

Rational Numbers

- The numbers, which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers. Rational numbers can be positive as well as negative. Rational numbers include all integers and fractions.

For example

$$-\frac{2}{7}, \frac{41}{366}, 2 = \frac{2}{1}, \text{ etc.}$$

- Equivalent rational numbers can be obtained by multiplying the numerator and denominator of a rational number by the same non-zero integer.

Example:

Write three equivalent rational numbers of $-\frac{2}{5}$.

Solution:

$$-\frac{2}{5} \times \frac{2}{2} = -\frac{4}{10}$$

$$-\frac{2}{5} \times \frac{3}{3} = -\frac{6}{15}$$

$$-\frac{2}{5} \times \frac{4}{4} = -\frac{8}{20}$$

$\therefore \frac{4}{10}, -\frac{6}{15}, -\frac{8}{20}$ are three equivalent rational numbers of $-\frac{2}{5}$.

- A rational number is said to be in **standard form** if its numerator and denominator have no common factor other than 1.

- Rational numbers can be reduced to the standard form by dividing their numerator and denominator by their HCF.

Example:

Reduce $-\frac{12}{54}$ into standard form.

Solution:

$$\text{HCF}(12, 54) = 6$$

$$-\frac{12}{54} = \frac{-12 \div 6}{54 \div 6} = -\frac{2}{9}$$

- **Rational numbers on number line**

Rational numbers can be represented on number line in the similar manner like fractions and integers.

Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

Example:

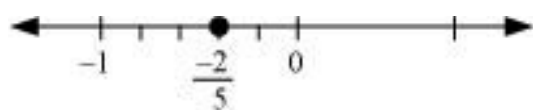
Represent $-\frac{2}{5}$ on number line.

Solution:

The given rational number is negative. Therefore, it will lie to the left of 0.

The space between -1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$.

Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.



- Two positive rational numbers can be compared as in fractions.
- Two negative rational numbers can be compared by ignoring their negative signs and then reversing their order.

Example: Compare $-\frac{2}{3}$ and $-\frac{1}{5}$

Solution: To compare $-\frac{2}{3}$ and $-\frac{1}{5}$, we first compare $\frac{2}{3}$ and $\frac{1}{5}$.

$$\text{HCF}(3, 5) = 15$$

$$\therefore \frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \text{ and } \frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

$$\text{Now, } \frac{10}{15} > \frac{3}{15}$$

$$\Rightarrow \frac{2}{3} > \frac{1}{5}$$

$$\Rightarrow -\frac{2}{3} < -\frac{1}{5}$$

- Rational numbers between two rational numbers can be found by first converting them to rational numbers with same denominator.

Example: Find 4 rational numbers between $-\frac{2}{3}$ and $-\frac{1}{5}$.

Solution:

$$-\frac{2}{3} \times \frac{5}{5} = -\frac{10}{15}$$

$$= \frac{1}{5} \times \frac{3}{3} = -\frac{3}{15}$$

$$\therefore \frac{10}{15} < -\frac{9}{15} < -\frac{8}{15} < -\frac{7}{15} < -\frac{6}{15} < -\frac{3}{15}$$

Thus, four rational numbers between

$$-\frac{2}{3} \text{ and } -\frac{1}{5} \text{ are } -\frac{9}{15}, -\frac{8}{15}, -\frac{7}{15} \text{ and } -\frac{6}{15}$$

- There are unlimited rational numbers between two rational numbers.

All the operations on rational numbers are performed as in fractions.

Example: Solve

1. $-\frac{3}{4} + \frac{5}{6}$

2. $\frac{2}{7} - \frac{3}{5}$

Solution:

$$1. -\frac{3}{4} + \frac{5}{6}$$

$$= \frac{-3 \times 3 + 5 \times 2}{12} = \frac{-9 + 10}{12} = \frac{1}{12}$$

$$2. \frac{2}{7} - \frac{3}{5}$$

$$= \frac{2 \times 5 - 3 \times 7}{35} = \frac{10 - 21}{35} = -\frac{11}{35}$$

- When 0 is added to any rational number, say $\frac{p}{q}$, the same rational number is obtained. Therefore, 0 is the additive identity of rational numbers.

$$\frac{p}{q} + 0 = \frac{p}{q} = 0 + \frac{p}{q}$$

- $-\frac{p}{q}$ is the additive inverse of the rational number $\frac{p}{q}$.

Example: $-\frac{4}{7}$ is the additive inverse of the rational number $\frac{4}{7}$.

Example: Solve

$$1. \frac{2}{9} \times \left(-\frac{4}{3}\right)$$

$$2. -\frac{3}{7} \div \frac{11}{21}$$

Solution:

$$1. \frac{2}{9} \times \left(-\frac{4}{3}\right)$$

$$= \frac{2 \times (-4)}{9 \times 3} = -\frac{8}{27}$$

$$2. -\frac{3}{7} \div \frac{11}{21}$$

$$= -\frac{3}{7} \times \frac{21}{11} = -\frac{9}{11}$$