

Congruency - Congruent Triangles

- **Similar and Congruent Figures**

- Two geometric figures having the same shape and size are said to be congruent figures.
- Two geometric figures having the same shape, but not necessarily the same size, are called similar figures.

Example:

- (1) All circles are similar.
- (2) All equilateral triangles are similar.
- (3) All congruent figures are similar. However, the converse is not true.

- **Similarity of Polygons**

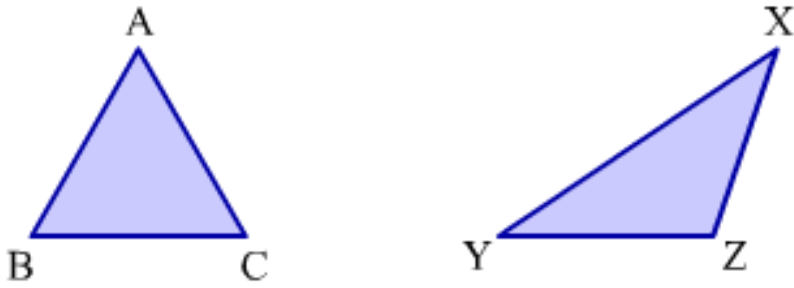
Two polygons with the same number of sides are similar, if

- their corresponding angles are equal
 - their corresponding sides are in the same ratio (or proportion)
- Two line segments are congruent, if they are equal in length.
 - Two angles are congruent, if they have the same measure.
 - **CPCT:**

CPCT stands for Corresponding Parts of Congruent Triangles.

If $\triangle ABC \cong \triangle PQR$, then corresponding sides are equal i.e., $AB = PQ$, $BC = QR$, and $CA = RP$ and corresponding angles are equal i.e., $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

1. In triangles, $\triangle ABC$ and $\triangle XYZ$, if vertices A and X corresponds to each other, then it is symbolized as ' $A \leftrightarrow X$ ' and read as '**There is one to one correspondence between A and X**'.

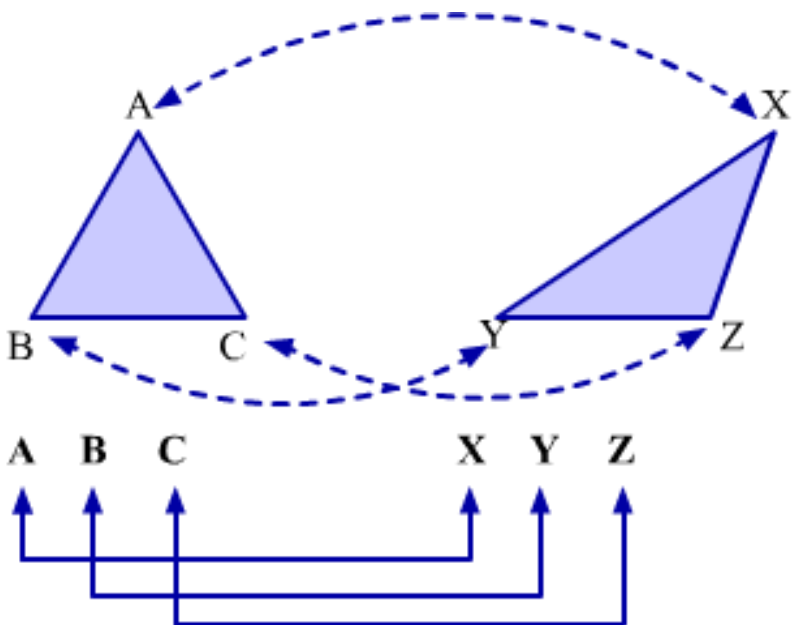


2. All of the correspondences can be represented together as ' $\triangle ABC \leftrightarrow \triangle XYZ$ '.

3. All the possible correspondences between $\triangle ABC$ and $\triangle XYZ$ are:

The correspondence between vertices	The correspondence written together
(1) $A \leftrightarrow X, B \leftrightarrow Y, C \leftrightarrow Z$	$ABC \leftrightarrow XYZ$
(2) $A \leftrightarrow X, B \leftrightarrow Z, C \leftrightarrow Y$	$ABC \leftrightarrow XZY$
(3) $A \leftrightarrow Y, B \leftrightarrow X, C \leftrightarrow Z$	$ABC \leftrightarrow YXZ$
(4) $A \leftrightarrow Y, B \leftrightarrow Z, C \leftrightarrow X$	$ABC \leftrightarrow YZX$
(5) $A \leftrightarrow Z, B \leftrightarrow X, C \leftrightarrow Y$	$ABC \leftrightarrow ZXY$
(6) $A \leftrightarrow Z, B \leftrightarrow Y, C \leftrightarrow X$	$ABC \leftrightarrow ZYX$

4. The arrow diagram to show the correspondence between the given triangles can be shown as follows:



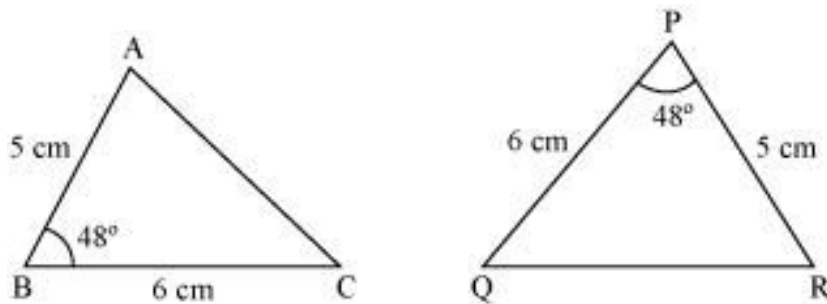
5. The correspondence between sides and angles of the given triangles can be written as follows:

Pairs of corresponding sides	Pairs of corresponding angles
(1) $AB \leftrightarrow XY$	(1) $\angle A \leftrightarrow \angle X$
(2) $BC \leftrightarrow YZ$	(2) $\angle B \leftrightarrow \angle Y$
(3) $AC \leftrightarrow XZ$	(3) $\angle C \leftrightarrow \angle Z$

- SAS congruence rule**

If two sides of a triangle and the angle included between them are equal to the corresponding two sides and included angle of another triangle, then the triangles are congruent by SAS congruence rule.

Example:



Are $\triangle ABC$ and $\triangle RPQ$ congruent?

Solution:

In $\triangle ABC$ and $\triangle RPQ$,

$$AB = RP$$

$$\angle ABC = \angle RPQ$$

$$BC = PQ$$

$$\therefore \triangle ABC \cong \triangle RPQ \quad (\text{By SAS congruence rule})$$

- CPCT**

CPCT stands for ‘corresponding parts of congruent triangles’.

‘Corresponding parts’ means corresponding sides and angles of triangles.

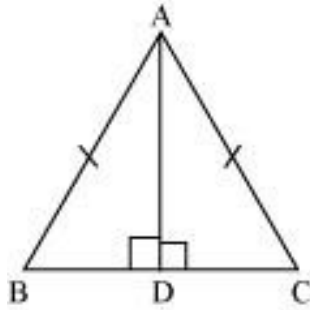
According to CPCT, if two or more triangles are congruent to one another, then all of their corresponding parts are equal.

- **ASA congruence rule**

If two angles and included side of a triangle are equal to the two corresponding angles and the included side of another triangle, then the triangles are congruent by ASA congruence rule.

Example:

In the following figure, AD is the median of $\triangle ABC$.



Are $\triangle ABD$ and $\triangle ACD$ congruent?

Solution:

In $\triangle ABC$,

$$AB = AC \quad (\text{Given})$$

$$\therefore \angle ACB = \angle ABC \quad (\text{Base angles of an isosceles triangle have equal measures})$$

Now, in $\triangle ABD$ and $\triangle ACD$,

$$\angle ABD = \angle ACD$$

$$BD = CD \quad (\text{AD is the median})$$

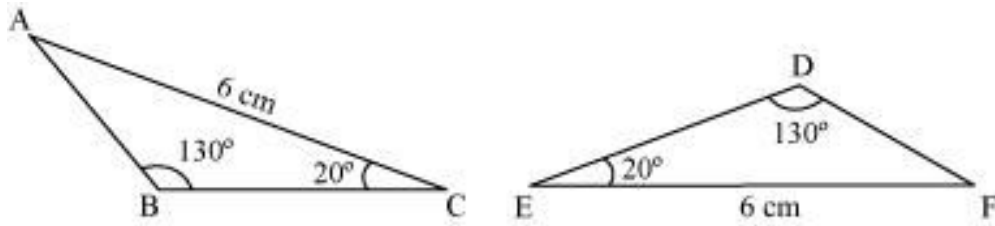
$$\angle ADB = \angle ADC = 90^\circ$$

$$\therefore \triangle ABD \cong \triangle ACD \quad (\text{By ASA congruence rule})$$

- **AAS congruence rule**

If two angles and one side of a triangle are equal to the corresponding angles and side of the other triangle then the two triangles are congruent to each other. This criterion is known as the **AAS congruence rule**.

For example, in the given triangles, $\angle B = \angle D = 130^\circ$, $\angle C = \angle E = 20^\circ$ and $AC = EF = 6 \text{ cm}$.

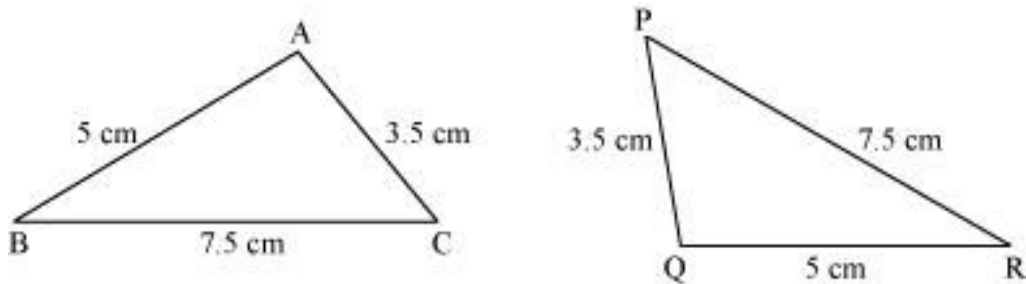


$\therefore$ By AAS congruence rule, $\triangle ABC \cong \triangle FDE$

- SSS congruence rule**

If three sides of a triangle are equal to the three sides of the other triangle, then the two triangles are congruent by SSS congruence rule.

Example:



Are $\triangle ABC$ and $\triangle QRP$ congruent?

Solution:

In $\triangle ABC$ and $\triangle QRP$

$$AB = QR = 5 \text{ cm}$$

$$BC = PR = 7.5 \text{ cm}$$

$$AC = PQ = 3.5 \text{ cm}$$

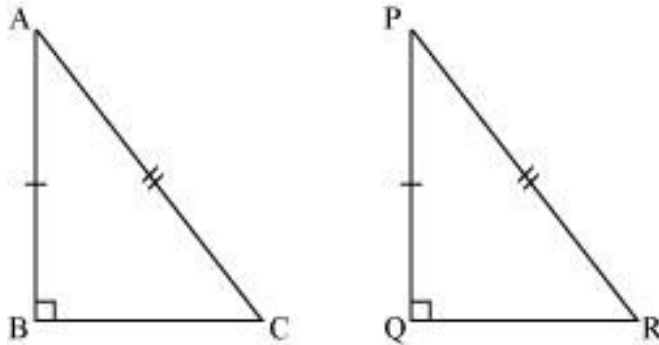
$$\therefore \triangle ABC \cong \triangle QRP$$

(By SSS congruence rule)

- **RHS congruence rule**

If the hypotenuse and one side of a right triangle are equal to the hypotenuse and one side of the other right triangle, then the two triangles are congruent to each other by RHS congruence rule.

Example:



If in the given figure, $\angle B = \angle Q = 90^\circ$, $AC = PR$, and $AB = PQ$,
 $\therefore$ By RHS congruence rule, $\triangle ABC \cong \triangle PQR$