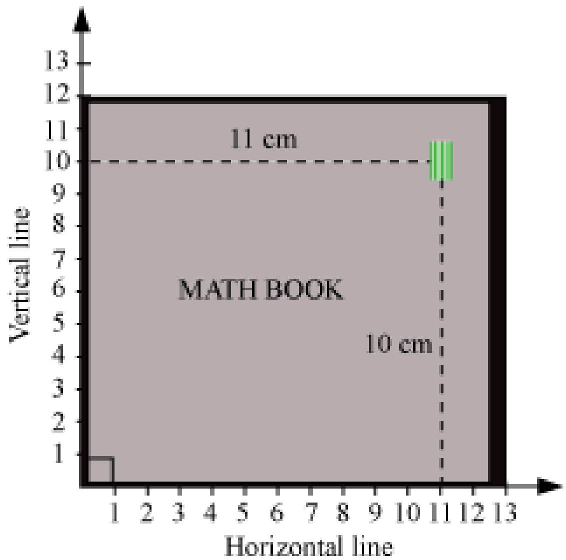
Coordinate Geometry

- **Cartesian plane and the terms associated with it**

To identify the position of an object or a point in a plane, we require two perpendicular lines: one of them is horizontal and the other is vertical.

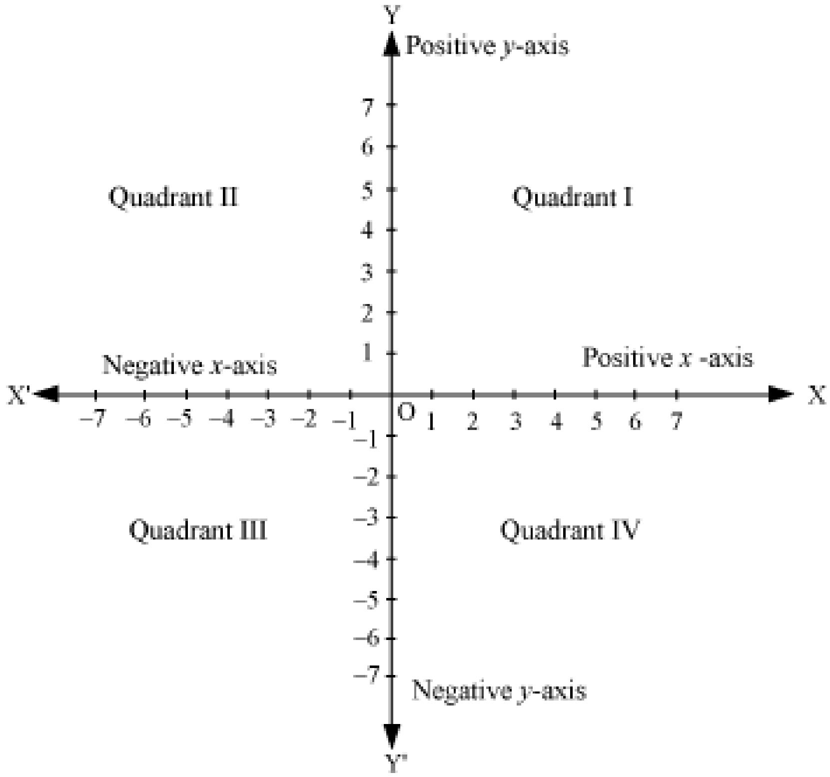
Example:
Put an eraser on a book and then describe the position of the eraser.
Solution:
In order to identify the position of the eraser on the book, we take the adjacent edges as perpendicular lines. Take 1 unit = 1 cm along the vertical and horizontal lines.
Now, it is seen that the eraser is at a distance of 11 cm from the vertical line and 10 cm from the horizontal line.



Thus, conventionally, the position of the eraser can be written as (11, 10).

- **Cartesian system**

A Cartesian system consists of two perpendicular lines: one of them is horizontal and the other is vertical. The horizontal line is called the x- axis and the vertical line is called the y -axis. The point of intersection of the two lines is called origin, and is denoted by O.



- XOX' is called the *x*-axis; YOY' is called the *y*-axis; the point O is called the origin.
- Positive numbers lie on the directions of OX and OY.
- Negative numbers lie on the directions of OX' and OY'.
- OX and OY are respectively called positive *x*-axis and positive *y*-axis.
- OX' and OY' are respectively called negative *x*-axis and negative *y*-axis. The axes divide the plane into four equal parts. The four parts are called quadrants, numbered I, II, III and IV, in anticlockwise from positive *x*-axis, OX.
- The plane is also called co-ordinate plane or Cartesian plane or *xy* -plane.

- **Coordinate Geometry**

  Example:
  Name the quadrant or the axis in which the points (5, –4), (2, 7) and (0, –9) lie?
  Solution
  The coordinates of the point (5, –4) are of the form (+, –).
  (5, –4) lie in quadrant IV
  The coordinates of the point (2, 7) are of the form (+, +).
  (2, 7) lie in quadrant I.
  The coordinates of the point (0, –9) are of the form (0, *b*).
  (0, –9) lie on the *y*-axis

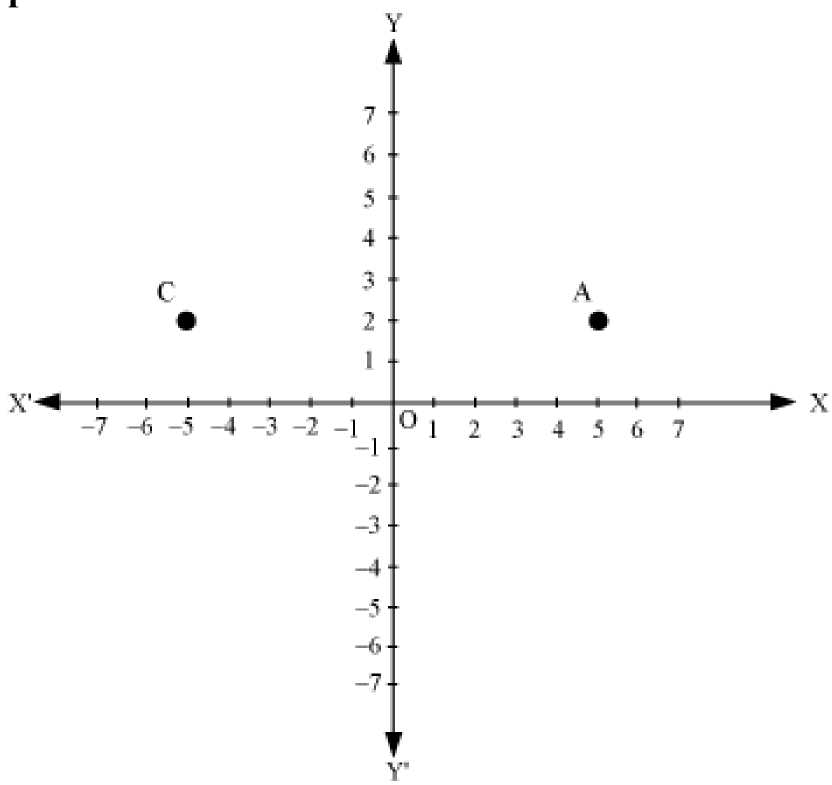
The coordinates of a point on the coordinate plane can be determined by the following conventions.

The x-coordinate of a point is its perpendicular distance from the y-axis, measured along the x-axis (positive along the positive x-axis and negative along the negative x-axis).

The x-coordinate is also called the abscissa.
The y-coordinate of a point is its perpendicular distance from the x-axis, measured along the y-axis (positive along the positive y-axis and negative along the negative y -axis)
The y-coordinate is also called the ordinate.

In stating the coordinates of a point in the coordinate plane, the x-coordinate comes first and then the y-coordinate. The coordinates are placed in brackets.

Example:



What are the coordinates of points A, B and C in the given figure?

Solution:

It is observed that
x-coordinate of point A is 5
y-coordinate of point A is 2
Coordinates of point A are (5, 2).
x-coordinate of point C is –5
y-coordinate of point C is 2
Coordinates of point C are (–5, 2).

Note: The coordinates of the origin are (0, 0). Since the origin has zero distance from both the axes, its abscissa and ordinate are both zero.

- **Relationship between the signs of the coordinates of a point and the quadrant of the point in which it lies:**

The 1st quadrant is enclosed by the positive *x*-axis and positive *y*-axis. So, a point in the 1st quadrant is in the form (+, +).The 2nd quadrant is enclosed by the negative *x*-axis and positive *y*-axis. So, a point in the 2nd quadrant is in the form (–, +).The 3rd quadrant is enclosed by the negative *x*-axis and the negative *y*-axis. So, the point in the 3rd quadrant is in the form (–, –).

The 4th quadrant is enclosed by the positive *x*-axis and the negative *y*-axis. So, the point in the 4th quadrant is in the form (+, –).

- **Location of a point in the plane when its coordinates are given**

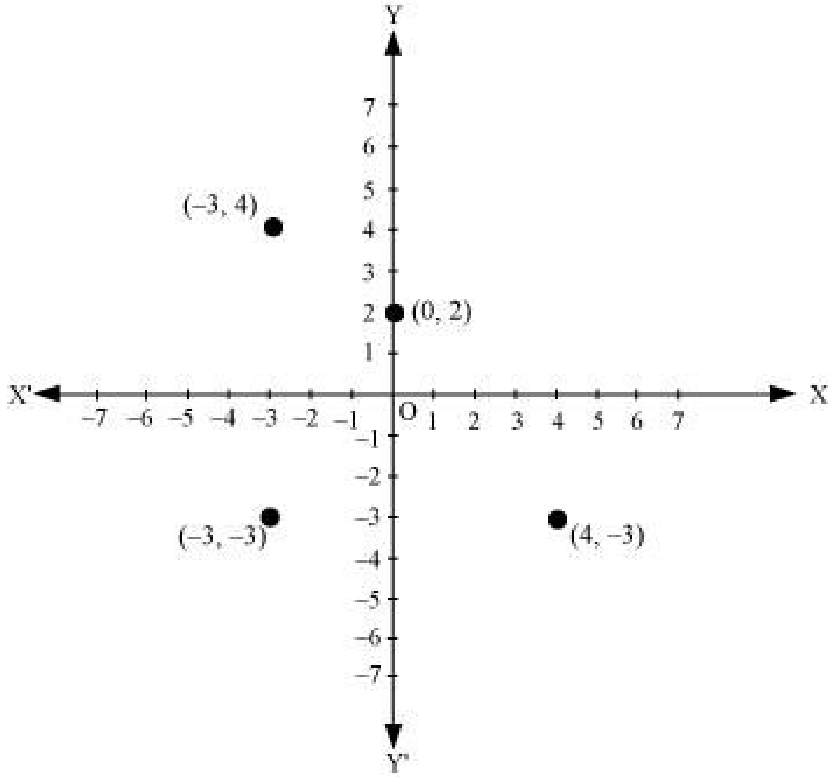
Example: Plot the following ordered pairs of numbers (*x*, *y*) as points in the coordinate plane.
[Use the scale 1 cm = 1 unit]

x	–3	4	–3	0
y	4	–3	–3	2

Solution:

x	–3	4	–3	0
y	4	–3	–3	2

Taking 1 cm = 1 unit, we draw the *x*-axis and *y*-axis.
The pairs of numbers in the given table can be represented as (–3, 4), (4, –3) and (–3, –3), (0, 2).
These points can be located in the coordinate plane as:



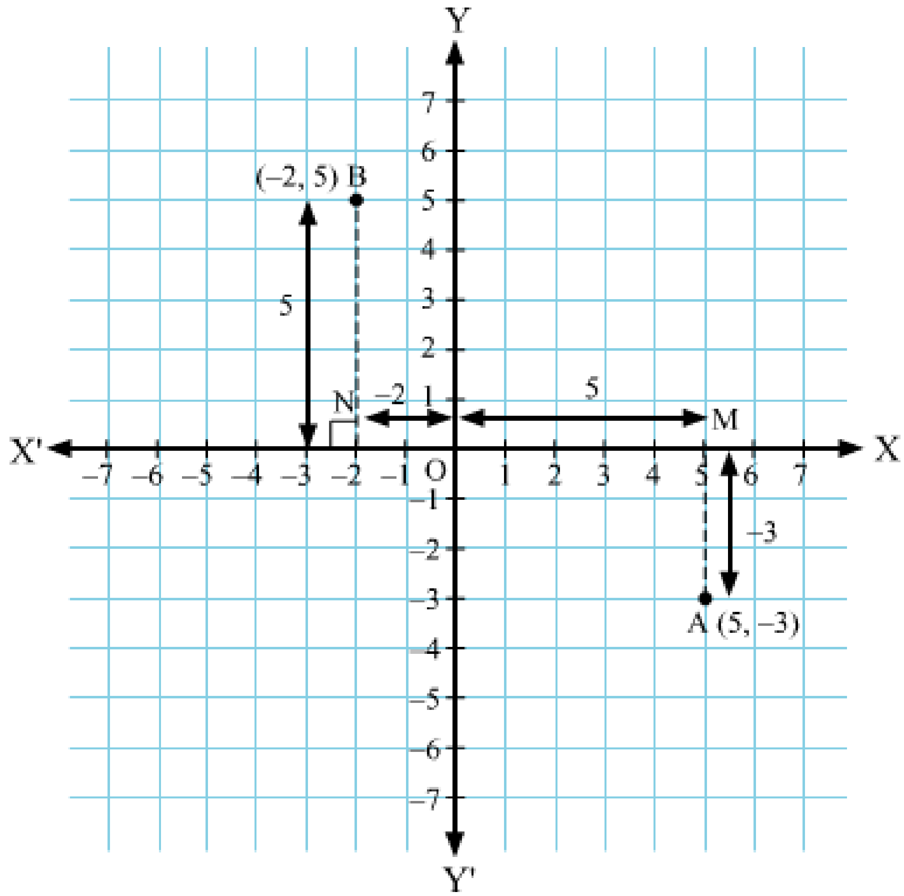
NB: The coordinates of the point on the *x*-axis are of the form (*a*, 0) and the coordinates of the point on the *y*-axis are of the form (0, *b*), where *a*, *b* are real numbers.

- We can plot a point in the Cartesian plane, if the coordinates of the points are given.

Example:
Plot the points A (5, –3) and B (–2, 5) on the Cartesian plane.
Solution:
To plot A (5, –3):
(1) Move 5 units along OX and mark the endpoint as M.
(2) From M and perpendicular to the *x*-axis, move 3 units along OY'. Mark the endpoint as A.
This is the location of the point (5, –3) on the Cartesian plane.

To plot B (–2, 5):
(1) Move 2 units along OX' and mark the endpoint as N.
(2) From N and perpendicular to the *x*-axis, move 5 units along OY. Mark the endpoint as B.
This is the location of the point (–2, 5) on the Cartesian plane.

Points A and B are plotted in the following graph.



- The graph of $x = a$ is a straight line parallel to the y-axis, situated at a distance of a units from y-axis.
- The graph of $y = b$ is a straight line parallel to the x-axis, situated at a distance of b units from x-axis.

Example:

Represent the equation $2y + 5 = 0$, on Cartesian plane.

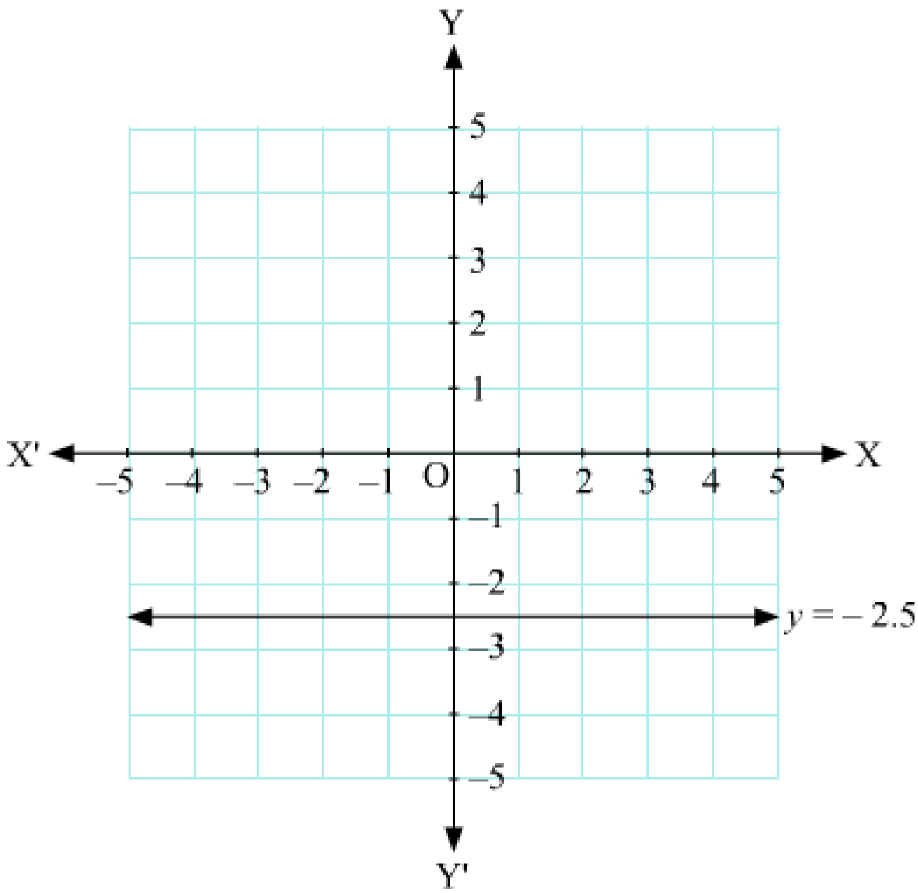
Solution:

$2y + 5 = 0$

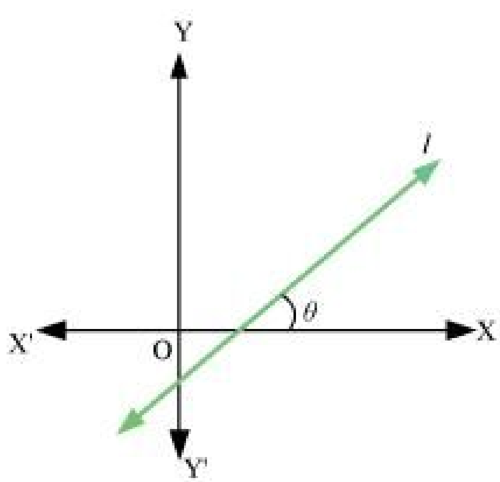
$\Rightarrow 2y = -5$

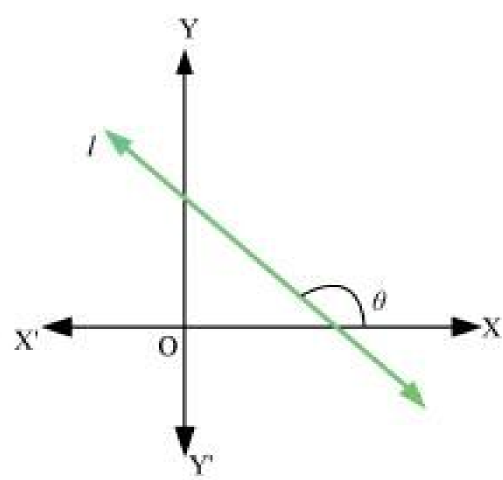
$\Rightarrow y = \frac{-5}{2} = -2.5$, which is of the form $y = b$.

The graph of this equation can be drawn as follows:



- **Slope of a line:** If θ is the inclination of a line *l* (the angle between positive *x*-axis and line *l*), then *m* = tan θ is called the slope or gradient of line *l*.





  - The slope of a line whose inclination is 90º is not defined. Hence, the slope of the vertical line, *y*-axis is undefined.
  - The slope of the horizontal line, *x*-axis is zero.

For example, the slope of a line making an angle of 135° with the positive direction of x-axis is $m = \tan 135° = \tan (180° – 45°) = – \tan 45° = –1$

- **Slope of line passing through two given points:**

  The slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$.

  For example, the slope of the line joining the points (–1, 3) and (4, –2) is given by,
  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 3}{4 - (-1)} = -\frac{5}{5} = -1$

- **Conditions for parallelism and perpendicularity of lines:**

  Suppose l_1 and l_2 are non-vertical lines having slopes m_1 and m_2 respectively.

  - l_1 is parallel to l_2 if and only if $m_1 = m_2$ i.e., their slopes are equal.
  - l_1 is perpendicular to l_2 if and only if $m_1 m_2 = –1$ i.e., the product of their slopes is –1.

Example:
Find the slope of the line which makes an angle of 45° with a line of slope 3.
Solution:
Let m be the slope of the required line.

$$\therefore \tan 45° = \left| \frac{m-3}{1+3m} \right|$$

$$\Rightarrow \left| \frac{m-3}{1+3m} \right| = 1$$

$$\Rightarrow \left| \frac{m-3}{1+3m} \right| = \pm 1$$

$$\Rightarrow \frac{m-3}{1+3m} = 1 \quad \text{or} \quad \frac{m-3}{1+3m} = -1$$

$$\Rightarrow m - 3 = 1 + 3m \quad \text{or} \quad m - 3 = -1 - 3m$$

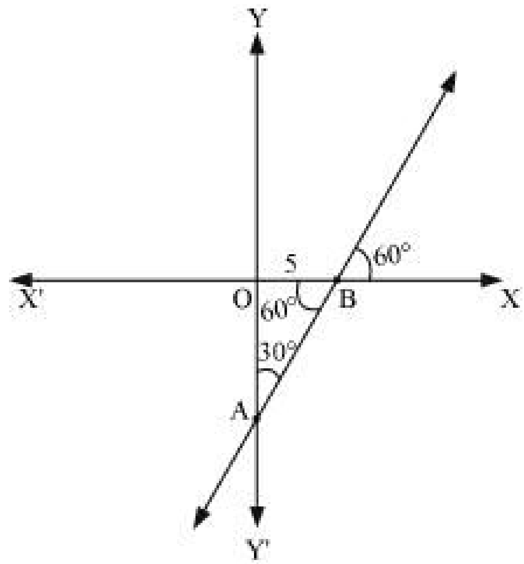
$$\Rightarrow -2m = 4 \quad \text{or} \quad 4m = 2$$

$$\Rightarrow m = -2 \quad \text{or} \quad m = \frac{1}{2}$$

- **Collinearity of three points:** Three points A, B and C are collinear if and only if slope of AB = slope of BC

- **Slope-intercept form of a line**
    - The equation of the line, with slope m, which makes y-intercept c is given by $y = mx + c$.
    - The equation of the line, with slope m, which makes x-intercept d is given by $y = m(x - d)$.

Example:
Find the equation of the line which cuts off an intercept 5 on the x-axis and makes an angle of 30° with the y-axis.
Solution:



Slope of the line, $m = \tan 60° = \sqrt{3}$
OB = 5
Intercept on the x-axis, $c = -\text{OB} = -5$ and $\tan 60° = -5\sqrt{3}$
Equation of the required line is $y = \sqrt{3}x + (-5\sqrt{3})$.

- **General equation of line**

Any equation of the form $Ax + By + C = 0$, where A and B are not zero simultaneously is called the general linear equation or general equation of line.

Slope of the line = $-\dfrac{\text{Coefficient of } x}{\text{Coefficient of } y} = -\dfrac{A}{B}$
y- intercept = $-\dfrac{C}{B}$

Example:
Find the slope and the y-intercept of the line $2x - 3y = -16$.
Solution:
The equation of the given line can be rewritten as $2x - 3y + 16 = 0$.
Here, $A = 2$, $B = -3$ and $C = 16$.
Slope of the line $= -\dfrac{A}{B} = -\dfrac{2}{(-3)} = \dfrac{2}{3}$

Intercept on the y-axis $= -\frac{C}{B} = -\frac{16}{(-3)} = \frac{16}{3}$

- **Graphical solution of linear equation in two variables:**

Every point on the graph of a linear equation in two variables is a solution of the linear equation and moreover, every solution of the linear equation is a point on the graph of the linear equation.

Example:

A bag contains some Re 1 coins and some Rs 2 coins. The total worth of coins is Rs 45. Find the number of Re 1 coins, if there are 10 coins of Rs 2.

Solution:

Let there be x coins of Re 1 and y coins of Rs 2.

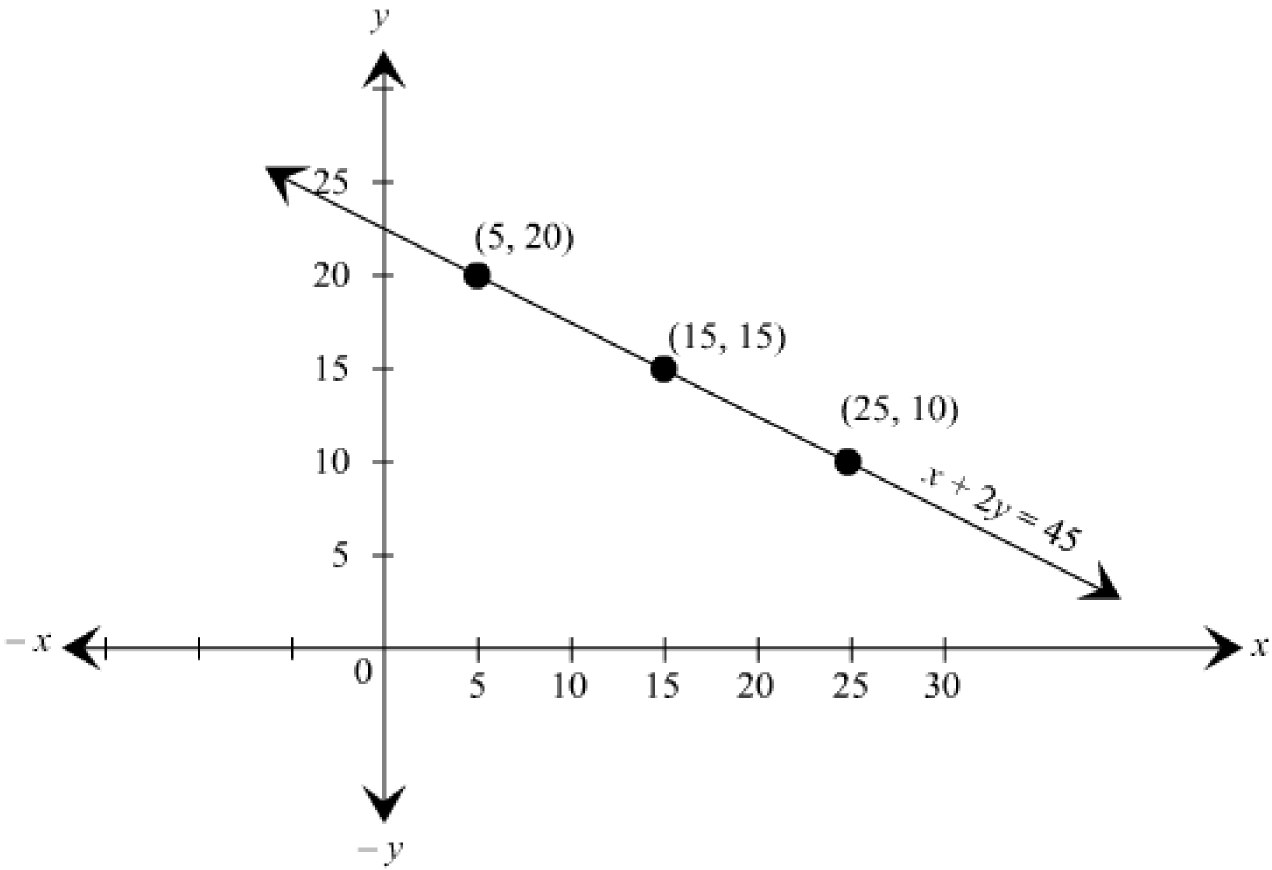
Thus, $1x + 2y = 45$

$\Rightarrow x + 2y = 45$

This is the required linear equation of the given information. The three solutions of this equation have been given in the tabular form as follows:

x	5	15	25
y	20	15	10

By plotting the points (5, 20), (15, 15) and (25, 10), we obtain the following graph.



From the above graph, it can be seen that the value of x corresponding to $y = 10$ is 25.

Therefore, there are 25 coins of Re 1, if there are 10 coins of Rs 2.

- **Solving given pairs of linear equations in two variables graphically:**

  Example:
  Solve the following system of linear equations graphically.
  $x + y + 2 = 0$, $2x - 3y + 9 = 0$
  Hence, find the area bounded by these two lines and the line $x = 0$
  Solution:
  The given equations are
  $x + y + 2 = 0$ (1)
  $2x - 3y + 9 = 0$ (2)

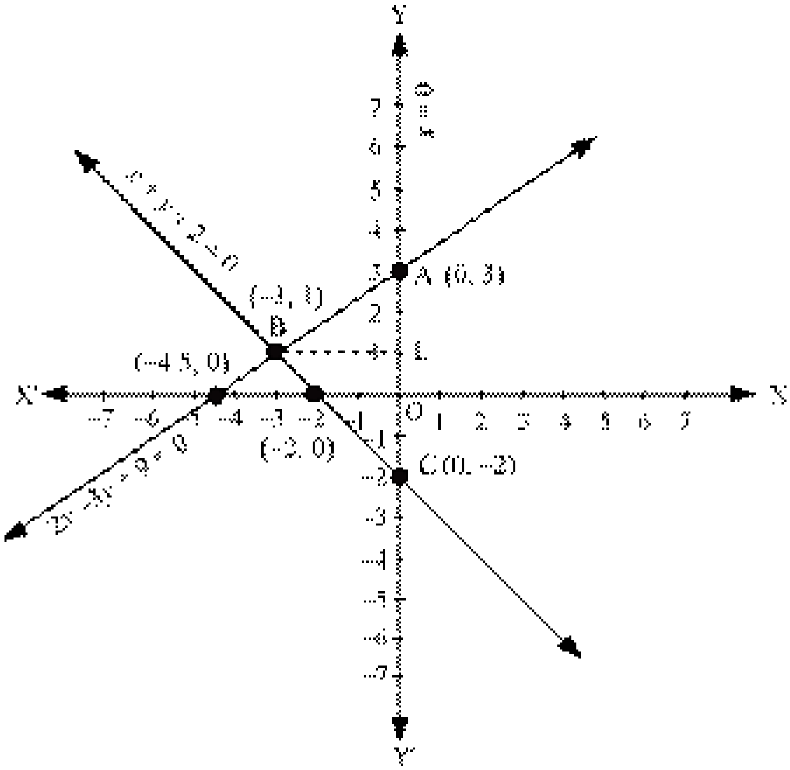


Table for the equations $x + y + 2 = 0$

x	0	–2
y	–2	0

Table for the equation $2x - 3y + 9 = 0$

x	0	–4.5
y	3	0

By plotting and joining the points (0, –2) and (–2, 0), the line representing equation (1) is obtained.
By plotting and joining the points (0, 3) and (–4.5, 0), the line representing equation (2) is obtained.
It is seen that the two lines intersect at point B (–3, 1).
Solution of the given system of equation is (–3, 1)
Area bound by the two lines and $x = 0$

= Area of ΔABC

$$= \frac{1}{2} \times AC \times BL = \frac{1}{2} \times 5 \times 3 \text{ square units} = 7.5 \text{ square units}$$

- **Distance formula**

The distance between the points P (x_1, y_1) and Q (x_2, y_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ PQ=x2-x12+y2-y12

Example 1:

Find the values of *l*, if the distance between the points (–5, 3) and (*l*, 6) is 5 units.

Solution:

The given points are A (–5, 3) and B (*l*, 6).

It is given that AB = 5 units

By distance formula we have

$$\sqrt{\{\lambda - (-5)\}^2 + (6-3)^2} = 5$$ λ--52+6-

$$\Rightarrow (\lambda + 5)^2 + 9 = 25$$
$$\Rightarrow \lambda^2 + 25 + 10\lambda + 9 = 25$$
$$\Rightarrow \lambda^2 + 10\lambda + 9 = 0$$
$$\Rightarrow (\lambda + 9)(\lambda + 1) = 0$$
$$\Rightarrow \lambda = -1, \text{ or } \lambda = -9$$

32=5⇒λ+52+9=25⇒λ2+25+10λ+9=25⇒λ2+10λ+9=0⇒λ+9λ+1=0⇒λ=-1, or λ=-9

Required values of *l* are –1 or –9.

- The distance of a point (x, y) from the origin O (0, 0) is given by OP = $\sqrt{x^2 + y^2}$ OP=x2+y2