

Rational Numbers

- The numbers, which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers. Rational numbers can be positive as well as negative. Rational numbers include all integers and fractions.

For example

$$-\frac{2}{7}, \frac{41}{366}, 2 = \frac{2}{1}, \text{ etc.}$$

All the operations on rational numbers are performed as in fractions.

Example: Solve

1. $-\frac{3}{4} + \frac{5}{6}$

2. $\frac{2}{7} - \frac{3}{5}$

Solution:

1. $-\frac{3}{4} + \frac{5}{6}$
 $= \frac{-3 \times 3 + 5 \times 2}{12} = \frac{-9 + 10}{12} = \frac{1}{12}$

2. $\frac{2}{7} - \frac{3}{5}$

$$= \frac{2 \times 5 - 3 \times 7}{35} = \frac{10 - 21}{35} = -\frac{11}{35}$$

- When 0 is added to any rational number, say $\frac{p}{q}$, the same rational number is obtained. Therefore, 0 is the additive identity of rational numbers.

$$\frac{p}{q} + 0 = \frac{p}{q} = 0 + \frac{p}{q}$$

- $-\frac{p}{q}$ is the additive inverse of the rational number $\frac{p}{q}$.

Example: $-\frac{4}{7}$ is the additive inverse of the rational number $\frac{4}{7}$.

Example: Solve

$$1. \frac{2}{9} \times \left(-\frac{4}{3}\right)$$

$$2. -\frac{3}{7} \div \frac{11}{21}$$

Solution:

$$1. \frac{2}{9} \times \left(-\frac{4}{3}\right)$$

$$= \frac{2 \times (-4)}{9 \times 3} = -\frac{8}{27}$$

$$2. -\frac{3}{7} \div \frac{11}{21}$$

$$= -\frac{3}{7} \times \frac{21}{11} = -\frac{9}{11}$$

- Natural numbers are a collection of all positive numbers starting from 1.
- Whole numbers are a collection of all natural numbers including 0.
- Integers are the set of numbers comprising of all the natural numbers 1, 2, 3 ... and their negatives $-1, -2, -3 \dots$, and the number 0.
- Rational numbers are the numbers that can be written in $\frac{p}{q}$ form, where p and q are integers and $q \neq 0$

- **Closure property**

- Whole numbers are closed under addition and multiplication. However, they are **not** closed under subtraction and division.
- Integers are also closed under addition, subtraction and multiplication. However, they are **not** closed under division.
- Rational numbers:
 1. Rational numbers are closed under addition.

Example: $\frac{2}{5} + \frac{3}{2} = \frac{19}{10}$ is a rational number.

2. Rational numbers are closed under subtraction.

Example: $\frac{1}{5} - \frac{3}{4} = \frac{-11}{20}$ is rational number.

3. Rational numbers are closed under multiplication.

Example: $\frac{2}{3} \times \left(\frac{-3}{5}\right) = \frac{-2}{5}$ is a rational number.

4. Rational numbers are **not** closed under division.

Example: $2 \div 0$ is not defined.

- **Commutativity**

- Whole numbers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
- Integers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
- Rational numbers:
 1. Rational numbers are commutative under addition.

Example:

$$\frac{2}{3} + \left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right) + \left(\frac{2}{3}\right) = \frac{-5}{6}$$

2. Rational numbers are not commutative under subtraction.

Example :

$$\left(\frac{3}{4}\right) - \left(\frac{5}{2}\right) = \left(\frac{-7}{4}\right) \text{ and } \frac{5}{2} - \frac{3}{4} = \frac{7}{4}$$
$$\therefore \left(\frac{3}{4}\right) - \left(\frac{5}{2}\right) \neq \left(\frac{5}{2}\right) - \left(\frac{3}{4}\right)$$

3. Rational numbers are commutative under multiplication.

Example:

$$\left(\frac{3}{4}\right) \times \left(\frac{-2}{6}\right) = \left(\frac{-2}{6}\right) \times \left(\frac{3}{4}\right) = \frac{-1}{4}$$

4. Rational numbers are not commutative under division.

$$2 \div 5 \neq 5 \div 2$$

- **Associativity**

- Whole numbers are associative under addition and multiplication.

However, they are **not** associative under subtraction and division.

- Integers are associative under addition and multiplication. However, they are **not** associative under subtraction and division.

- Rational numbers:

- 1. Rational numbers are associative under addition.

Example:

$$\left(\frac{2}{3} + \frac{1}{3}\right) + 1 = \frac{2}{3} + \left(\frac{1}{3} + 1\right) = 2$$

- 2. Rational numbers are **not** associative under subtraction.

Example:

$$\begin{aligned}\left(\frac{2}{3} - \frac{1}{3}\right) - 1 &= \frac{-2}{3} \\ \frac{2}{3} - \left(\frac{1}{3} - 1\right) &= \frac{4}{3} \\ \therefore \left(\frac{2}{3} - \frac{1}{3}\right) - 1 &\neq \frac{2}{3} - \left(\frac{1}{3} - 1\right)\end{aligned}$$

- 3. Rational numbers are associative under multiplication.

Example:

$$\left(\frac{2}{3} \times \frac{1}{3}\right) \times 1 = \frac{2}{3} \times \left(\frac{1}{3} \times 1\right) = \frac{2}{9}$$

- 4. Rational numbers are **not** associative under division.

Example:

$$\left\{ \frac{2}{7} \div \left(\frac{-1}{14} \right) \right\} \div \frac{3}{7} = \frac{-28}{3}$$

$$\frac{2}{7} \div \left\{ \left(\frac{-1}{14} \right) \div \frac{3}{7} \right\} = \frac{-12}{7}$$

$$\therefore \frac{2}{7} \div \left\{ \left(\frac{-1}{14} \right) \right\} \div \frac{3}{7} \neq \frac{2}{7} \div \left\{ \left(\frac{-1}{14} \right) \div \frac{3}{7} \right\}$$

- 0 is the additive identity of whole numbers, integers, and rational numbers.

$$\therefore 0 + a = a + 0 = a, \text{ where } a \text{ is a rational number}$$

- 1 is the multiplicative identity of whole numbers, integers, and rational numbers.

$$a \times 1 = 1 \times a = a$$

- Additive inverse of a number is the number, which when added to a number, gives 0. It is also called the negative of a number.

$$a + (-a) = (-a) + a = 0$$

$$\therefore \text{Additive inverse of } \frac{2}{5} \text{ is } \left(\frac{-2}{5} \right)$$

- Reciprocal or multiplicative inverse of a number is the number, which when multiplied by the number, gives 1. Therefore, the reciprocal of a is $\frac{1}{a}$

$$\left[a \times \frac{1}{a} = 1 \right]$$

$$\therefore \text{Reciprocal of } \frac{-2}{3} \text{ is } \frac{-3}{2}$$

- Rational numbers are distributive over addition and subtraction.

$$\text{i.e., for any rational numbers } a, b, \text{ and } c, a(b + c) = ab + ac, a(b - c) = ab - ac$$

- **Rational numbers on number line**

Rational numbers can be represented on number line in the similar manner like fractions and integers.

Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

Example:

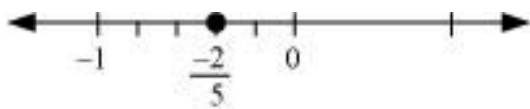
Represent $-\frac{2}{5}$ on number line.

Solution:

The given rational number is negative. Therefore, it will lie to the left of 0.

The space between -1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$.

Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.



- To find rational numbers between any two given rational numbers, firstly we have to make their denominators same and then find the respective rational numbers.

Example:

Find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$.

Solution:

The L.C.M. of 6 and 8 is 24.

Now, we can write

$$\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$$

$$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

Therefore, some of the rational numbers between $\frac{4}{24} \left(\frac{1}{6} \right)$ and $\frac{21}{24} \left(\frac{7}{8} \right)$ are

$$\frac{5}{24}, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}, \frac{9}{24}, \frac{10}{24}, \frac{11}{24}, \frac{12}{24}, \frac{13}{24}, \frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{17}{24}, \frac{18}{24}, \frac{19}{24}, \frac{20}{24}$$