

Find the area of a triangle whose sides are 9 cm, 28 cm and 35 cm.

Solution:

Let $a = 9 \text{ cm}$, $b = 28 \text{ cm}$ and $c = 35 \text{ cm}$

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2} = \frac{9+28+35}{2} \text{ cm} = 36 \text{ cm}$$

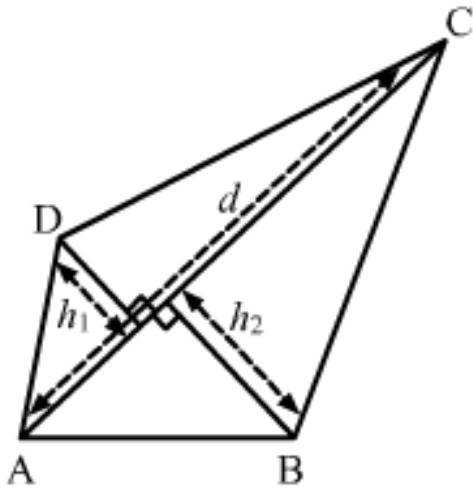
$$\text{Area of triangle} = \sqrt{36(36-9)(36-28)(36-35)} \text{ cm}^2$$

$$= \sqrt{36 \times 27 \times 8 \times 1} \text{ cm}^2$$

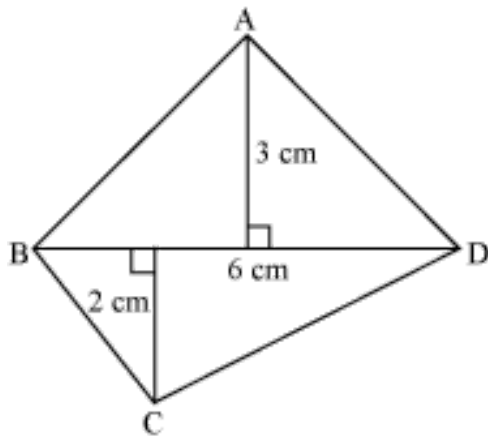
$$= 36\sqrt{6} \text{ cm}^2$$

- **Area of quadrilateral ABCD = Area of $\triangle ABC$ + Area of $\triangle ACD$**

- $$= 12 \times d \times h_1 + 12 \times d \times h_2 = 12 \times d \times h_1 + h_2$$



Example: Find the area of the quadrilateral ABCD.



Solution:

Area of the quadrilateral ABCD = Area of $\triangle ABD$ + Area of $\triangle BCD$

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{corresponding height}$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times 6\text{ cm} \times 3\text{ cm} = 9\text{ cm}^2$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times 6\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2$$

$$\therefore \text{Area of quadrilateral ABCD} = 9\text{ cm}^2 + 6\text{ cm}^2 = 15\text{ cm}^2$$

- Area of rhombus = $\frac{1}{2}$ (Product of its diagonals)

- Area of a rectangle is given by the formula:

Area of a rectangle = length $\times$ breadth

Example: How much carpet is required to cover a rectangular floor of length

25 m and breadth 18 m?

Solution: Area of the carpet required = Area of rectangular floor
$$= 25 \text{ m} \times 18 \text{ m} = 450 \text{ m}^2$$

- Area of a square is given by the formula:

Area of a square = side $\times$ side

Example: What is the area of a square park of side 10 m 20 cm?

Solution: Length of park = 10 m 20 cm = 10.2 m

Area of park = $10.2 \text{ m} \times 10.2 \text{ m} = 104.04 \text{ m}^2$

- Perimeter of a rectangle = $2 (\text{length} + \text{breadth})$

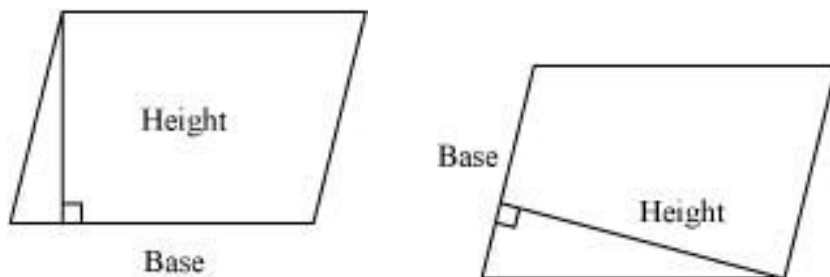
Example:

What is the perimeter of a rectangular field whose length and breadth are 15 m and 8 m respectively?

Solution:

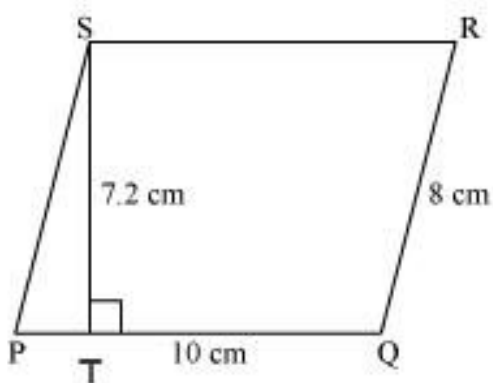
Perimeter of rectangular field = $2 (15 \text{ m} + 8 \text{ m}) = (2 \times 23) \text{ m} = 46 \text{ m}$

- Area of a parallelogram:
 - The perpendicular dropped on a side from its opposite vertex is known as the height and the side is known as the base.
 - Area of a parallelogram = Base $\times$ Height



Example:

Find the height of the parallelogram PQRS corresponding to the base RQ.



Solution:

Let the height corresponding to the base RQ be x cm.

$$\begin{aligned}\text{Area of the parallelogram PQRS} &= PQ \times ST \\ &= 10 \text{ cm} \times 7.2 \text{ cm} \\ &= 72 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the parallelogram} &= RQ \times x \\ &= 8 \text{ cm} \times x \text{ cm} \\ &= 8x \text{ cm}^2\end{aligned}$$

$$\therefore 8x = 72$$

$$\Rightarrow x = 9$$

Thus, the height of the parallelogram corresponding to the base RQ is 9 cm.

- Area and perimeter of various shapes:

Shape	Area	Perimeter
1. Rectangle with adjacent sides a and b	$a \times b$	$2(a + b)$
2. Square with side a	a^2	$4a$

3. Circle with radius r	πr^2	$2\pi r$
4. Triangle with base b and its corresponding height h	$\frac{1}{2} \times b \times h$	Sum of the three sides
5. Parallelogram with base b and its corresponding height h	$b \times h$	Sum of the four sides

Area of trapezium = $\frac{1}{2}$ (Sum of the lengths of the parallel sides) $\times$ (Perpendicular distance between them)

- The distance around a circular region is known as its circumference.
- The circumference of a circle = $\pi \times \text{Diameter} = 2\pi \times \text{Radius}$

The value of pi (π) is $\frac{22}{7}$ or 3.14.

- Area of a circle = $\pi \times (\text{Radius})^2$

Example: What is the area of a circle whose circumference is 44 cm? $\left(\pi = \frac{22}{7}\right)$

Solution:

$$\text{Circumference} = 2\pi r = 44 \text{ cm}$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow r = 44 \times \frac{7}{22 \times 2} = 7 \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$