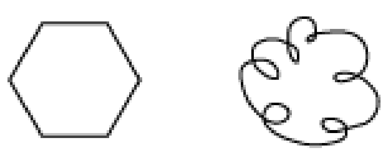
Understanding Shapes

Curve:

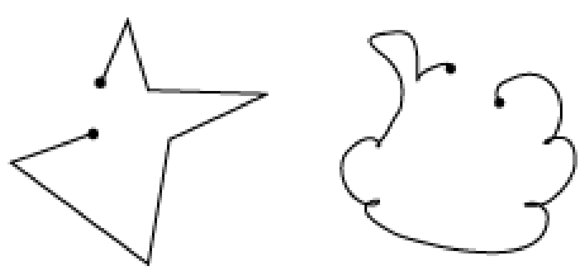
Any drawing (straight or non-straight) done without lifting the pencil is called a **curve**. Line is also a curve.

The curve which does not intersect itself is called a **simple curve**.

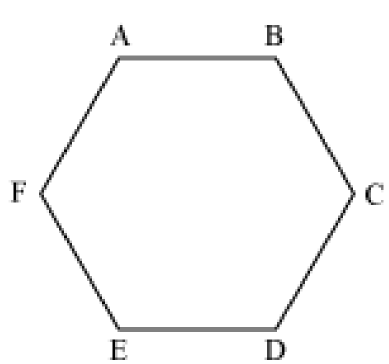
2. A curve is said to be **closed**, if it has no starting or ending point.



3. A curve is said to be **open**, if its end points are not joined.

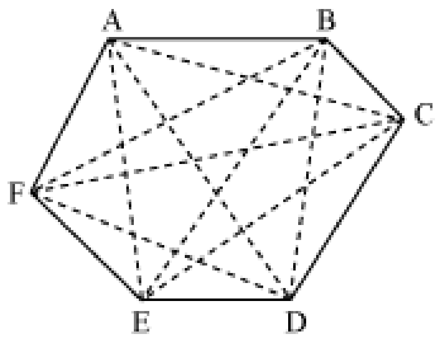


- A **polygon** is a simple closed curve made up of line segments. ABCDEF is a polygon.



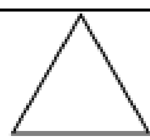
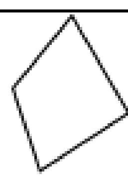
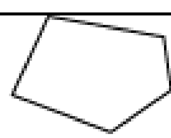
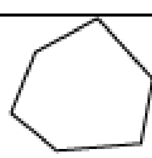
The attributes with respect to polygon ABCDEF are:

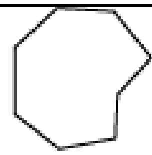
1. The line segments AB, BC, CD, DE, EF, and FA are known as the **sides of the polygon** ABCDEF.
2. Any two sides with common end points are called **adjacent sides**. AB and BC are adjacent sides with common end point B.
3. The meeting point of a pair of sides of a polygon is known as **vertex**. In the polygon ABCDEF, sides AB and BC meets at point B. So, point B is called the vertex of the polygon. Similarly, the other vertices are A, C, D, E, and F.
4. The line joining any two non-adjacent vertices of a polygon is known as its **diagonal**.



In the polygon ABCDEF, the diagonals are AC, AD, AE, BD, BE, BF, CE, CF, and DF.

- A polygon's name is based on the number of its sides.

Number of sides	**Figure**	**Name**
3		Triangle
4		Quadrilateral
5		Pentagon
6		Hexagon

8		Octagon

- **Polygons**
  - A simple closed curve made up of line segments only is called a **polygon.**
  - **Polygons can be classified according to their number of sides (or vertices).**

Number of side/vertices	Classification
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
.	.
.	.
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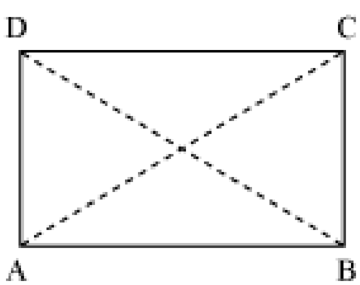
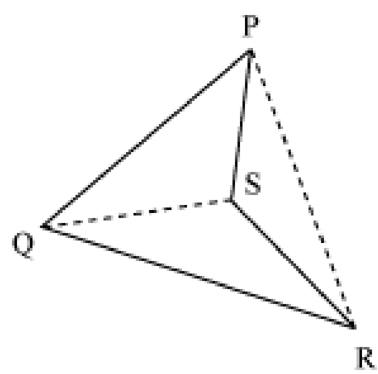
- The line segment connecting two non-consecutive vertices of a polygon are called **diagonals.**





For polygon ABCD, AC and BD are diagonals and for polygon PQRS, QS and PR are diagonals.

- The polygon, none of whose diagonals lie in its exterior, is called a **convex polygon.** In the given figure, ABCD is a convex polygon.

The polygon whose atleast one of the diagonals lie in its exterior is called a **concave polygon.** PQRS is a concave polygon.

- The sum of all the interior angles of an *n*-sided polygon is given by, $(n - 2) \times 180°$.

  Example: What is the number of sides of a polygon whose sum of all interior angles is 720°?

  Solution: It is known that,

  $$(n-2)180° = 720°$$

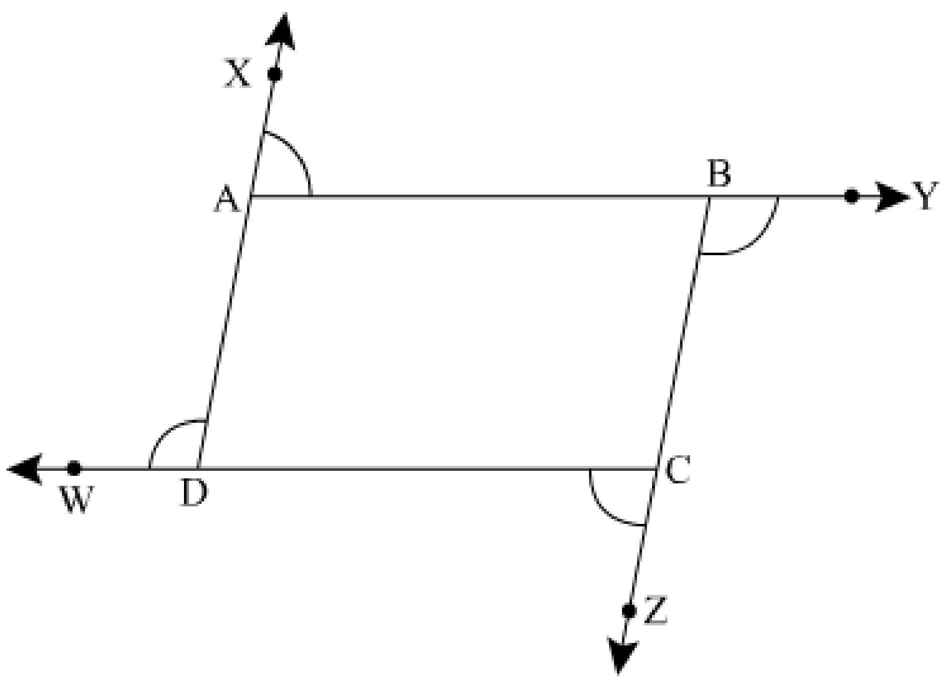
  $$\Rightarrow (n-2) = \frac{720°}{180°} = 4$$

  $$\Rightarrow n = 6$$

  Thus, the required polygon is six-sided.

- The sum of measures of all exterior angles of a polygon is 360°.

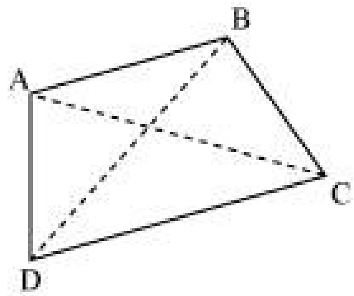
  For example, in the quadrilateral given below,
  $\angle XAB + \angle YBC + \angle ZCD + \angle WDA = 360°$



- A polygon, which is both equiangular and equilateral, is called a **regular polygon.** Otherwise, it is an **irregular polygon.**

  Example: Square is a regular polygon but rectangle is an irregular polygon.

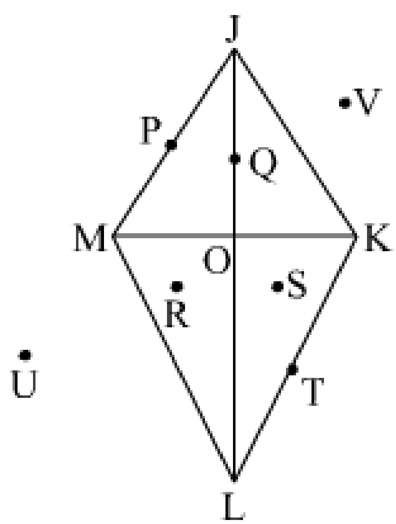
- **Quadrilateral:** A quadrilateral is a four-sided polygon.



For the given quadrilateral ABCD:

  - Possible names of quadrilateral are □ABCD, □BCDA, □CDAB and □ DABC.
  - AB, CD and BC, DA are the pairs of **opposite sides**.
  - AB, BC; BC, CD; CD, DA and DA, AB are the pairs of **adjacent sides**.
  - A, C and B, D are the pairs of **opposite vertices**.
  - AC and BD are the **diagonals** of quadrilateral ABCD.
  - ∠A, ∠C and ∠B, ∠D are pairs of **opposite angles**.
  - ∠B, ∠C; ∠A, ∠B; ∠C, ∠D and ∠D, ∠A are the pairs of **adjacent angles**.

For the given quadrilateral JKLM:



  - Points lying in the **interior** of the quadrilateral are Q, R, S and O.
  - Points lying in the **exterior** of the quadrilateral are V and U.
  - Points lying on the **boundary** of the quadrilateral are P and T.
  - The interior and boundary together form the **region** of the quadrilateral.

1. **Angle sum property of a quadrilateral** states that the sum of measure of the four angles of a quadrilateral is 360°.

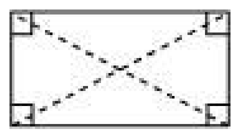
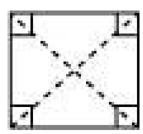
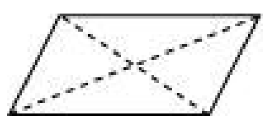
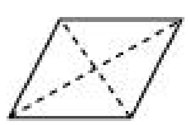
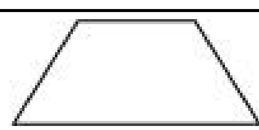
For example, in a □ABCD, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

2. The angle sum property can be verified:

(i) by measuring the angles of a quadrilateral

(ii) by dividing a quadrilateral into two triangles

- Quadrilaterals are classified according to their properties.

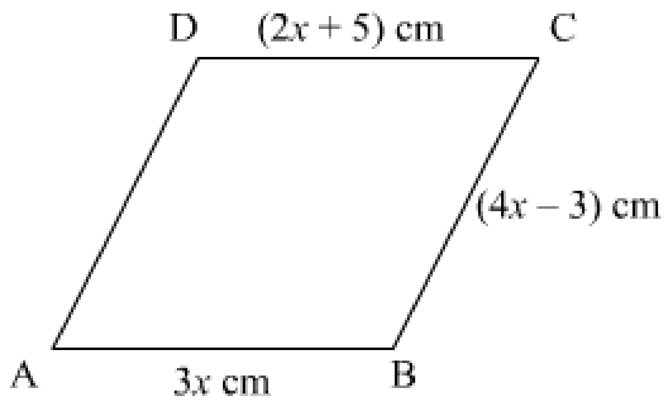
Name of the quadrilateral	Figure	Properties
Rectangle		1. Opposite sides are equal. 2. Each angle is 90°. 3. Diagonals are equal. 4. Opposite sides are parallel.
Square		1. All sides are equal. 2. Each angle is 90°. 3. Diagonals are equal. 4. Opposite sides are parallel.
Parallelogram		1. Opposite sides are parallel. 2. Opposite sides are equal. 3. Diagonals are not equal.
Rhombus		1. Opposite sides are parallel. 2. All sides are equal. 3. Diagonals may or may not be equal.
Trapezium		1. One pair of opposite sides is parallel.

- A parallelogram is a rhombus if all sides are equal.
- A parallelogram is a rectangle if all angles are 90°.

- A parallelogram is a square if all sides are equal and all angles are 90°.
- A rhombus is a square if all angles are 90°.
- A Rectangle is a square if all sides are equal.

- Opposite sides in a parallelogram are equal. Conversely, in a quadrilateral, if each pair of opposite sides are equal then the quadrilateral is a parallelogram.

Example:
In the following figure, ABCD is a parallelogram. Find the length of each sides.



Solution:

We know, the opposite sides of a parallelogram are equal in length.

Therefore, AB = CD

$3x = 2x + 5$

$\Rightarrow 3x - 2x = 5$

$\therefore x = 5$

Thus, AB = $3x = 3 \times 5 = 15$ cm

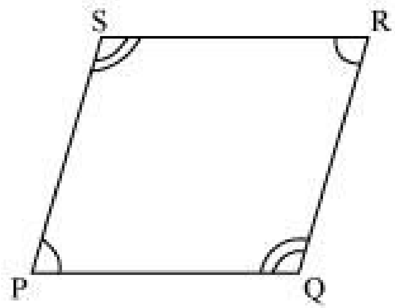
BC = $4x - 3 = 4 \times 5 - 3 = 17$ cm

CD = $2x + 5 = 2 \times 5 + 5 = 15$ cm

Also, BC = AD [opposite sides of parallelogram]

$\therefore$ AD = 17 cm

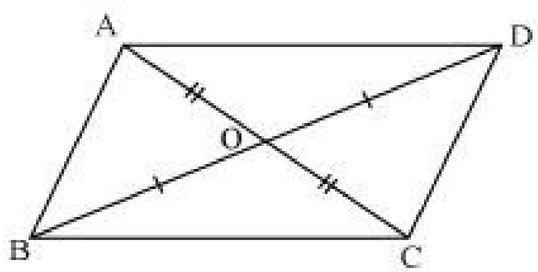
- In a parallelogram, opposite angles are equal. Conversely in a quadrilateral, if pair of opposite angles is equal, then the quadrilateral is a parallelogram.



If in the quadrilateral PQRS, ∠P = ∠R and ∠Q = ∠S as shown in the above figure, then the quadrilateral is a parallelogram.

- The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

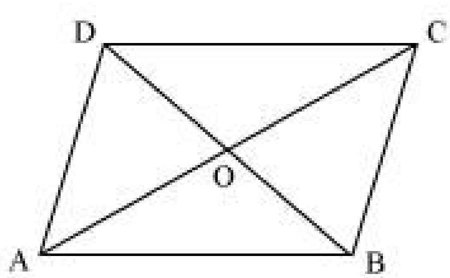
  Suppose ABCD is a quadrilateral. The diagonals of the quadrilateral intersect at O such that AO = OC and DO = OB



Therefore, ABCD is a parallelogram.

Example:

In the given figure, ABCD is a parallelogram. If OD = $(3x - 2)$ cm and OB = $(2x + 3)$ cm, then find x and length of diagonal BD.



Solution:

We know that the diagonals of a parallelogram bisect each other.

$\therefore$ OD = OB

$\Rightarrow 3x - 2 = 2x + 3$

$\Rightarrow 3x - 2x = 3 + 2$

$\Rightarrow x = 5$

Thus, the value of x is 5.

Length of BD = OD + OB

$= (3x - 2) + (2x + 3)$

$= (3\times5 - 2) + (2\times5 + 3)$

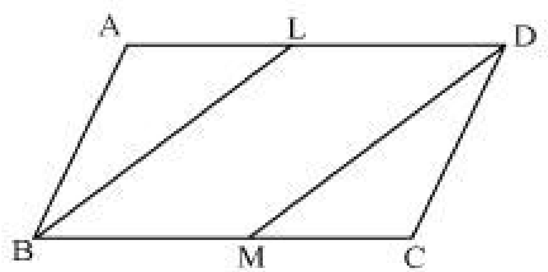
$= 13 + 13$

$= 26$ cm

- A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Example:

In the given figure, ABCD is a parallelogram and L and M are the mid-points of AD and BC respectively. Prove that BMDL is a parallelogram.



Solution:

As L and M are the mid-points of AD and BC respectively.

Therefore, $BM = \frac{1}{2}BC$ and $LD = \frac{1}{2}AD$ … (1)

As BC = AD (Since ABCD is a parallelogram)

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}AD$

$\Rightarrow$ BM = LD … (2) (From (1))

Also, BC || AD

$\Rightarrow$ BM || LD

Hence, BMDL is a parallelogram.