

Ex 3.1

Q1

Function = Let A and B be two non-empty sets. A relation f from A to B , i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B , if

- (i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$
- (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$

If $(a, b) \in f$, then ' b ' is called the image of ' a ' under f

If a function f is expressed as the set of ordered pairs, the domain f is the set of all first components of members of f and the range of f is the set of second components of members of f .

Q2

Function = Let A and B be two non-empty sets. Then a function ' f ' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

- (i) all elements of set A are associated to element in set B .
- (ii) an element of set A is associated to a unique element in set B .

In other words, a function ' f ' from a set A to set B associates each element of set A to a unique element of set B .

Q3

Function is a type of relation. But in a function no two ordered pairs have the same first element. For eg: R_1 and R_2 are two relations.

Clearly, R_1 is a function, but R_2 is not a function because two ordered pairs $(1, 2)$ and $(1, 4)$ have the same first element.

This means every function is a relation but every relation is not a function.

Q4

We have,

$$f(x) = x^2 - 2x - 3$$

Now,

$$\begin{aligned} f(-2) &= (-2)^2 - 2(-2) - 3 \\ &= 4 + 4 - 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^2 - 2(-1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(0) &= (0)^2 - 2 \times 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(1) &= (1)^2 - 2 \times 1 - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 2 \times 2 - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

$$(a) \text{ Rang}(f) = \{-4, -3, 0, 5\}$$

(b) Clearly, pre-images of 6, -3 and 5 is $\emptyset$, $\{0, 2\}$, -2 respectively.

Q5

We have,

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

Now,

$$\begin{aligned} f(1) &= 4 \times 1 + 1 = 5, \\ f(-1) &= 3 \times (-1) - 2 = -3 - 2 = -5, \\ f(0) &= 1, \end{aligned}$$

$$\text{and, } f(2) = 4 \times 2 + 1 = 9$$

$$\therefore \begin{aligned} f(1) &= 5, & f(-1) &= -5, \\ f(0) &= 1, & f(2) &= 9, \end{aligned}$$

Q6

We have,

$$f(x) = x^2 \quad \text{--- (i)}$$

(a) clearly range of $f = \mathbb{R}^+$ (set of all real numbers greater than or equal to zero)

(b) we have,

$$\begin{aligned} & \{x : f(x) = 4\} \\ \Rightarrow & f(x) = 4 \quad \text{--- (ii)} \end{aligned}$$

Using equation (i) and equation (ii), we get

$$\begin{aligned} & x^2 = 4 \\ \Rightarrow & x = \pm 2 \\ \therefore & \{x : f(x) = 4\} = \{-2, 2\} \end{aligned}$$

$$\begin{aligned} \text{(c)} & \{y : f(y) = -1\} \\ \Rightarrow & f(y) = -1 \quad \text{--- (iii)} \end{aligned}$$

Clearly, $x^2 \neq -1$ or $x^2 \geq 0$

$$\Rightarrow f(y) \neq -1$$

$$\therefore \{y : f(y) = -1\} = \emptyset$$

Q7

We have,

$$f: R^+ \rightarrow R$$

$$\text{and } f(x) = \log_e x \quad \text{--- (i)}$$

(a) Now,

$$f: R^+ \rightarrow R$$

$\therefore$ the image set of the domain of $f = R$

(b) Now,

$$\{x : f(x) = -2\}$$

$$\Rightarrow f(x) = -2 \quad \text{--- (ii)}$$

Using equation (i) and equation (ii), we get

$$\log_e x = -2$$

$$\Rightarrow x = e^{-2}$$

$$[\because \log_a b = c \Rightarrow b = a^c]$$

$$\therefore \{x : f(x) = -2\} = \{e^{-2}\}$$

(c) Now,

$$f(xy) = \log_e(xy)$$

$$= \log_e x + \log_e y$$

$$f(x) + f(y)$$

$$\therefore f(xy) = f(x) + f(y)$$

$$[f(x) = \log_e x]$$

$$[\because \log mn = \log m + \log n]$$

$$\text{Yes, } f(xy) = f(x) + f(y).$$

Q8

(a) we have,

$$\{(x, y) = y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$$

Putting $x = 1, 2, 3$ in $y = 3x$, we get

$$y = 3, 6, 9 \text{ respectively}$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9)\}$$

Yes, it is a function.

(b) we have,

$$\{(x, y) : y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$$

Putting $x = 1, 2$ in $y > x + 1$, we get

$$y > 2, y > 3 \text{ respectively.}$$

$$\therefore R = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$$

It is not a function from A to B because two ordered pairs in R have the same first element.

(c) we have,

$$\{(x, y) = x + y = 3, x, y \in \{0, 1, 2, 3\}\}$$

Now,

$$y = 3 - x$$

Putting $x = 0, 1, 2, 3$, we get

$$y = 3, 2, 1, 0 \text{ respectively}$$

$$\therefore R = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$$

Yes, this relation is a function.

Q9

We have,

$$f : R \rightarrow R \text{ and } g : C \rightarrow C$$

$$\therefore \text{Domain } (f) = R \text{ and Domain } (g) = C$$

$$\therefore \text{Domain } (f) \neq \text{Domain } (g) = C$$

$$\therefore f(x) \text{ and } g(x) \text{ are not equal functions.}$$

Q10

(i) We have,

$$f(x) = x^2$$

Range of $f(x) = \mathbb{R}^+$ (set of all real numbers greater than or equal to zero)
 $= \{x \in \mathbb{R} \mid x \geq 0\}$

(ii) We have,

$$g(x) = \sin x$$

Range of $g(x) = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$

(iii) We have,

$$h(x) = x^2 + 1$$

Range of $h(x) = \{x \in \mathbb{R} : x \geq 1\}$

Q11

(a) We have,

$$f_1 = \{(1, 1), (2, 11), (3, 1), (4, 15)\}$$

f_1 is a function from X to Y .

(b) We have,

$$f_2 = \{(1, 1), (2, 7), (3, 5)\}$$

f_2 is not a function from X to Y because there is an element $4 \in X$ which is not associated to any element of Y .

(c) We have,

$$f_3 = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$$

f_3 is not a function from X to Y because an element $2 \in X$ is associated to two elements 9 and 11 in Y .

Q12

We have,

$$f(x) = \text{highest prime factor of } x.$$

$$\begin{aligned}\therefore \quad 12 &= 3 \times 4, \\ 13 &= 13 \times 1, \\ 14 &= 7 \times 2, \\ 15 &= 5 \times 3, \\ 16 &= 2 \times 8, \\ 17 &= 17 \times 1.\end{aligned}$$

$$\therefore \quad f = \{(12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$$

$$\therefore \quad \text{Range}(f) = \{3, 13, 7, 5, 2, 17\}$$

Q13

We know that,

$$\text{if } f: A \rightarrow B$$

such that $y \in B$. Then,

$$f^{-1}(y) = \{x \in A : f(x) = y\}. \text{ In other words, } f^{-1}(y) \text{ is the set of pre-images of } y.$$

$$\text{Let } f^{-1}\{17\} = x. \text{ Then, } f(x) = 17$$

$$\Rightarrow x^2 + 1 = 17$$

$$\Rightarrow x^2 = 17 - 1 = 16$$

$$\Rightarrow x = \pm 4$$

$$\text{Let } f^{-1}\{-3\} = x. \text{ Then, } f(x) = -3$$

$$\Rightarrow x^2 + 1 = -3$$

$$\Rightarrow x^2 = -3 - 1 = -4$$

$$\Rightarrow x = \sqrt{-4}$$

$$\therefore \quad f^{-1}\{-3\} = \emptyset$$

Q14

We have,

$$A = \{p, q, r, s\} \text{ and } B = \{1, 2, 3\}$$

(a) Now,

$$R_1 = \{(p, 1), (q, 2), (r, 1), (s, 2)\}$$

R_1 is a function

(b) Now,

$$R_2 = \{(p, 1), (q, 1), (r, 1), (s, 1)\}$$

R_2 is a function

(c) Now,

$$R_3 = \{(p, 1), (q, 2), (p, 2), (s, 3)\}$$

R_3 is not a function because an element $p \in A$ is associated to two elements 1 and 2 in B .

(d) Now,

$$R_4 = \{(p, 2), (q, 3), (r, 2), (s, 2)\}$$

R_4 is a function.

Q15

We have,

$f(n)$ = the highest prime factor of n .

Now,

$$9 = 3 \times 3,$$

$$10 = 5 \times 2,$$

$$11 = 11 \times 1,$$

$$12 = 3 \times 4,$$

$$13 = 13 \times 1$$

$$\therefore f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13)\}$$

Clearly, $\text{range}(f) = \{3, 5, 11, 13\}$

Q16

We have,

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

$$\text{and, } g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

$$\text{Now, } f(3) = (3)^2 = 9 \text{ and } f(3) = 3 \times 3 = 9$$

$$\text{and, } g(2) = (2)^2 = 4 \text{ and } g(2) = 3 \times 2 = 6$$

We observe that $f(x)$ takes unique value at each point in its domain $[0, 10]$. However $g(x)$ does not takes unique value at each point in its domain $[0, 10]$.

Hence, $g(x)$ is not a function.

Q17

$$\text{Given } f(x) = x^2$$

$$f(1.1) = 1.21$$

$$f(1) = 1$$

$$\begin{aligned} \frac{f(1.1) - f(1)}{(1.1) - 1} &= \frac{1.21 - 1}{1.1 - 1} \\ &= \frac{0.21}{0.1} \\ &= 2.1 \end{aligned}$$

Q18

$$f: X \rightarrow \mathbb{R} \text{ given by } f(x) = x^3 + 1$$

$$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

$$f(0) = (0)^3 + 1 = 0 + 1 = 1$$

$$f(3) = (3)^3 + 1 = 27 + 1 = 28$$

$$f(9) = (9)^3 + 1 = 81 + 1 = 82$$

$$f(7) = (7)^3 + 1 = 343 + 1 = 344$$

Set of ordered pairs are $\{(-1, 0), (0, 1), (3, 28), (9, 82), (7, 344)\}$

Ex 3.2

Q1

We have,

$$f(x) = x^2 - 3x + 4$$

Now,

$$\begin{aligned} f(2x+1) &= (2x+1)^2 - 3(2x+1) + 4 \\ &= 4x^2 + 1 + 4x - 6x - 3 + 4 \\ &= 4x^2 - 2x + 2 \end{aligned}$$

It is given that

$$f(x) = f(2x+1)$$

$$\begin{aligned} \Rightarrow x^2 - 3x + 4 &= 4x^2 - 2x + 2 \\ \Rightarrow 0 &= 4x^2 - x^2 - 2x + 3x + 2 - 4 \\ \Rightarrow 3x^2 + x - 2 &= 0 \\ \Rightarrow 3x^2 + 3x - 2x - 2 &= 0 \\ \Rightarrow 3x(x+1) - 2(x+1) &= 0 \\ \Rightarrow (x+1)(3x-2) &= 0 \\ \Rightarrow x+1 = 0 \quad \text{or} \quad 3x-2 &= 0 \\ \Rightarrow x = -1 \quad \text{or} \quad x = \frac{2}{3} \end{aligned}$$

Q2

We have,

$$f(x) = (x-a)^2(x-b)^2$$

Now,

$$\begin{aligned} f(a+b) &= (a+b-a)^2(a+b-b)^2 \\ &= b^2a^2 \end{aligned}$$

$$\Rightarrow f(a+b) = a^2b^2$$

Q3

We have,

$$y = f(x) = \frac{ax - b}{bx - a}$$

$$\Rightarrow y = \frac{ax - b}{bx - a}$$

$$\Rightarrow y(bx - a) = ax - b$$

$$\Rightarrow xyb - ay = ax - b$$

$$\Rightarrow xyb - ax = ay - b$$

$$\Rightarrow x(by - a) = ay - b$$

$$\Rightarrow x = \frac{ay - b}{by - a}$$

$$\Rightarrow x = f(y)$$

Hence, proved

Q4

We have,

$$f(x) = \frac{1}{1-x}$$

Now,

$$\begin{aligned} f\{f(x)\} &= f\left\{\frac{1}{1-x}\right\} \\ &= \frac{1}{1 - \frac{1}{1-x}} \\ &= \frac{1}{\frac{1-x-1}{1-x}} \\ &= \frac{1-x}{-x} \\ &= \frac{x-1}{x} \end{aligned}$$

$$\begin{aligned} \therefore f[f(x)] &= f\left\{\frac{x-1}{x}\right\} \\ &= \frac{1}{1 - \left(\frac{x-1}{x}\right)} \\ &= \frac{1}{\frac{x-x+1}{x}} \\ &= \frac{x}{1} \\ &= x \end{aligned}$$

$$\therefore f[f(x)] = x \text{ Hence, proved.}$$

Q5

We have,

$$f(x) = \frac{x+1}{x-1}$$

Now,

$$\begin{aligned} f[f(x)] &= f\left(\frac{x+1}{x-1}\right) \\ &= \frac{\left(\frac{x+1}{x-1}\right) + 1}{\left(\frac{x+1}{x-1}\right) - 1} \\ &= \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-1(x-1)}{x-1}} \\ &= \frac{2x}{x+1-x+1} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

∴ $f[f(x)] = x$ Hence, proved.

Q6

We have,

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

(a) $f(1/2) = \frac{1}{2}$

(b) $f(-2) = (-2)^2 = 4$

(c) $f(1) = \frac{1}{1} = 1$

(d) $f(\sqrt{3}) = \frac{1}{\sqrt{3}}$

(e) $f(\sqrt{-3})$ does not exist because $\sqrt{-3} \notin \text{domain}(f)$.

Q7

We have,

$$f(x) = x^3 - \frac{1}{x^3} \quad \text{---(i)}$$

Now,

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$

$$= \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} f(x) + f\left(\frac{1}{x}\right) &= \left(x^3 - \frac{1}{x^3}\right) + \left(\frac{1}{x^3} - x^3\right) \\ &= x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 \\ &= 0 \end{aligned}$$

$$\therefore f(x) + f\left(\frac{1}{x}\right) = 0 \quad \text{Hence, proved.}$$

Q8

We have,

$$f(x) = \frac{2x}{1+x^2}$$

Now,

$$\begin{aligned} f(\tan \theta) &= \frac{2(\tan \theta)}{1 + \tan^2 \theta} \\ &= \sin 2\theta \end{aligned}$$

$$\left[\because \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$$\therefore f(\tan \theta) = \sin 2\theta \quad \text{Hence, proved.}$$

Q9

$$i. \quad = \frac{x-1}{x+1}$$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x} = -f(x)$$

$$ii. f(x) = \frac{x-1}{x+1}$$

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{1}{\frac{1+x}{x-1}} = -\frac{1}{f(x)}$$

Q10

We have,

$$f(x) = (a - x^n)^{1/n}, \quad a > 0$$

Now,

$$\begin{aligned} f(f(x)) &= f(a - x^n)^{1/n} \\ &= \left[a - \left\{ (a - x^n)^{1/n} \right\}^n \right]^{1/n} \\ &= \left[a - (a - x^n) \right]^{1/n} \\ &= \left[a - a + x^n \right]^{1/n} \\ &= (x^n)^{1/n} \\ &= (x)^{n \times \frac{1}{n}} \\ &= x \end{aligned}$$

$\therefore f(f(x)) = x$ Hence, proved.

Q11

We have,

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \quad \text{--- (i)}$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = \frac{1}{\frac{1}{x}} - 5$$

$$= x - 5$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = x - 5 \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$af(x) + bf(x) + bf\left(\frac{1}{x}\right) + af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 + x - 5$$

$$\Rightarrow \quad (a+b)f(x) + f\left(\frac{1}{x}\right)(a+b) = \frac{1}{x} + x - 10$$

$$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \frac{1}{a+b} \left[\frac{1}{x} + x - 10 \right] \quad \text{--- (iii)}$$

Subtracting equation (ii) from equation (i), we get

$$af(x) - bf(x) + bf\left(\frac{1}{x}\right) - af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 - x + 5$$

$$\Rightarrow \quad (a-b)f(x) - f\left(\frac{1}{x}\right)(a-b) = \frac{1}{x} - x$$

$$\Rightarrow \quad f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a-b} \left[\frac{1}{x} - x \right]$$

Adding equations (iii) and (iv), we get

$$\begin{aligned}
 2f(x) &= \frac{1}{a+b} \left[\frac{1}{x} + x - 10 \right] + \frac{1}{a-b} \left[\frac{1}{x} - x \right] \\
 \Rightarrow 2f(x) &= \frac{(a-b) \left[\frac{1}{x} + x - 10 \right] + (a+b) \left[\frac{1}{x} - x \right]}{(a+b)(a-b)} \\
 \Rightarrow 2f(x) &= \frac{\frac{a}{x} + ax - 10a - \frac{b}{x} - bx + 10b + \frac{a}{x} - ax + \frac{b}{x} - bx}{a^2 - b^2} \\
 \Rightarrow 2f(x) &= \frac{\frac{2a}{x} - 10a + 10b - 2bx}{a^2 - b^2} \\
 \Rightarrow f(x) &= \frac{1}{a^2 - b^2} \times \frac{1}{2} \left[\frac{2a}{x} - 10a + 10b - 2bx \right] \\
 &= \frac{1}{a^2 - b^2} \left[\frac{a}{x} - 5a + 5b - bx \right] \\
 \therefore f(x) &= \frac{1}{a^2 - b^2} \left[\frac{a}{x} - bx - 5a + 5b \right] \\
 &= \frac{1}{a^2 - b^2} \left[\frac{a}{x} - bx \right] - \frac{5(a-b)}{a^2 - b^2} \\
 &= \frac{1}{a^2 - b^2} \left[\frac{a}{x} - bx \right] - \frac{5(a-b)}{(a-b)(a+b)} \\
 &= \frac{1}{a^2 - b^2} \left[\frac{a}{x} - bx \right] - \frac{5}{a+b}
 \end{aligned}$$

Ex 3.3

Q1

We have,

$$f(x) = \frac{1}{x}$$

Clearly, $f(x)$ assumes real values for all real values for all x except for the values of $x = 0$

Hence, $\text{Domain}(f) = R - \{0\}$

We have,

$$f(x) = \frac{1}{x-7}$$

Clearly, $f(x)$ assumes real values for all real values for all x except for the values of x satisfying $x-7=0$ i.e., $x=7$

Hence, $\text{Domain}(f) = R - \{7\}$

We have,

$$f(x) = \frac{3x-2}{x+1}$$

We observe that $f(x)$ is a rational function of x as $\frac{3x-2}{x+1}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which $x+1=0$ i.e., $x=-1$

Hence, $\text{Domain} = R - \{-1\}$

We have,

$$\begin{aligned} f(x) &= \frac{2x+1}{x^2-9} \\ &= \frac{2x+1}{(x^2-3^2)} \\ &= \frac{2x+1}{(x-3)(x+3)} \end{aligned} \quad \left[\because a^2 - b^2 = (a-b)(a+b) \right]$$

We observe that $f(x)$ is a rational function of x as $\frac{2x+1}{x^2-9}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which $x^2-9=0$ i.e., $x=-3, 3$

Hence, $\text{Domain}(f) = R - \{-3, 3\}$.

We have,

$$\begin{aligned}f(x) &= \frac{x^2 + 2x + 1}{x^2 - 8x + 12} \\&= \frac{x^2 + 2x + 1}{x^2 - 6x - 2x + 12} \\&= \frac{x^2 + 2x + 1}{x(x - 6) - 2(x - 6)} \\&= \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}\end{aligned}$$

Clearly, $f(x)$ is a rational function of x as $\frac{x^2 + 2x + 1}{x^2 - 8x + 12}$ is a rational expression in x .

We observe that $f(x)$ assumes real values for all x except for all those values of x for which $x^2 - 8x + 12 = 0$ i.e., $x = 2, 6$

$$\therefore \text{Domain}(f) = \mathbb{R} - \{2, 6\}$$

Q2

(i) We have,

$$f(x) = \sqrt{x-2}$$

Clearly, $f(x)$ assumes real values, if

$$x-2 \geq 0$$

$$\Rightarrow x \geq 2$$

$$\Rightarrow x \in [2, \infty)$$

Hence, Domain $(f) = [2, \infty]$

(ii) We have,

$$f(x) = \frac{1}{\sqrt{x^2-1}}$$

Clearly, $f(x)$ assumes real values, if

$$x^2-1 > 0$$

$$\Rightarrow (x-1)(x+1) > 0$$

$$\Rightarrow x < -1 \text{ or } x > 1$$

$$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$$

$$[\because a^2 - b^2 = (a-b)(a+b)]$$

Hence, domain $(f) = (-\infty, -1) \cup (1, \infty)$

(iii) We have,

$$f(x) = \sqrt{9-x^2}$$

Clearly, $f(x)$ assumes real values, if

$$9-x^2 \geq 0$$

$$\Rightarrow 9 \geq x^2$$

$$\Rightarrow x^2 \leq 9$$

$$\Rightarrow -3 \leq x \leq 3$$

$$\Rightarrow x \in [-3, 3]$$

Hence, domain $(f) = [-3, 3]$

(iv) We have,

$$f(x) = \sqrt{\frac{x-2}{3-x}}$$

Clearly, $f(x)$ assumes real values, if

$$\begin{aligned} & x-2 \geq 0 \quad \text{and} \quad 3-x > 0 \\ \Rightarrow & x \geq 2 \quad \text{and} \quad 3 > x \\ \Rightarrow & x \in [2, 3) \end{aligned}$$

Hence, $\text{domain}(f) = [2, 3)$.

Q3

We have,

$$f(x) = \frac{ax+b}{bx-a}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax+b}{bx-a}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which $bx-a=0$ i.e., $bx=a$

$$\Rightarrow x = \frac{a}{b}$$

$$\therefore \text{Domain}(f) = \mathbb{R} - \left\{ \frac{a}{b} \right\}$$

Range of f : Let $f(x) = y$

$$\Rightarrow \frac{ax+b}{bx-a} = y$$

$$\Rightarrow ax+b = y(bx-a)$$

$$\Rightarrow ax+b = bxy-ax$$

$$\Rightarrow b+ay = bxy-ax$$

$$\Rightarrow b+ay = x(by-a)$$

$$\Rightarrow \frac{b+ay}{b-ay} = x$$

$$\Rightarrow x = \frac{b+ay}{by-a}$$

Clearly, x will take real value for all $x \in \mathbb{R}$ except for

$$by-a=0$$

$$\Rightarrow by=a$$

$$\Rightarrow y = \frac{a}{b}$$

$$\therefore \text{Range}(f) = \mathbb{R} - \left\{ \frac{a}{b} \right\}.$$

We have,

$$f(x) = \frac{ax - b}{cx - d}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax - b}{cx - d}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which $cx - d = 0$ i.e., $cx = d$

$$\Rightarrow x = \frac{d}{c}$$

$$\therefore \text{Domain}(f) = R - \left\{ \frac{d}{c} \right\}$$

Range: Let $f(x) = y$

$$\Rightarrow \frac{ax - b}{cx - d} = y$$

$$\Rightarrow ax - b = y(cx - d)$$

$$\Rightarrow ax - b = cxy - dy$$

$$\Rightarrow dy - b = cxy - ax$$

$$\Rightarrow dy - b = x(cy - a)$$

$$\Rightarrow \frac{dy - b}{cy - a} = x$$

Clearly, x assumes real values for all y except

$$cy - a = 0 \text{ i.e., } y = \frac{a}{c}$$

$$\text{Hence, range}(f) = R - \left\{ \frac{a}{c} \right\}$$

We have,

$$f(x) = \sqrt{x-1}$$

Clearly, $f(x)$ assumes real values, if

$$x-1 \geq 0$$

$$\Rightarrow x \geq 1$$

$$\Rightarrow x \in [1, \infty)$$

Hence, $\text{domain}(f) = [1, \infty)$

Range: For $x \geq 1$, we have,

$$x-1 \geq 0$$

$$\Rightarrow \sqrt{x-1} \geq 0$$

$$\Rightarrow f(x) \geq 0$$

Thus, $f(x)$ takes all real values greater than zero.

Hence, $\text{range}(f) = [0, \infty)$

We have,

$$f(x) = \sqrt{x-3}$$

Clearly, $f(x)$ assumes real values, if

$$x-3 \geq 0$$

$$\Rightarrow x \geq 3$$

$$\Rightarrow x \in [3, \infty)$$

Hence, $\text{domain}(f) = [3, \infty)$

Range: For $x \geq 3$, we have,

$$x-3 \geq 0$$

$$\Rightarrow \sqrt{x-3} \geq 0$$

$$\Rightarrow f(x) \geq 0$$

Thus, $f(x)$ takes all real values greater than zero.

Hence, $\text{range}(f) = [0, \infty)$

We have,

$$f(x) = \frac{x-2}{2-x}$$

Domain of f : Clearly, $f(x)$ is defined for all $x \in \mathbb{R}$ except for which

$$2-x \neq 0 \text{ i.e., } x \neq 2$$

Hence, $\text{domain}(f) = \mathbb{R} - \{2\}$

Range of f : Let $f(x) = y$

$$\Rightarrow \frac{x-2}{2-x} = y$$

$$\Rightarrow \frac{-1(2-x)}{2-x} = y$$

$$\Rightarrow -1 = y$$

$$\Rightarrow y = -1$$

$$\therefore \text{Range}(f) = \{-1\}$$

We have,

$$f(x) = |x-1|$$

Clearly, $f(x)$ is defined for all $x \in \mathbb{R}$

$$\Rightarrow \text{Domain}(f) = \mathbb{R}$$

Range: Let $f(x) = y$

$$\Rightarrow |x-1| = y$$

$$\Rightarrow f(x) \geq 0 \quad \forall x \in \mathbb{R}$$

It follows from the above relation that y takes all real values greater or equal to zero.

$$\therefore \text{Range}(f) = [0, \infty)$$

As $|x|$ is defined for all real numbers, its domain is $\mathbb{R}$ and range is only negative numbers because, $|x|$ is always positive real number for all real numbers and $-|x|$ is always negative real numbers.

In order to have $F(x)$ has defined value, term inside square root should always be greater than or equal to zero which gives domain as $-3 \leq x \leq 3$

Where as Range of above function is limited to $[0, 3]$

Ex 3.4

Q1

We have,

$$f(x) = x^3 + 1 \text{ and } g(x) = x + 1$$

Now,

$$f + g : \mathbb{R} \rightarrow \mathbb{R} \text{ given by } (f + g)(x) = x^3 + x + 2$$

$$\begin{aligned} f - g : \mathbb{R} \rightarrow \mathbb{R} \text{ given by } (f - g)(x) &= x^3 + 1 - (x + 1) \\ &= x^3 - x \end{aligned}$$

$$cf : \mathbb{R} \rightarrow \mathbb{R} \text{ given by } (cf)(x) = c(x^3 + 1)$$

$$\begin{aligned} fg : \mathbb{R} \rightarrow \mathbb{R} \text{ given by } (fg)(x) &= (x^3 + 1)(x + 1) \\ &= x^4 + x^3 + x + 1 \end{aligned}$$

$$\frac{1}{f} : \mathbb{R} - \{-1\} \rightarrow \mathbb{R} \text{ given by } \left(\frac{1}{f}\right)(x) = \frac{1}{x^3 + 1}$$

$$\begin{aligned} \frac{f}{g} : \mathbb{R} - \{-1\} \rightarrow \mathbb{R} \text{ given by } \left(\frac{f}{g}\right)(x) &= \frac{(x + 1)(x^2 - x + 1)}{x + 1} \\ &= x^2 - x + 1 \end{aligned}$$

We have,

$$f(x) = \sqrt{x - 1} \text{ and } g(x) = \sqrt{x + 1}$$

Now,

$$f + g : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } (f + g)(x) = \sqrt{x - 1} + \sqrt{x + 1},$$

$$f - g : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } (f - g)(x) = \sqrt{x - 1} - \sqrt{x + 1},$$

$$cf : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } (cf)(x) = c\sqrt{x - 1},$$

$$\begin{aligned} fg : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } (fg)(x) &= (\sqrt{x - 1})(\sqrt{x + 1}) \\ &= \sqrt{x^2 - 1} \end{aligned}$$

$$\frac{1}{f} : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x - 1}}$$

$$\frac{f}{g} : (1, \infty) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x - 1}{x + 1}}$$

Q2

We have,

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

We observe that $f(x) = 2x + 5$ is defined for all $x \in R$.

So, $\text{domain}(f) = R$

Clearly $g(x) = x^2 + x$ is defined for all $x \in R$

So, $\text{domain}(g) = R$

$$\therefore \text{Domain}(f) \cap \text{Domain}(g) = R$$

(i) Clearly, $(f+g): R \rightarrow R$ is given by

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= 2x + 5 + x^2 + x \\ &= x^2 + 3x + 5\end{aligned}$$

$$\text{Domain}(f+g) = R$$

(ii) We find that $f - g : R \rightarrow R$ is defined as

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= 2x + 5 - (x^2 + x) \\ &= 2x + 5 - x^2 - x \\ &= -x^2 + x + 5\end{aligned}$$

$$\text{Domain}(f-g) = R$$

(iii) We find that $fg : R \rightarrow R$ is given by

$$\begin{aligned}(fg)(x) &= f(x) \times g(x) \\ &= (2x + 5) \times (x^2 + x) \\ &= 2x^3 + 2x^2 + 5x^2 + 5x \\ &= 2x^3 + 7x^2 + 5x\end{aligned}$$

$$\text{Domain}(fg) = R$$

(iv) We have,

$$g(x) = x^2 + x$$

$$\therefore f(x) = 0 \Rightarrow x^2 + x = 0$$

$$\Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0 \text{ or, } x = -1$$

$$\begin{aligned} \text{So, } \text{domain}\left(\frac{f}{g}\right) &= \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\} \\ &= R - \{-1, 0\} \end{aligned}$$

$$\text{We find that, } \frac{f}{g} : R - \{-1, 0\} \rightarrow R \text{ is given by } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+5}{x^2+x}$$

$$\text{Domain}\left(\frac{f}{g}\right) = R - \{-1, 0\}$$

Q3

We have,

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$$

Now,

$$f(|x|) = |x| - 1, \text{ where } -2 \leq x \leq 2$$

$$\text{and } |f(x)| = \begin{cases} 1, & -2 \leq x \leq 0 \\ -(x-1), & 0 \leq x \leq 1 \\ (x-1), & 1 \leq x \leq 2 \end{cases}$$

$$\begin{aligned} \therefore g(x) &= f(|x|) + |f(x)| \\ &= \begin{cases} -x & -2 \leq x \leq 0 \\ 0, & 0 < x < 1 \\ 2(x-1), & 1 \leq x \leq 2 \end{cases} \end{aligned}$$

Q4

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$f+g: [-1, 3] \rightarrow \mathbb{R}$ is given by $(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$g-f: [-1, 3] \rightarrow \mathbb{R}$ is given by $(g-f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty)$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty) \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

$$\begin{aligned} fg : [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (fg)(x) &= f(x) \times g(x) = \sqrt{x+1} \times \sqrt{9-x^2} \\ &= \sqrt{9+9x-x^2-x^3} \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

We have, $g(x) = \sqrt{9-x^2}$

$$\therefore 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$$

$$\Rightarrow (x-3)(x+3) = 0$$

$$\Rightarrow x = \pm 3$$

$$\text{So, } \text{domain}\left(\frac{f}{g}\right) = [-1, 3] - [-3, 3] = [-1, 3]$$

$$\therefore \frac{f}{g} : [-1, 3] \rightarrow \mathbb{R} \text{ is given by } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1}$$

$$\therefore \sqrt{x+1} = 0$$

$$\Rightarrow x+1 = 0$$

$$\Rightarrow x = -1$$

$$\begin{aligned} \text{So, } \text{domain}\left(\frac{g}{f}\right) &= [-1, 3] - \{-1\} \\ &= [-1, 3] \end{aligned}$$

$$\therefore \frac{g}{f} : [-1, 3] \rightarrow \mathbb{R} \text{ is given by } \frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

$$\begin{aligned} 2f - \sqrt{5}g : [-1, 3] \rightarrow \mathbb{R} \text{ defined by } (2f - \sqrt{5}g)(x) &= 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2} \\ &= 2\sqrt{x+1} - \sqrt{45-5x^2}. \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

$$\begin{aligned} f^2 + 7f : [-1, \infty] \rightarrow \mathbb{R} \text{ defined by } (f^2 + 7f)(x) &= f^2(x) + 7f(x) & [\because D(f) = [-1, \infty]] \\ &= (\sqrt{x+1})^2 + 7\sqrt{x+1} \\ &= x+1 + 7\sqrt{x+1} \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

We have,

$$g(x) = \sqrt{9-x^2}$$

$$\therefore 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$$

$$\Rightarrow (x-3)(x+3) = 0$$

$$\Rightarrow x = \pm 3$$

$$\begin{aligned} \text{So, } \text{domain}\left(\frac{1}{g}\right) &= [-3, 3] - \{-3, 3\} \\ &= (-3, 3) \end{aligned}$$

$$\therefore \frac{5}{g} = (-3, 3) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$$

Q5

We have,

$$f(x) = \log_e(1-x)$$

and $g(x) = [x]$

$f(x) = \log_e(1-x)$ is defined, if $1-x > 0$

$$\Rightarrow 1 > x$$

$$\Rightarrow x < 1$$

$$\Rightarrow x \in (-\infty, 1)$$

$$\therefore \text{Domain}(f) = (-\infty, 1)$$

$$g(x) = [x] \text{ is defined for all } x \in \mathbb{R}$$

$$\therefore \text{Domain}(g) = \mathbb{R}$$

$$\therefore \text{Domain}(f) \cap \text{Domain}(g) = (-\infty, 1) \cap \mathbb{R} \\ = (-\infty, 1)$$

$$(i) \ f+g : (-\infty, 1) \rightarrow \mathbb{R} \text{ defined by } (f+g)(x) = f(x) + g(x) \\ = \log_e(1-x) + [x]$$

$$(ii) \ fg : (-\infty, 1) \rightarrow \mathbb{R} \text{ defined by } (fg)(x) = f(x) \times g(x) \\ = \log_e(1-x) \times [x] \\ = [x] \log_e(1-x)$$

$$(iii) \ g(x) = [x]$$

$$\therefore [x] = 0$$

$$\Rightarrow x \in (0, 1)$$

$$\text{So, } \text{domain}\left(\frac{f}{g}\right) = \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\} \\ = (-\infty, 0)$$

$$\therefore \frac{f}{g} : (-\infty, 0) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$$

(iv) We have,

$$f(x) = \log_e(1-x)$$

$$\Rightarrow \frac{1}{f(x)} = \frac{1}{\log_e(1-x)}$$

$\therefore \frac{1}{f(x)}$ is defined if $\log_e(1-x)$ is defined and $\log_e(1-x) \neq 0$

$$\Rightarrow 1-x > 0 \quad \text{and} \quad 1-x \neq 0$$

$$\Rightarrow x < 1 \quad \text{and} \quad x \neq 0$$

$$\Rightarrow x \in (-\infty, 0) \cup (0, 1)$$

$$\therefore \text{domain}\left(\frac{g}{f}\right) = (-\infty, 0) \cup (0, 1)$$

$$\frac{g}{f} : (-\infty, 0) \cup (0, 1) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$$

Now,

$$\begin{aligned}(f+g)(-1) &= f(-1) + g(-1) \\ &= \log_e(1-(-1)) + [-1] \\ &= \log_e 2 - 1\end{aligned}$$

$$\Rightarrow (f+g)(-1) = \log_e 2 - 1$$

$$\begin{aligned}\text{(v)} \quad fg(0) &= \log_e(1-0) \times [0] \\ &= 0\end{aligned}$$

$$\text{(vi)} \quad \left(\frac{f}{g}\right)\left(\frac{1}{2}\right) = \text{does not exist}$$

$$\text{(vii)} \quad \left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{\left[\frac{1}{2}\right]}{\log_e\left(1-\frac{1}{2}\right)} = 0$$

Q6

We have,

$$f(x) = \sqrt{x+1}, g(x) = \frac{1}{x}$$

$$\text{and } h(x) = 2x^2 - 3$$

Clearly, $f(x)$ is defined for $x+1 \geq 0$

$$\Rightarrow x \geq -1$$

$$\Rightarrow x \in [-1, \infty]$$

$$\therefore \text{Domain}(f) = [-1, \infty]$$

$g(x)$ is defined for $x \neq 0$

$$\Rightarrow x \in \mathbb{R} - \{0\}$$

and, $h(x)$ is defined for all $x \in \mathbb{R}$

$$\therefore \text{Domain}(f) \cap \text{Domain}(g) \cap \text{Domain}(h) = [-1, \infty] - \{0\}$$

Clearly,

$2f + g - h : [-1, \infty] - \{0\} \rightarrow \mathbb{R}$ is given by

$$(2f + g - h)(x) = 2f(x) + g(x) - h(x)$$

$$= 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3$$

$$\therefore (2f + g - h)(1) = 2\sqrt{1+1} + \frac{1}{1} - 2 \times (1)^2 + 3$$

$$= 2\sqrt{2} + 1 - 2 + 3$$

$$= 2\sqrt{2} + 4 - 2$$

$$= 2\sqrt{2} + 2$$

and, $(2f + g - h)(0)$ does not exist, it is not lies in the domain $x \in [-1, \infty] - \{0\}$.

Q7

Let,

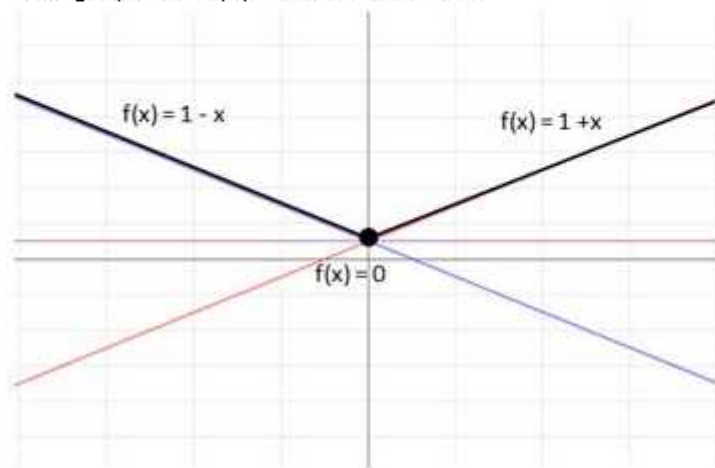
$$y = f(x) = \begin{cases} 1-x, & x < 0 \\ 1, & x = 0 \\ x+1, & x > 0 \end{cases}$$

The graph of $f(x)$ for $x < 0$ is the part of the line $y = 1-x$ that lies to the left of origin.

The graph of $f(x)$ for $x > 0$ is the part of the line $y = 1+x$ that lies to the right of origin.

For $x = 0$, the graph of $f(x)$ represents the point $(0,1)$

The graph of $f(x)$ is shown below.



Q8

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $(f+g)(x) = 3x-2$

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $(f-g)(x) = -x+4$

$f: \mathbb{R} - \left\{ \frac{3}{2} \right\} \rightarrow \mathbb{R}$ defined by $\frac{f}{g}(x) = \frac{x+1}{2x-3}$

Q9

$f+g: [0, \infty) \rightarrow \mathbb{R}$ defined by $(f+g)(x) = \sqrt{x} + x$;

$f-g: [0, \infty) \rightarrow \mathbb{R}$ defined by $(f-g)(x) = \sqrt{x} - x$;

$fg: [0, \infty) \rightarrow \mathbb{R}$ defined by $(fg)(x) = x^{3/2}$;

$\frac{f}{g}: [0, \infty) \rightarrow \mathbb{R}$ defined by $\left(\frac{f}{g} \right)(x) = \frac{1}{\sqrt{x}}$;

Q10

$(f + g): \mathbb{R} \rightarrow [0, \infty)$ defined by $(f + g)(x) = x^2 + 2x + 1 = (x + 1)^2$

$(f - g): \mathbb{R} \rightarrow \mathbb{R}$ defined by $(f - g)(x) = x^2 - 2x - 1$

$(fg): \mathbb{R} \rightarrow \mathbb{R}$ defined by $(fg)(x) = 2x^3 + x^2$

$\left(\frac{f}{g}\right): \mathbb{R} \rightarrow \mathbb{R}$ defined by $\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x + 1}$