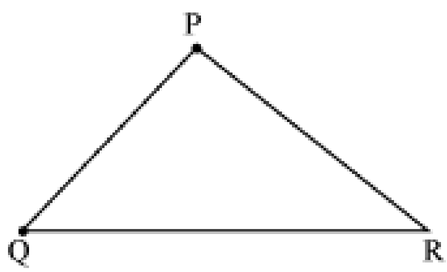
Triangles

- **Triangle:** A triangle is a three-sided polygon. It is the polygon with the least number of sides.



We denote this triangle as ΔPQR. Here, $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$ are the sides of ΔPQR. The points P, Q and R are the vertices of ΔPQR and the angles are ∠RPQ, ∠PQR and ∠QRP.

- A triangle can be classified on the basis of the measures of its angles and sides.

- Classification of triangles on the basis of the measures of its angles:

Name	Nature of the angle
Acute-angled triangle	Each angle is acute
obtuse-angled triangle	One angle is obtuse
Right-angled triangle	One angle is a right angle

- Classification of triangles on the basis of the lengths of its sides:

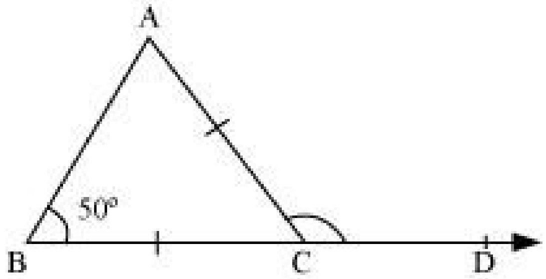
Name	Nature of the angle
Scalene triangle	All three sides are of unequal length
Isosceles triangle	Any two sides are of equal length

Equilateral triangle	All sides are of equal length

- Isosceles triangle

  In an isosceles triangle, (1) two sides are of equal length and (2) the base angles opposite to the equal sides are equal.

  Example:
  In the following figure, AC = BC. Find m∠ACD.



Solution: In ΔABC, AC = BC
∴ ∠CBA = ∠CAB
∴ ∠ACD = ∠CAB + ∠CBA = 2 × 50° = 100°

- Equilateral triangle

  In an equilateral triangle, (1) all sides are equal and (2) each angle is of measure 60°.

- A triangle can be constructed if all its sides are known.

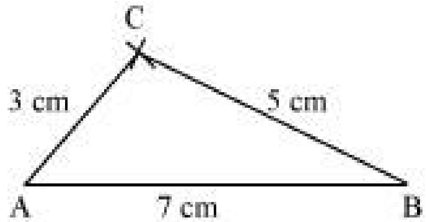
  Example:
  Construct a triangle whose sides are 3 cm, 5cm and 7 cm.

  Solution:

  1. Draw a line segment AB of length 7 cm. With A as centre and radius equal to 3 cm, draw an arc.

  2. With B as centre and radius 5 cm, draw another arc cutting the earlier drawn arc at C.

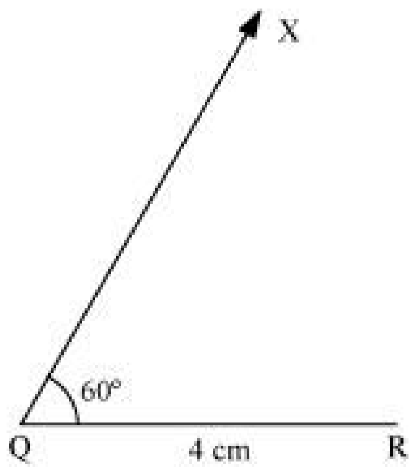
  3. Join AC and BC to get ΔABC.



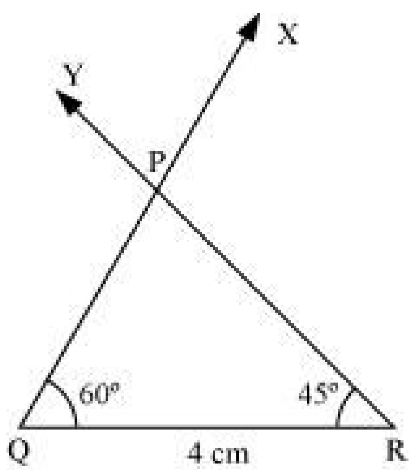
Example: Construct ΔPQR, where ∠PQR = 60°, ∠PRQ = 45° and QR = 4 cm.

Solution:

1. Draw a line segment QR of length 4 cm and draw a ray QX, making an angle of 60° with QR



2. Now, draw ray RY, making an angle of 45° with QR and intersecting QX at P. The resulting ΔPQR is the required triangle.



- A triangle can be constructed if the length of two sides and angle between them are given.

Example:

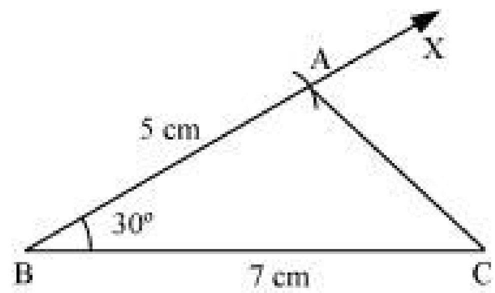
Construct ΔABC where $BC = 7$ cm, $AB = 5$ cm and $\angle ABC = 30°$

Solution:

1. Draw a line segment BC of length 7 cm and at B draw a ray BX, making an angle of 30° with BC.

2. With B as centre and radius equal to 5 cm, draw an arc cutting BX at A.

3. Join AC to get the required ΔABC.

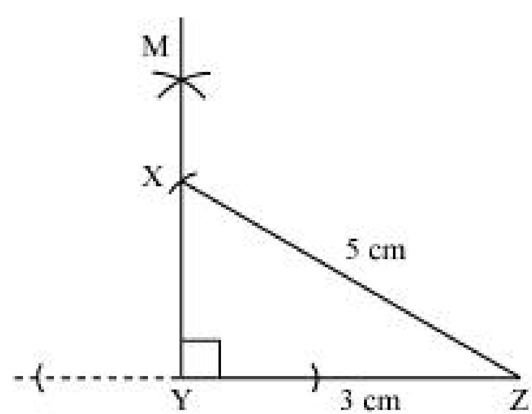


- A right-angled triangle can be constructed if the length of one of its sides or arms and the length of its hypotenuse are known.

Example:
Construct ΔXYZ, right-angled at Y, with $XZ = 5$ cm and $YZ = 3$ cm.
Solution:

1. Draw a line segment YZ of length 3 cm. At Y, draw $MY \perp YZ$.

2. With Z as centre and radius equal to 5 cm, draw an arc intersecting MY at X. Join XZ to get the required ΔXYZ.



- **Construction of circumcircle of given triangle:**

Example:

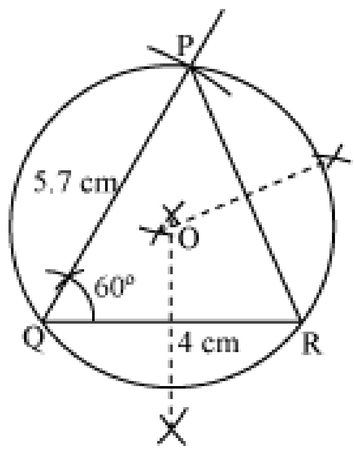
Construct the circumcircle of ΔPQR such that ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm.

Solution:

Step 1: Draw a triangle PQR with ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm

Step 2: Draw perpendicular bisector of any two sides, say QR and PR. Let these perpendicular bisectors meet at point O.

Step 3: With O as centre and radius equal to OP, draw a circle.



The circle so drawn passes through the points P, Q, and R, and is the required circumcircle of ΔPQR.

- **Construction of incircle of given triangle:**

  Example:

  Construct incircle of a right ΔPQR, right angled at Q, such that QR = 4 cm and PR = 6 cm.

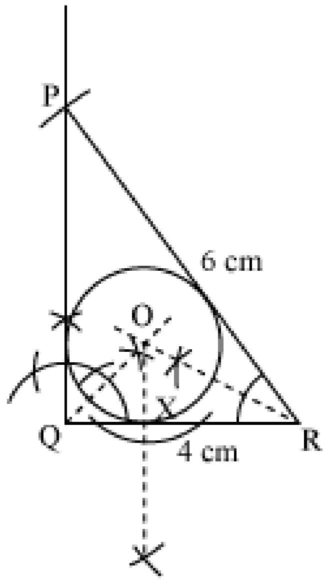
  Solution:

  Step 1: Draw a ΔPQR right-angled at Q with QR = 4 cm and PR = 6 cm.

  Step 2: Draw bisectors of ∠Q and ∠R. Let these bisectors meet at the point O.

  Step 3: From O, draw OX perpendicular to the side QR.

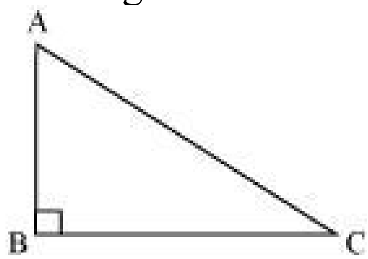
  Step 4: With O as centre and radius equal to OX, draw a circle.



The circle so drawn touches all the sides of ΔPQR and is the required incircle of ΔPQR.

- Right-angled triangle

  A triangle with one of the angles as 90°, is called a right-angled triangle.

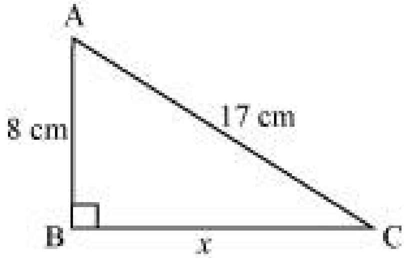


  ΔABC is a right-angled triangle with ∠B = 90°

- The side opposite to the right angle is called its hypotenuse. The other two sides are called the legs of the right-angled triangle, i.e., in the given figure, AC is the hypotenuse and AB, BC are the legs.
- In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides, i.e., $AC^2 = AB^2 + BC^2$.

  This property is called Pythagoras Theorem.

  Example: Find the value of *x* in the following figure.



Solution: ΔABC is a right-angled triangle with $\angle B = 90°$.

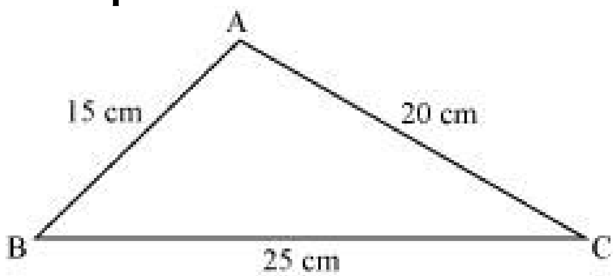
$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem]

$\Rightarrow (17\text{ cm})^2 = (8\text{ cm})^2 + x^2$

$\Rightarrow x = \sqrt{289 - 64}\text{cm} = 15\text{cm}$

- In a triangle, if Pythagoras property holds, then it is a right-angled triangle.

Example:



Prove that ΔABC is right-angled.

Solution: In ΔABC,

$AB = 15\text{ cm} \Rightarrow AB^2 = 225\text{ cm}^2$

$AC = 20\text{ cm} \Rightarrow AC^2 = 400\text{ cm}^2$

$BC = 25\text{ cm} \Rightarrow BC^2 = 625\text{ cm}^2$

$BC^2 = AB^2 + AC^2$

Therefore, Pythagoras property holds in ΔABC. Thus, it is a right-angled triangle.