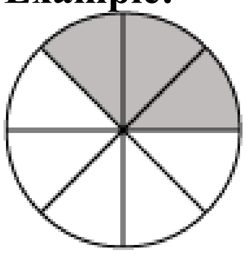
Fractions and Decimals

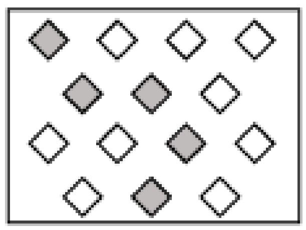
- A fraction is a number representing a part of a whole.

  The whole may be a single object or a group of objects.

  Example:



$\frac{3}{8}$



$\frac{5}{14}$

- A **common fraction** is written in the form $\frac{a}{b}$, where *a* and *b* both are integers and $b \neq 0$. Here, *a* is numerator and *b* is denominator. Common fractions are also known as **vulgar** or **simple fractions**.
- While expressing a situation where parts have to be counted to write a fraction, it must be ensured that all parts are equal.
- For a fraction $\frac{2}{13}$, 2 is called its numerator and 13 is called its denominator.
- Fractions are categorized into three types: proper, improper, and mixed fraction.
- **Proper fractions** are those fractions in which the numerator is less than the denominator. These fractions are always less than 1.
- For example, $\frac{17}{24}$ is a proper fraction since the numerator, 17, is less than the denominator, 24.
- **Improper fractions** are those fractions in which the numerator is greater than the denominator. These fractions are always greater than 1.
- For example, $\frac{15}{7}$ is an improper fraction since the numerator (15) > denominator (7).
- A **mixed fraction** is a combination of a whole number and a part.
- For example, $9\frac{5}{13}$ $\left[9\frac{5}{13} = 9 + \frac{5}{13}\right]$

- If numerator and denominator of a fraction are equal, then the fraction is 1.
- For example, $\frac{5}{5} = 1$
-
- A mixed fraction can be converted into an improper fraction as $\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$
- For example, $8\frac{2}{23} = \frac{(8\times 23)+2}{23} = \frac{184+2}{23} = \frac{186}{23}$

- To convert an improper fraction into a mixed fraction, first of all, the quotient and remainder are obtained by just dividing the numerator by the denominator. Then, the mixed fraction corresponding to the given improper fraction is written as

  $\text{Quotient}\frac{\text{Remainder}}{\text{Divisor(Denominator)}}$

  For example, to find the mixed fraction corresponding to the improper fraction $\frac{182}{7}$ first of all, 182 is divided by 17.

  Here, divisor = 17, quotient = 10, and remainder = 12

  $\therefore \frac{182}{17} = 10\frac{12}{17}$

- To find an equivalent fraction of a given fraction, both the numerator and denominator of the given fraction are multiplied or divided by the same number.

  Example:

  $\frac{12}{18} = \frac{12\times 2}{18\times 2} = \frac{24}{26}, \frac{12}{18} = \frac{12\times 5}{18\times 5} = \frac{60}{90}$

  $\frac{12}{18} = \frac{12\div 2}{18\div 2} = \frac{6}{9}, \frac{12}{18} = \frac{12\div 3}{18\div 3} = \frac{4}{6}$

  $\therefore \frac{12}{18} = \frac{24}{36} = \frac{60}{90} = \frac{6}{9} = \frac{4}{6}$

- Two fractions are equivalent, if the product of the numerator of the first fraction and the denominator of the second fraction is equal to the product of the numerator of the second fraction and the denominator of the first fraction.

  Example: To check the equivalence of $\frac{3}{5}$ and $\frac{18}{30}$, the following calculation is carried out.

  3 × 30 = 90 and 5 × 18 = 90

Since, $3 \times 30 = 5 \times 18$
$\Rightarrow \frac{3}{5} = \frac{18}{30}$

- Fractions with same denominators are called **like fractions**.

  For example, $\frac{9}{5}, \frac{2}{5}, \frac{11}{5}, \frac{7}{5}$ are like fractions.

- Fractions with different denominators are called **unlike fractions.**

  For example, $\frac{9}{8}, \frac{11}{3}, \frac{12}{7}$ are unlike fractions.

- If two or more fractions are like fractions, then greater the numerator, greater is the fraction.

  For example, among the fractions, $\frac{9}{17}, \frac{25}{17}, \frac{21}{17}$, and $\frac{6}{17}$, it can be observed that,
  $25 > 21 > 9 > 6$
  $\therefore \frac{25}{17} > \frac{21}{17} > \frac{9}{17} > \frac{6}{17}$

- If two or more fractions have the same numerator, then smaller the denominator, greater is the fraction.

  For example, among the fractions, $\frac{17}{6}, \frac{17}{3}$, and $\frac{17}{11}$, it can be observed that, $3 < 5 < 11$
  $\therefore \frac{17}{3} > \frac{17}{5} > \frac{17}{11}$

- To compare two unlike fractions (without same numerator), first of all, these fractions are converted into their equivalent fractions of same denominator, which is the LCM of the denominators of the fractions. Then, like fractions are obtained, which can be compared easily.

  For example, $\frac{5}{6}$ and $\frac{20}{21}$ can be compared as:
  LCM of 6 and 21 = 42

  $$\therefore \frac{5}{6} = \frac{5 \times 7}{6 \times 7} = \frac{35}{42}, \frac{20}{21} = \frac{20 \times 2}{21 \times 2} = \frac{40}{42}$$

  Here, $\frac{35}{42}$ and $\frac{40}{42}$ are like fractions.

  Since $\frac{40}{42} > \frac{35}{42}$, we obtain $\frac{20}{21} > \frac{5}{6}$

- Addition of two like fractions can be performed just by adding the numerators and retaining the denominator of the fractions.

  For example, $\frac{17}{25}+\frac{3}{25}=\frac{17+3}{25}=\frac{20}{25}=\frac{20\div 5}{25\div 5}=\frac{4}{5}$

- Subtraction of two like fractions can be performed just by subtracting the numerators and retaining the denominator of the fractions.

  For example, $\frac{31}{15}-\frac{4}{15}=\frac{31-4}{15}=\frac{27}{15}=\frac{27\div 3}{15\div 3}=\frac{9}{5}$

- To perform the addition and subtraction of unlike fractions, first of all, they are converted into their equivalent fractions with the denominator as the LCM of their denominators. Then, addition or subtraction can be performed easily.

  Example:
  Find the sum of $\frac{4}{3}$ and $\frac{5}{12}$.
  Solution:
  LCM of 3 and 12 = 12

  $\therefore \frac{4}{3}+\frac{5}{12}=\frac{4\times 4}{3\times 4}+\frac{5\times 1}{12\times 1}=\frac{16}{12}+\frac{5}{12}=\frac{21}{12}=\frac{21\div 3}{12\div 3}=\frac{7}{4}$

  Example:
  Subtract $\frac{4}{33}$ from $\frac{3}{22}$.
  Solution:
  LCM of 33 and 22 = 66

  ∴322-433=3×322×3-4×233×2=966-866=166

- To add or subtract mixed fractions, first of all, they are converted into improper fractions. Then, they can be added or subtracted easily.

  For example,

$$7\frac{2}{5} + 3\frac{4}{9} = \frac{37}{5} + \frac{31}{9}$$

$$= \frac{37 \times 9}{5 \times 9} + \frac{31 \times 5}{9 \times 5} \qquad (\text{LCM of 5 and 9 is 45})$$

$$= \frac{333}{45} + \frac{155}{45}$$

$$= \frac{333+155}{45}$$

$$= \frac{448}{45}$$

$$= 10\frac{38}{45}$$

- **Multiplication of fractions with whole number**
  - A whole number is multiplied with a proper or improper fraction by multiplying the whole number with the numerator of the fraction, keeping the denominator same. For example, $\frac{4}{3} \times 2 = \frac{8}{3}$
  - A mixed fraction is first converted into an improper fraction and then multiplied with the whole number. For example, $1\frac{2}{3} \times 5 = \frac{5}{3} \times 5 = \frac{25}{3}$

Multiplication of fraction by fraction

When two fractions are multiplied, the product is obtained as

$$\frac{\text{Product of numerators}}{\text{Product of denominators}}$$

For example, $\frac{2}{9} \times \frac{7}{3} = \frac{2 \times 7}{9 \times 3} = \frac{14}{27}$

The product of two proper fractions is always less than each of the fractions.

For example, $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$ Here $\frac{8}{21} < \frac{2}{3}$ and $\frac{8}{21} < \frac{4}{7}$

The product of two improper fractions is greater than each of the fractions.

For example, $\frac{3}{2} \times \frac{7}{4} = \frac{21}{8}$ Here, $\frac{21}{8} > \frac{3}{2}$ and $\frac{21}{8} > \frac{7}{4}$

The product of a proper fraction and an improper fraction is greater than the proper fraction, but less than the improper fraction.

For example, $\frac{2}{3} \times \frac{7}{4} = \frac{2}{3} \times \frac{7}{4} = \frac{7}{6}$ Here, $\frac{7}{6} > \frac{2}{3}$ and $\frac{7}{6} < \frac{7}{4}$

- **Reciprocal of a number** is obtained by interchanging the numerator and denominator of that number.

  For example, reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4.

- **Division of fraction by whole number or fraction**

  The fraction is multiplied with the reciprocal of the divisor. For example,

  $\frac{2}{9} \div \frac{4}{5} = \frac{2}{9} \times \frac{5}{4} = \frac{10}{36} = \frac{5}{9}$

  $\frac{3}{11} \div 3 = \frac{3}{11} \times \frac{1}{3} = \frac{3}{33} = \frac{1}{11}$

Division of whole number by fraction

The whole number is multiplied with the reciprocal of the fraction. For example, $2 \div \frac{1}{5} = 2 \times 5 = 10$

- We represent a unit by a block to understand the parts of one whole. If one block is divided into 10 equal parts, then it means that each part is $\left(\frac{1}{10}\right)$ (one-tenth) of a unit. It can be written as 0.1 in decimal notation. Similarly, 8 equal parts out of 10 equal parts is written as 0.8 (read as zero point eight). The dot represents the decimal point and it comes between the units place and the tenths place.

  For example: The place value table of 295.3 can be compiled as:

Hundreds (100)	**Tens** (10)	**Ones** (1)	**Tenths** $\left(\frac{1}{10}\right)$

2	9	5	3	We can write 295.3 in expanded form as $2 \times 100 + 9 \times 10 + 5 \times 1 + 3 \times \left(\frac{1}{10}\right)$

.

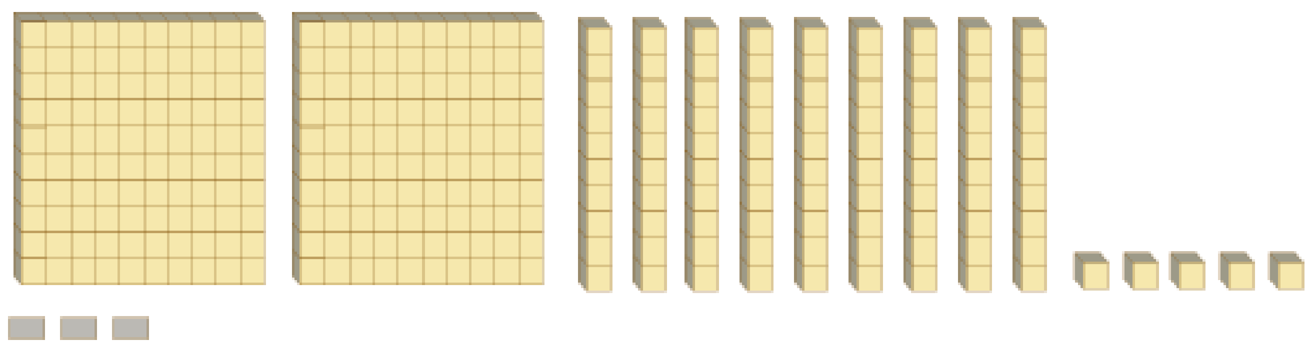
- We can represent the number 295.3 by using blocks.

295.3
$= 2 \times 100 + 9 \times 10 + 5 \times 1 + 3 \times \left(\frac{1}{10}\right)$.

= 2 hundreds, 9 tens, 5 ones, and 3 tenths

Therefore, 295.3 contains 2 hundreds, 9 tens, 5 ones, and 3 tenths. Thus, 295.3 can be represented by blocks as follows.

(Here, 1 block represents 1 unit.)



- The decimal point always comes between ones place and tenths place. If we move to the left of the decimal point, then we find ones, tens, hundreds, thousands place, etc. Similarly, if we move to the right of the decimal point, then we find tenths, hundredths, thousandths place, etc.

Example: The place value table of the number 8570.216 can be compiled as:

Thousands	8

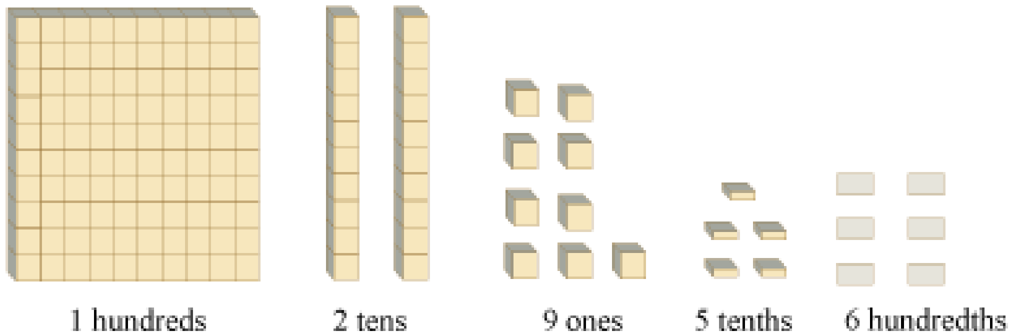
(1000)	
Hundreds (100)	5
Tens (10)	7
Ones (1)	0
Tenths $\left(\frac{1}{10}\right)$	2
Hundredths $\left(\frac{1}{100}\right)$	1
Thousandths $\left(\frac{1}{1000}\right)$	6

Using this table, we can expand 8570.216 according to its place value as

$$8570.216 = 8 \times 1000 + 5 \times 100 + 7 \times 10 + 2 \times \frac{1}{10} + 1 \times \frac{1}{100} + 6 \times \frac{1}{100}$$

- We can represent a decimal number (upto hundredth place) using blocks.

  Example: To express 129.56 by blocks, the blocks have to be arranged as follows.



- Every decimal can be written as a fraction.

  Example:

  $$2.96 = 2 + \frac{96}{100} = 2 + \frac{96 \div 4}{100 \div 4} = 2 + \frac{24}{25} = 2\frac{24}{25} = \frac{74}{25}$$

  $$8.8 = 8 + 0.8 = 8 + \frac{8}{10} = 8 + \frac{8 \div 2}{10 \div 2} = 8 + \frac{4}{5} = 8\frac{4}{5} = \frac{44}{5}$$

- Every fraction with denominator 10 or 100 can be converted into decimal form easily.

  Example:

  $$\frac{56}{10} = \frac{50+6}{10} = 5 + \frac{6}{10} = 5.6$$

  $$\frac{291}{100} = \frac{200+91}{100} = 2 + \frac{91}{100} = 2.91$$

- A fraction whose denominator is 10 or 100 can be converted into decimal form by multiplying the numerator and denominator by the same number such that the denominator is 10 or 100.

  Example:

  $$\frac{41}{20} = \frac{41 \times 5}{20 \times 5} = \frac{205}{100} = \frac{200+5}{100} = \frac{5}{100} = 2.05$$

  $$\frac{9}{5} = \frac{9 \times 2}{5 \times 2} = \frac{18}{10} = \frac{10+8}{10} = 1 + \frac{8}{10} = 1.8$$

- We use decimals in our day to day lives in many ways, for example, in representing units of money, weight, length, volume, etc.

  Example: If we want to represent 6 kg 5g into kg, then we may proceed as follows.

$$6\text{kg } 5\text{ g} = 6\text{ kg} + 5 \times \frac{1}{1000}\text{kg} = \left(6 + \frac{5}{1000}\right)\text{kg} = 6.005\text{ kg}$$

$$\left(1\text{g} = \frac{1}{1000}\text{kg}\right)$$

- We can add or subtract decimals in the same way as whole numbers by placing decimal points one above the other.

  Example:

  - If 9.56 and 17.15 are to be added, then we proceed as:

Tens	Ones	Tenths	Hundredths
 + 1	9 7	5 1	6 5
2	6	7	1

$\therefore$ 9.56 + 17.15 = 26.71

  - If 72.18 has to be subtracted from 92, then we proceed as:

Tens	Ones	Tenths	Hundredths
9 – 7	2 2	0 1	0 8
1	9	8	2

$\therefore$ 92 – 72.18 = 19.82

Multiplication of decimals

To multiply two decimal numbers, the numbers have to be first multiplied as whole numbers. Then, decimal is put in the product by counting the digits from the rightmost digit equal to the sum of the number of digits to the right of the decimal in both the numbers. For example,

0.32×0.4
Here, the number of digits to the right of the decimal in 0.32 is 2 and in 0.4 is 1.
$32 \times 4 = 128$
Putting the decimal in 128 by counting $(2 + 1) = 3$ places to the left of 8, we obtain
$0.32 \times 0.4 = 0.128$

- When a decimal number is multiplied by 10, 100, or 1000, the digits in the product are same as in the decimal number, but the decimal point in the product is shifted to the right to as many places as there are zeroes i.e.,

$0.42 \times 10 = 4.2$
$0.42 \times 100 = 42$
$0.42 \times 1000 = 420$

- To divide a decimal number by a whole number, it is first divided as whole numbers and then the decimal is put in the quotient to as many places from the right as in the decimal number. For example,

$$\frac{8.4}{7} = 1.2$$

- To divide a decimal number by another decimal number, firstly both the divisor and dividend are changed into fractional forms and then the dividend is multiplied with the reciprocal of the divisor.

  For example, 0.96 can be divided by 0.8 as follows:

$$\begin{aligned} 0.96 \div 0.8 &= \frac{96}{100} \div \frac{8}{10} \\ &= \frac{96}{100} \times \frac{10}{8} \\ &= \frac{96 \times 1}{10 \times 8} \\ &= \frac{12}{10} \\ &= 1.2 \end{aligned}$$

- **Division of decimal numbers**

When a decimal number is divided by 10, 100, or 1000, the quotient is same as the decimal number, but the decimal point in the quotient shifts to the left by as many places as there are zeroes. For example,

$$\frac{43.2}{10} = 4.32$$

$$\frac{43.2}{100} = 0.432$$

$$\frac{43.2}{1000} = 0.0432$$