Algebraic Expressions

- Algebraic expressions are formed by combining variables with constants using operations of addition, subtraction, multiplication and division.

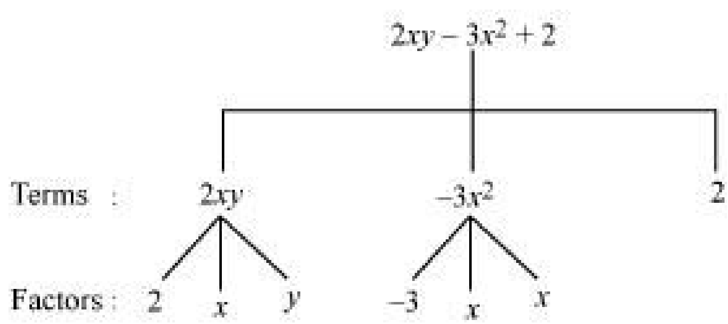
  For example: 4xy, 2x2 – 3, 7xy + 2x, etc.

  In an algebraic expression, say 2xy – 3x2 + 2; 2xy, (–3x2), 2 are known as the terms of the expression.

  The expression 2xy – 3x2 +2 is formed by adding the terms 2xy, (–3x2) and 2 where 2, x, y are factors of the term 2xy; (–3), x, x are factors of the term (–3x2); 2 is the factor of the term 2.

  For an expression, the terms and its factors can be represented easily and elegantly by a tree diagram.

  Tree diagram for the expression 2xy – 3x2 +2:



  Note: In an expression, 1 is not taken as separate factor.

- The numerical factor of a term is known as its coefficient. For example, for the term –3x2y, the coefficient is (–3).
- The terms having the same algebraic factors are called like terms, while the terms having different algebraic factors are called unlike terms.

  For example: 13x2y, – 23x2y are like terms; 12xy, 3x2 are unlike terms

- **Polynomial**

An algebraic expression in which the exponents of the variables are non-negative integers are called polynomials. For example, $3x^4 + 2x^3 + x + 9$, $3x^4$ etc are polynomials.

- **Constant polynomial:** A constant polynomial is of the form $p(x) = k$, where k is a real number. For example, –9, 10, 0 are constant polynomials.
- **Zero polynomial:** A constant polynomial '0' is called zero polynomial.

General form of a polynomial:

A polynomial of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, where $a_0, a_1 \ldots a_r$ are constants and $a_n \neq 0$.
Here, $a_0, a_1 \ldots a_n$ are the respective coefficients of $x^0, x^1, x^2, \ldots x^n$ and n is the power of the variable x.
$a_n x^n, a_{n-1} x^{n-1} - a_0$ and $a_0 \neq 0$ are called the terms of $p(x)$.

- **Classification of polynomials on the basis of number of terms**
    - A polynomial having one term is called a monomial e.g. $3x$, $25t^3$ etc.
    - A polynomial having two terms is called a binomial e.g. $2t - 6$, $3x^4 + 2x$ etc.
    - A polynomial having three terms is called a trinomial. e.g. $3x^4 + 8x + 7$ etc.

- **Degree**

The degree of a polynomial is the highest exponent of the variable of the polynomial. For example, the degree of polynomial $3x^4 + 2x^3 + x + 9$ is 4.

The degree of a term of a polynomial is the value of the exponent of the term.

- **Classification of polynomial according to their degrees**

- A polynomial of degree one is called a linear polynomial e.g. $3x + 2$, $4x$, $x + 9$.
- A polynomial of degree two is called a quadratic polynomial. e.g. $x^2 + 9$, $3x^2 + 4x + 6$ ·
- A polynomial of degree three is called a cubic polynomial e.g. $10x^3 + 3$, $9x^3$.

Note: The degree of a non-zero constant polynomial is zero and the degree of a zero polynomial is not defined.

- Addition and subtraction of algebraic expressions:
    - The sum or difference of two like terms is a like term, with its numerical coefficient equal to the sum or difference of the numerical coefficients of the two like terms.
    - When algebraic expressions are added, the like terms are added and unlike terms are left as they were.

    Example : Subtract (x^2–$2y^2$+y) from the sum of (–$2x^2$+$3x$+2) and

    $\left(-2y+3x^2+5x\right)$

    Solution:

    $$\begin{aligned}&\left(-2x^2+3x+2\right)+\left(-2y+3x^2+5x\right)\\&=\left(-2x^2+3x^2\right)+\left(3x+5x\right)-2y+2 && \left[\text{Rearranging terms}\right]\\&=x^2+8x-2y+2\end{aligned}$$

    $$\begin{aligned}&\therefore\left(x^2+8x-2y+2\right)-\left(x^2-2y^2+y\right)\\&=x^2+8x-2y+2-x^2+2y^2-y\\&=\left(x^2-x^2\right)+2y^2+8x+\left(-2y-y\right)+2 && \left[\text{Rearranging terms}\right]\\&=2y^2+8x-3y+2\end{aligned}$$

- Addition and subtraction of linear algebraic expressions can be done using some algebraic properties and the concept of addition and subtraction of like terms.
- Some properties used in the addition and subtraction of algebraic expressions are:

  $x-(y+z)=(x-y)-z$

  $x-(y-z)=x-y+z$

  For example, $(2a + 5b) - (a + b)$ can be simplified as follows:

$(2a + 5b) - (a + b) = (2a + 5b - a) - b$ $\{x - (y + z) = (x - y) - z$, where $x = (2a + 5b)$, $y = a$ and $z = b\}$

$= (2a - a + 5b) - b$

$= a + 5b - b$

$= a + 4b$

- The value of an algebraic expression depends upon the value of its variables.

  For example, the value of $(3a + 2b) - (2a + b)$ at $a = 1$ and $b = 3$ can be calculated as follows:

  $(3a + 2b) - (2a + b)$

  $= 3a + 2b - 2a - b$ $\{x - (y - z) = x - y + z$, where $x = (3a + 2b)$, $y = 2a$ and $z = b]$

  $= 3a - 2a + 2b - b$

  $= a + b$

  When $a = 1$ and $b = 3$ then the value of the given expression is:

  $1 + 3 = 4$

- The multiplication of a monomial by a monomial gives a monomial. While performing multiplication, the coefficients of the two monomials are multiplied and the powers of different variables in the two monomials are multiplied by using the rules of exponents and powers.

$(-2ab^2c) \times (3abc^2) = (-2 \times 3) \times (a \times a \times b^2 \times b \times c \times c^2) = -6a^2b^3c^3$

The multiplication of three or more monomials is also performed similarly.

$(xy) \times (3yz) \times (3x^2z^2)$

$= (3 \times 3) \times (x \times x^2) \times (y \times y) \times (z \times z^2)$

$= 9x^3y^2z^3$

- There are two ways of arrangement of multiplication while multiplying a monomial by a binomial or trinomial or polynomial. These are horizontal arrangement and vertical arrangement.

  Multiplication in **horizontal arrangement** can be performed as follows:

  Here, we arrange monomial and polynomial both horizontally and multiply every term in the polynomial by the monomial by making use of distributive law.

  $5a \times (2b + a - 3b + c)$

  $= (5a \times 2b) + (5a \times a) + (5a \times (-3b)) + (5a \times c)$

  $= 10ab + 5a^2 - 15ab + 5ac$

  $= 5a^2 - 5ab + 5ac$

  Multiplication in **vertical arrangement** can be performed as follows:

  $$\begin{array}{r} 4x^2 + 2x \\ \times \quad 3x \\ \hline 12x^3 + 6x \\ \hline \end{array}$$

  Here, we have first multiplied $3x$ with $2x$ and wrote the product with sign at the bottom. After doing this, we have multiplied $3x$ with $4x^2$ and wrote the product with sign at the bottom.

  Similarly, we can multiply a trinomial with monomial as follows:

  $$\begin{array}{r} 2y^3 - 5y + 1 \\ \times \qquad\quad 2y \\ \hline 4y^4 - 10y^2 + 2y \\ \hline \end{array}$$

- While multiplying a polynomial by a binomial (or trinomial) in horizontal arrangement, we multiply it term by term. That is, every term of the polynomial is multiplied by every term of the binomial (or trinomial).

Example:

Simplify $(x + 2y)(x + 3) – (2x + 1)(y + x + 1)$.

Solution:

$(x + 2y)(x + 3) = x(x + 3) + 2y(x + 3)$

$= x^2 + 3x + 2xy + 6y$

$(2x + 1)(y + x + 1) = 2x(y + x + 1) + 1(y + x + 1)$

$= 2xy + 2x^2 + 2x + y + x + 1$

$= 2xy + 2x^2 + 3x + y + 1$

$\therefore (x + 2y)(x + 3) – (2x + 1)(y + x + 1) = x^2 + 3x + 2xy + 6y – 2xy – 2x^2 – 3x – y – 1$

$= – x^2 + 5y – 1$

- We can also perform multiplication of two polynomials using vertical arrangement.

  For example,

$$\begin{array}{r} l + 6m + 7n \\ \times \qquad\qquad l + 3m \\ \hline 3lm + 18m^2 + 21mn \\ +l^2 + 6lm + 7nl \qquad\qquad \\ \hline l^2 + 9lm + 18m^2 + 21mn + 7nl \\ \hline \end{array}$$

- Division of any polynomial by a monomial is carried out either by dividing each term of the polynomial by the monomial or by the common factor method.

  For example, $(8x^3 + 4x^2y + 6xy^2)$ can be divided by $2x$ as follows:

$$\left(8x^3 + 4x^2y + 6xy^2\right) \div 2x = \frac{8x^3+4x^2y+6xy^2}{2x}$$

$$= \frac{8x^3}{2x} + \frac{4x^2y}{2x} + \frac{6xy^2}{2x}$$

$$= 4x^2 + 2xy + 3y^2$$

Or,

$$\left(8x^3 + 4x^2y = 6xy^2\right) \div 2x = \frac{2\times x\left(4x^2+2xy+3y^2\right)}{2\times x} = 4x^2 + 2xy$$

- **Value of an expression at given values of variables:**
  - The value of an expression depends on the values of the variables forming the expression.
  - The value of an expression at particular values of variables can be found by substituting the variables by the corresponding values given.

  Example:

  What is the value of the expression $-a^2b + 2ab + b$ at $a = -1$ and $b = 2$?

  Solution:

  The given expression is $-a^2b + 2ab + b$.

  Substituting $a = -1$ and $b = 2$ we get,

  $-a^2b + 2ab + b = -(-1)^2(2) + 2(-1)(2) + (2)$

  $= -2 - 4 + 2$

  $= -4$

- An **algebraic expression** may contain some brackets, namely line bracket, common bracket, curly bracket, or rectangular brackets, and some mathematical operations. An expression enclosed within a bracket is considered as a single quantity even though it may consist of many terms.
- For simplifying an expression, we remove the bracket by the following rules:

  (i) If '+' sign occurs before a bracket, then the signs of all the terms inside the bracket do not change.

(ii) If '−' sign occurs before a bracket, then the signs of all the terms inside the bracket change.

Brackets are removed in the order of

(a) line brackets

(b) common brackets

(c) curly brackets

(d) rectangular brackets

Example:

Simplify $3e^2 - \left[d^2 - 4\left\{f^2 - \left(2e^2 - \overline{f^2 + d^2}\right)\right\}\right]$

Solution:

$3e^2 - \left[d^2 - 4\left\{f^2 - \left(2e^2 - \overline{f^2 + d^2}\right)\right\}\right]$

$= 3e^2 - [d^2 - 4\ \{f^2 - (2e^2 - f^2 - d^2)\}]$ [Line bracket is removed]

$= 3e^2 - [d^2 - 4\ \{f^2 - 2e^2 + f^2 + d^2\}]$ [Common bracket is removed]

$= 3e^2 - [d^2 - 4\ \{2f^2 - 2e^2 + d^2\}]$

$= 3e^2 - [d^2 - 8f^2 + 8e^2 - 4d^2]$ [Curly bracket is removed]

$= 3e^2 - [-3d^2 - 8f^2 + 8e^2]$

$= 3e^2 + 3d^2 + 8f^2 - 8e^2$ [Rectangular bracket is removed]

$= 3d^2 - 5e^2 + 8f^2$

- An algebraic equation is an equality involving variables. In an equation, the value of expression on the left hand side (LHS) is equal to the value of expression on the right hand side (RHS).