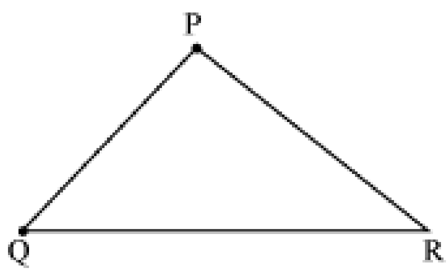
Triangles

- **Triangle:** A triangle is a three-sided polygon. It is the polygon with the least number of sides.



We denote this triangle as ΔPQR. Here, $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$ are the sides of ΔPQR. The points P, Q and R are the vertices of ΔPQR and the angles are ∠RPQ, ∠PQR and ∠QRP.

- A triangle can be classified on the basis of the measures of its angles and sides.

- Classification of triangles on the basis of the measures of its angles:

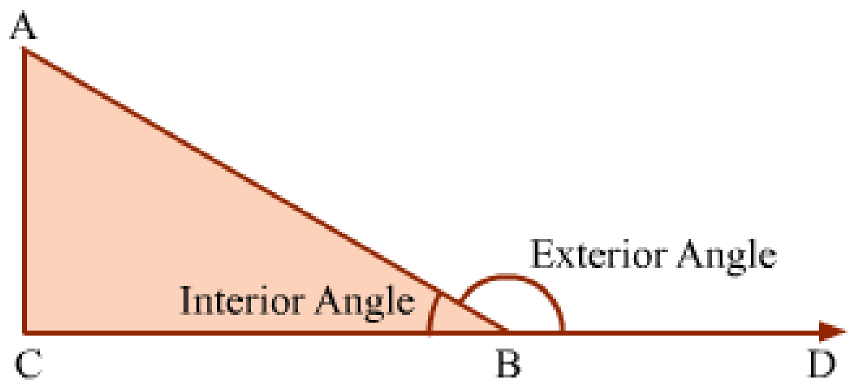
Name	**Nature of the angle**
Acute-angled triangle	Each angle is acute
obtuse-angled triangle	One angle is obtuse
Right-angled triangle	One angle is a right angle

- Classification of triangles on the basis of the lengths of its sides:

Name	**Nature of the angle**
Scalene triangle	All three sides are of unequal length
Isosceles triangle	Any two sides are of equal length

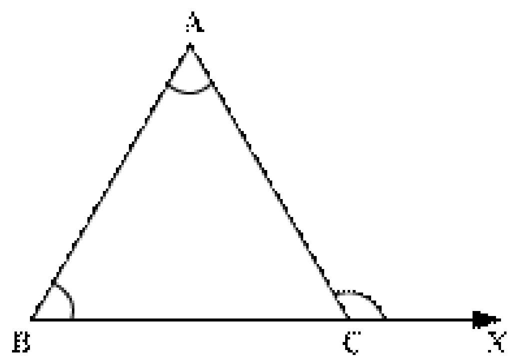
Equilateral triangle	All sides are of equal length

- The angle formed by a side of a triangle with an extended adjacent side is called an **exterior angle of the triangle**.



It can be seen that in ΔABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, **∠ABD is an exterior angle of ΔABC.**

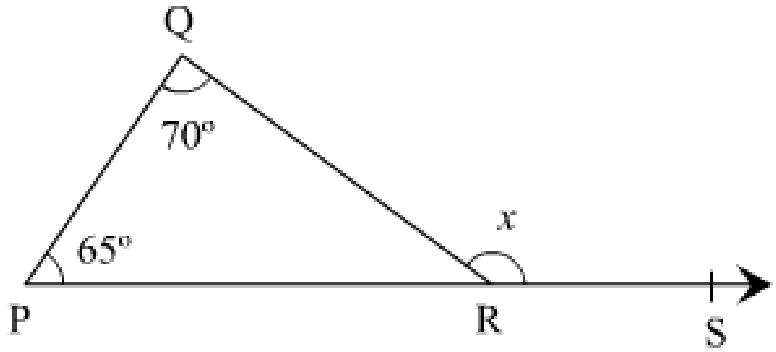
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.



∠ACX = ∠BAC + ∠ABC
This property is known as exterior angle property of a triangle.

Example:

Find the value of *x* in the following figure.



Solution:

∠QRS is an exterior angle of ΔPQR. It is thus equal to the sum of its interior opposite angles.

∴∠QRS = ∠QPR + ∠PQR

⇒ $x = 65° + 70° = 135°$

Thus, the value of x is 135°.

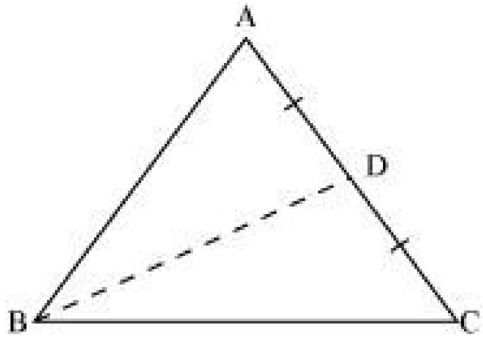
- Two exterior angles can be drawn at each vertex of triangle. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.

- A triangle is a simple closed curve made up of three line segments.

  It has three vertices, three sides and three angles.

- Triangles can be classified on the basis of their sides as:
  1. Scalene – No side of the triangle is equal
  2. Isosceles – Exactly two sides of the triangle are equal
  3. Equilateral – All the sides of the triangle are equal
- On the basis of angles, triangles can be classified as:
  1. Acute-angled – All the angles of the triangle are less than 90°
  2. Obtuse-angled – Any one of the angles of the triangle is greater than 90°
  3. Right-angled – Any one of the angles of the triangle is 90°
- Median of a triangle

  A median is a line segment joining the vertex of a triangle to the mid-point of the opposite side.

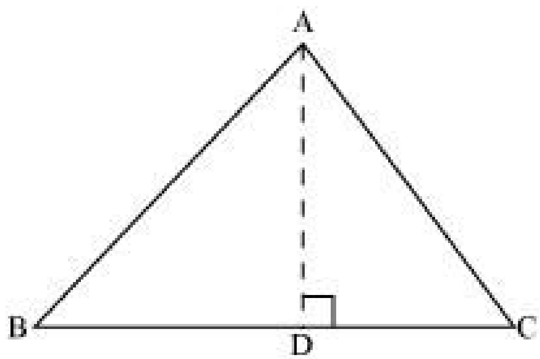


In the given ΔABC, if AD = DC, then BD is the median of ΔABC with respect to the side AC.

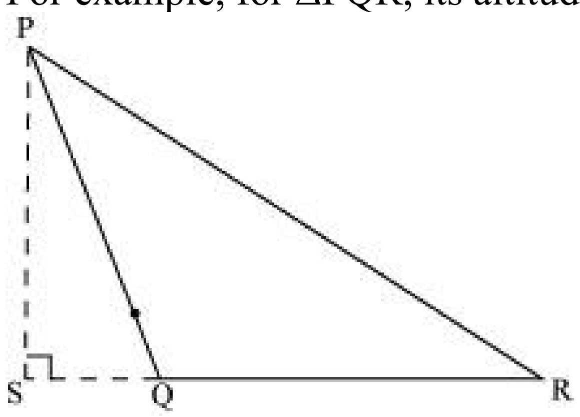
A triangle has three medians, one for each side.

- Altitude of a triangle

  An altitude is the perpendicular drawn from the vertex of a triangle to its opposite side.



In the given figure, AD is the altitude of ΔABC with respect to side BC.
A triangle has three altitudes, one from each vertex.
The altitude of a triangle may or may not lie inside the triangle.
For example, for ΔPQR, its altitude lies outside it.

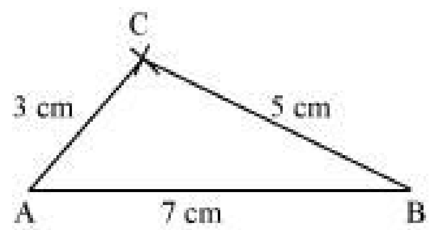


- A triangle can be constructed if all its sides are known.

  Example:
  Construct a triangle whose sides are 3 cm, 5cm and 7 cm.

Solution:

1. Draw a line segment AB of length 7 cm. With A as centre and radius equal to 3 cm, draw an arc.
2. With B as centre and radius 5 cm, draw another arc cutting the earlier drawn arc at C.
3. Join AC and BC to get ΔABC.

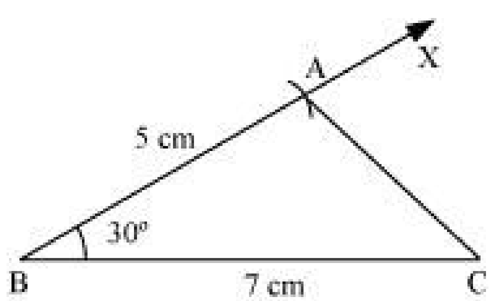


- A triangle can be constructed if the length of two sides and angle between them are given.

Example:
Construct ΔABC where BC = 7 cm, AB = 5 cm and ∠ABC = 30°

Solution:

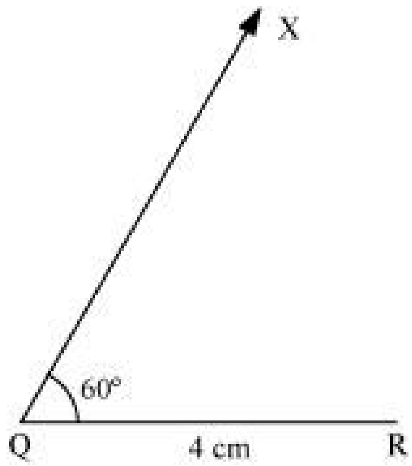
1. Draw a line segment BC of length 7 cm and at B draw a ray BX, making an angle of 30° with BC.
2. With B as centre and radius equal to 5 cm, draw an arc cutting BX at A.
3. Join AC to get the required ΔABC.



Example: Construct ΔPQR, where ∠PQR = 60°, ∠PRQ = 45° and QR = 4 cm.

Solution:

1. Draw a line segment QR of length 4 cm and draw a ray QX, making an angle of 60° with QR



2. Now, draw ray RY, making an angle of 45° with QR and intersecting QX at P. The resulting ΔPQR is the required triangle.