Exponents

- We use exponents to write very large numbers.

  For example, 1000000000 can be written as $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

  It is read as ten raised to the power nine, where 10 is known as base and 9 as the exponent.

  The number 10^9 is known as exponential form of 1000000000.

- If the exponent of a negative base is odd, then the value of the exponential form is negative. However, if the exponent of a negative base is even, then the value of the exponential form is positive.

  For example, $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
  $(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$

- There are certain rules which helps us in comparing exponential numbers.
  - An exponential form having a negative base and an even exponent will always be greater than an exponential form having a negative base and an odd exponent.
  - An exponential form having a positive base and any exponent will always be greater than an exponential form having a negative base and an odd exponent.
  - If two exponential forms have the same positive base, then the number with greater exponent will be greater.

  Example:

  Arrange 5^4, $(-4)^6$, and 6^3 in decreasing order.

  Solution:

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

$(-4)^6 = (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) = 4096$

$6^3 = 6 \times 6 \times 6 = 216$

Now, $4096 > 625 > 216$

$\therefore (-4)^6 > 5^4 > 6^3$

Thus, the given numbers can be arranged in descending order as $(-4)^6, 5^4, 6^3$.

- **Laws of exponents** (Here, *a* and *b* are non-zero integers and *m* and *n* are integers)
  1. $a^m \times a^n = a^{m+n}$
  2. $\frac{a^m}{a^n} = a^{m-n} \qquad (a \neq 0)$
  3. $(a^m)^n = a^{mn}$
  4. $a^m \times b^m = (ab)^m$
  5. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
  6. $a^{-m} = \frac{1}{a^m}$
  7. $a^0 = 1 \quad (a \neq 0)$

For example, $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$ can be simplified using laws of exponents as:

$$\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$$

$$= \frac{1^{-2}}{6^{-2}} + \frac{1^{-1}}{7^{-1}} + \frac{1^{-1}}{11^{-1}} \qquad \left(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right)$$

$$= \frac{6^2}{1^2} + \frac{7^1}{1^1} + \frac{11^1}{1^1} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$$

$= 36 + 7 + 11$

$= 54$