

Ex 5.1

Q1

$$\begin{aligned}\text{LHS} &= \sec^4 \theta - \sec^2 \theta \\ &= \sec^2 \theta (\sec^2 \theta - 1) \\ &= (1 + \tan^2 \theta) \tan^2 \theta && [\because \sec^2 \theta = 1 + \tan^2 \theta] \\ &= \tan^2 \theta + \tan^4 \theta \\ &= \tan^4 \theta + \tan^2 \theta \\ &= \text{RHS} \\ \text{LHS} &= \text{RHS} \\ &\text{Proved}\end{aligned}$$

Q2

$$\begin{aligned}\text{LHS} &= \sin^6 \theta + \cos^6 \theta \\ &= (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\ &= (\sin^2 \theta + \cos^2 \theta) \left[(\sin^2 \theta)^2 - \sin^2 \theta \cos^2 \theta + (\cos^2 \theta)^2 \right] && (\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)) \\ &= (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta \\ &&& \left[\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \text{ and} \right. \\ &&& \left. \text{using identity } \sin^2 \theta + \cos^2 \theta = 1 \right] \\ &= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\ &= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\ &= 1^2 - 3 \sin^2 \theta \cos^2 \theta \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= 1 - 3 \sin^2 \theta \cos^2 \theta \\ &= \text{RHS} \\ \therefore \quad \text{LHS} &= \text{RHS} \\ &\text{Proved}\end{aligned}$$

Q3

$$\text{LHS} = (\cos \text{csc} \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta)$$

$$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right)$$

$$= \frac{\cos^2 \theta \cdot \sin^2 \theta \cdot 1}{\sin^2 \theta \cdot \cos^2 \theta} \left(\begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \Rightarrow 1 - \sin^2 \theta = \cos^2 \theta, \text{ and} \\ 1 - \cos^2 \theta = \sin^2 \theta \end{array} \right)$$

$$= 1$$

$$= \text{RHS}$$

Proved

$$\left[\begin{array}{l} \because \cos \text{csc} \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \\ \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right]$$

Q4

$$\text{LHS} = \cos \text{csc} \theta (\sec \theta - 1) - \cot \theta (1 - \cos \theta)$$

$$= \frac{1}{\sin \theta} \left(\frac{1}{\cos \theta} - 1 \right) - \frac{\cos \theta}{\sin \theta} (1 - \cos \theta)$$

$$= \frac{(1 - \cos \theta)}{\sin \theta \cos \theta} - \frac{\cos \theta (1 - \cos \theta)}{\sin \theta}$$

$$= \frac{(1 - \cos \theta) - \cos^2 \theta (1 - \cos \theta)}{\sin \theta \cos \theta}$$

$$= \frac{(1 - \cos \theta)(1 - \cos^2 \theta)}{\sin \theta \cos \theta}$$

$$= \frac{(1 - \cos \theta) \sin^2 \theta}{\sin \theta \cos \theta} \quad (\because 1 - \cos^2 \theta = \sin^2 \theta)$$

$$= (1 - \cos \theta) \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} - \sin \theta$$

$$= \tan \theta - \sin \theta \quad (\because \tan \theta = \frac{\sin \theta}{\cos \theta})$$

$$= \text{RHS}$$

Proved

$$\left[\because \cos \text{csc} \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

Q5

$$\begin{aligned}
 \text{LHS} &= \frac{1 - \sin A \cos A}{\cos A (\sec A - \csc A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^2 A + \cos^2 A} \\
 &= \frac{1 - \sin A \cos A}{\cos A \left(\frac{1}{\cos A} - \frac{1}{\sin A} \right)} \cdot \frac{(\sin A - \cos A)(\sin A + \cos A)}{(\sin A - \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)} \\
 &= \frac{(1 - \sin A \cos A)}{\cos A \left(\frac{\sin A - \cos A}{\cos A \sin A} \right)} \cdot \frac{(\sin A - \cos A)}{(1 - \sin A \cos A)} \quad \left(\because \sin^2 A + \cos^2 A = 1 \right) \\
 &= \frac{\cos A \sin A}{\cos A} \\
 &= \sin A \\
 &\text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

$\left\{ \begin{aligned} \text{Using } a^2 - b^2 &= (a-b)(a+b) \\ \text{and } a^2 + b^2 &= (a+b)(a^2 - b^2 - ab) \end{aligned} \right.$

Q6

$$\begin{aligned}
 \text{LHS} &= \frac{\tan A}{1 - \cot^2 A} + \frac{\cot A}{1 - \tan^2 A} \\
 &= \frac{(\sin A / \cos A)}{\left(1 - \frac{\cos^2 A}{\sin^2 A} \right)} + \frac{(\cos A / \sin A)}{\left(1 - \frac{\sin^2 A}{\cos^2 A} \right)} \\
 &= \frac{\sin A}{\cos A \left(\frac{\sin^2 A - \cos^2 A}{\sin^2 A} \right)} + \frac{\cos A}{\sin A \left(\frac{\cos^2 A - \sin^2 A}{\cos^2 A} \right)} \\
 &= \frac{\sin^3 A}{\cos A (\sin^2 A - \cos^2 A)} + \frac{\cos^3 A}{\sin A (\cos^2 A - \sin^2 A)} \\
 &= \frac{\sin^3 A - \cos^3 A}{\cos A \sin A (\sin^2 A - \cos^2 A)} \\
 &= \frac{(\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\cos A \sin A (\sin^2 A - \cos^2 A)} \quad \left\{ \text{Using } a^3 - b^3 = (a-b)(a^2 + b^2 + ab) \right\} \\
 &= \frac{1 + \sin A \cos A}{\cos A \sin A} \quad \left\{ \because \sin^2 A + \cos^2 A = 1 \right\} \\
 &= \frac{1}{\cos A \sin A} + \frac{\cos A \sin A}{\cos A \sin A} \\
 &= \sec A \csc A + 1 \\
 &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

Q7

$$\begin{aligned} \text{LHS} &= \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} \\ &= \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{(\sin A - \cos A)} \\ &\quad \left(\text{Using } a^3 + b^3 = (a+b)(a^2 + b^2 - ab) \text{ and } a^3 - b^3 = (a-b)(a^2 + b^2 + ab) \right) \\ &= (1 - \sin A \cos A) + (1 + \sin A \cos A) \quad \left(\because \sin^2 A + \cos^2 A = 1 \right) \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Q8

$$\begin{aligned} \text{LHS} &= (\sec A \sec B + \tan A \tan B)^2 - (\sec A \tan B + \tan A \sec B)^2 \\ &= (\sec A \sec B)^2 + (\tan A \tan B)^2 + 2 \sec A \sec B \tan A \tan B \\ &\quad - \left((\sec A \tan B)^2 + (\tan A \sec B)^2 + 2 \sec A \tan B \tan A \sec B \right) \quad \left[\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \right] \\ &= \sec^2 A \sec^2 B + \tan^2 A \tan^2 B + 2 \sec A \sec B \tan A \tan B \\ &\quad - \sec^2 A \tan^2 B - \tan^2 A \sec^2 B - 2 \sec A \sec B \tan A \tan B \quad \left[\text{Using } (ab)^2 = a^2 b^2 \right] \\ &= \sec^2 A \sec^2 B - \sec^2 A \tan^2 B + \tan^2 A \tan^2 B - \tan^2 A \sec^2 B \\ &= \sec^2 A (\sec^2 B - \tan^2 B) + \tan^2 A (\tan^2 B - \sec^2 B) \\ &= \sec^2 A \cdot 1 - \tan^2 A \cdot 1 \quad \left[\begin{array}{l} \because \sec^2 \theta = 1 + \tan^2 \theta \\ \Rightarrow \sec^2 \theta - \tan^2 \theta = 1 \end{array} \right] \\ &= 1 + \tan^2 A - \tan^2 A \\ &= 1 \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q9

$$\text{RHS} = \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta}$$

$$= \frac{((1 + \cos \theta) + \sin \theta)}{(1 + \cos \theta) - \sin \theta} \times \frac{((1 + \cos \theta) + \sin \theta)}{(1 + \cos \theta + \sin \theta)}$$

$$= \frac{((1 + \cos \theta) + \sin \theta)^2}{(1 + \cos \theta)^2 - \sin^2 \theta}$$

$$\left(\begin{array}{l} \text{Using } (a+b)(a+b) = (a+b)^2 \\ \text{\& } (a+b)(a-b) = a^2 - b^2 \end{array} \right)$$

$$= \frac{(1 + \cos \theta)^2 + \sin^2 \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - \sin^2 \theta}$$

$$\left(\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \right)$$

$$= \frac{1 + \cos^2 \theta + 2 \cdot 1 \cos \theta + \sin^2 \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - (1 - \cos^2 \theta)}$$

$$\left(\text{Using } \sin^2 \theta = 1 - \cos^2 \theta \right)$$

$$= \frac{1 + 1 + 2 \cos \theta + 2 \sin \theta (1 + \cos \theta)}{1 - 1 + \cos^2 \theta + \cos^2 \theta + 2 \cos \theta}$$

$$\left(\text{Using } \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$= \frac{2 + 2 \cos \theta + 2 \sin \theta (1 + \cos \theta)}{2 \cos^2 \theta + 2 \cos \theta}$$

$$= \frac{2(1 + \cos \theta) + 2 \sin \theta (1 + \cos \theta)}{2 \cos \theta (\cos \theta + 1)}$$

$$= \frac{(1 + \cos \theta)(2 + 2 \sin \theta)}{2 \cos \theta (1 + \cos \theta)}$$

$$= \frac{1 + \sin \theta}{\cos \theta}$$

$$= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$= \frac{\cos \theta}{1 - \sin \theta}$$

Q10

$$\text{LHS} = \frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 - \cot^2 \theta}$$

$$= \frac{\sin^3 \theta}{\cos^3 \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)} + \frac{\cos^3 \theta}{\sin^3 \theta \left(1 - \frac{\cos^2 \theta}{\sin^2 \theta}\right)}$$

$$\left(\begin{array}{l} \therefore \tan \theta = \frac{\sin \theta}{\cos \theta} \\ \text{and } \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right)$$

$$= \frac{\sin^3 \theta \cos^2 \theta}{\cos^3 \theta (\cos^2 \theta + \sin^2 \theta)} + \frac{\cos^3 \theta \sin^2 \theta}{\sin^3 \theta (\sin^2 \theta + \cos^2 \theta)}$$

$$= \frac{\sin^5 \theta}{\cos \theta} + \frac{\cos^5 \theta}{\sin \theta}$$

$$(\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$= \frac{\sin^4 \theta + \cos^4 \theta}{\sin \theta \cos \theta}$$

$$= \frac{(\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta} \quad (\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta)$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1^2 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

= RHS

Proved

Q11

$$\text{LHS} = 1 - \frac{\sin^2 \theta}{1 + \cot \theta} - \frac{\cos^2 \theta}{1 + \tan \theta}$$

$$= 1 - \frac{\sin^2 \theta}{1 + \frac{\cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{1 + \frac{\sin \theta}{\cos \theta}} \left(\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right)$$

$$= 1 - \frac{\sin^2 \theta}{\frac{\sin \theta + \cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{\frac{\cos \theta + \sin \theta}{\cos \theta}}$$

$$= 1 - \frac{\sin^3 \theta}{\sin \theta + \cos \theta} - \frac{\cos^3 \theta}{\cos \theta + \sin \theta}$$

$$= \frac{\sin \theta + \cos \theta - (\sin^3 + \cos^3 \theta)}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta + \cos \theta - (\sin \theta + \cos \theta)(\sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta)}{\sin \theta + \cos \theta}$$

$$\left(\text{Using } a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \right)$$

$$\frac{(\sin \theta + \cos \theta)(1 - (1 - \sin \theta \cos \theta))}{\sin \theta + \cos \theta}$$

$$\left(\text{Using } \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$= \sin \theta \cos \theta$$

$$= \text{RHS}$$

Proved

Q12

$$\text{LHS} = \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\csc^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta$$

$$= \left(\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\left(\frac{1}{\sin^2 \theta} - \sin^2 \theta \right)} \right) \sin^2 \theta \cos^2 \theta$$

$$= \left(\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right) \sin^2 \theta \cos^2 \theta$$

$$= \left(\frac{\cos^2 \theta}{(1 - \cos^2 \theta)(1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{(1 - \sin^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta$$

$$\left(\begin{array}{l} \text{Using } 1 - a^4 = 1 - (a^2)^2 \\ \quad - (1 - a^2)(1 + a^2) \end{array} \right)$$

$$= \left(\frac{\cos^2 \theta}{\sin^2 \theta (1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{\cos^2 \theta (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta$$

$$\left(\begin{array}{l} \text{Using } 1 - \cos^2 \theta = \sin^2 \theta \\ \quad \& \quad 1 - \sin^2 \theta = \cos^2 \theta \end{array} \right)$$

$$= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 - \cos^2 \theta)}{\sin^2 \theta \cos^2 \theta (1 + \cos^2 \theta) (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta$$

$$= \frac{\cos^4 \theta + \sin^2 \theta \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta) (1 + \sin^2 \theta)}$$

$$= \frac{(\cos^2 \theta)^2 + (\sin^2 \theta)^2 + 2\cos^2 \theta \sin^2 \theta - 2\cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta) (1 + \sin^2 \theta)}$$

(adding and subtracting $2\cos^2 \theta \sin^2 \theta$)

$$= \frac{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{1 + \sin^2 \theta + \cos^2 \theta + \sin^2 \theta \cos^2 \theta}$$

$$= \frac{1^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta \cdot 1}{1 + 1 + \sin^2 \theta \cos^2 \theta}$$

$$= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}$$

= RHS

Proved

Q13

$$\text{LHS} = (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2$$

$$= 1 + (\tan \alpha + \tan \beta)^2 + 2 \cdot 1 \tan \alpha \tan \beta + (\tan \alpha)^2 + (\tan \beta)^2 - 2 \tan \alpha \cdot \tan \beta$$

(Using $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$)

$$= 1 + \tan^2 \alpha + \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta$$

$$= 1 + \tan^2 \alpha + \tan^2 \alpha + \tan^2 \beta + \tan^2 \beta$$

$$= \sec^2 \alpha + \tan^2 \beta (1 + \tan^2 \alpha) \quad (\because 1 + \tan^2 \alpha = \sec^2 \alpha)$$

$$= \sec^2 \alpha + \tan^2 \beta \cdot \sec^2 \alpha$$

$$= \sec^2 \alpha (1 + \tan^2 \beta)$$

$$= \sec^2 \alpha \cdot \sec^2 \beta \quad (\because 1 + \tan^2 \beta = \sec^2 \beta)$$

= RHS

Proved

Q14

$$\text{LHS} = \frac{(1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta)}{\sec^3 \theta - \operatorname{cosec}^3 \theta}$$

$$= \frac{\left(1 + \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}\right)}{\left(\frac{1}{\cos^3 \theta} - \frac{1}{\sin^3 \theta}\right)}$$

$$= \frac{\left(1 + \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}\right)}{\left(\frac{\sin^3 \theta - \cos^3 \theta}{\cos^3 \theta \sin^3 \theta}\right)} (\sin \theta - \cos \theta)$$

$$= \frac{(\sin \theta \cos \theta + 1) \sin^3 \theta \cos^3 \theta}{\sin \theta \cos \theta (\sin^3 \theta - \cos^3 \theta)} (\sin \theta - \cos \theta)$$

$$= \frac{(1 + \sin \theta \cos \theta) \sin^2 \theta \cos^2 \theta (\sin \theta - \cos \theta)}{(\sin \theta - \cos \theta) (\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}$$

$$= \frac{(1 + \sin \theta \cos \theta) \cdot \sin^2 \theta \cos^2 \theta}{(1 + \sin \theta \cos \theta)}$$

$$= \sin^2 \theta \cos^2 \theta$$

= RHS

Proved

$$\left(\begin{array}{l} \because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \\ \sec \theta = \frac{1}{\cos \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} \end{array} \right)$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$(\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab))$$

Q15

$$\text{LHS} = \frac{2 \sin \theta \cos \theta - \cos \theta}{1 - \sin \theta + \sin^2 \theta - \cos^2 \theta}$$

$$= \frac{\cos \theta (2 \sin \theta - 1)}{1 - \cos^2 \theta + \sin^2 \theta - \sin \theta}$$

$$= \frac{\cos \theta (2 \sin \theta - 1)}{\sin^2 \theta + \sin^2 \theta - \sin \theta} \quad (\because 1 - \cos^2 \theta = \sin^2 \theta)$$

$$= \frac{\cos \theta (2 \sin \theta - 1)}{2 \sin^2 \theta - \sin \theta}$$

$$= \frac{\cos \theta (2 \sin \theta - 1)}{\sin \theta (2 \sin \theta - 1)}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta$$

$$= \text{RHS}$$

Proved

Q16

$$\begin{aligned} \text{LHS} &= \cos \theta (\tan \theta + 2) (2 \tan \theta + 1) \\ &= \cos \theta \left(\frac{\sin \theta}{\cos \theta} + 2 \right) \left(\frac{2 \sin \theta}{\cos \theta} + 1 \right) \left(\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right) \\ &= \cos \frac{(\sin \theta + 2 \cos \theta) (2 \sin \theta + \cos \theta)}{\cos \theta \cdot \cos \theta} \\ &= \frac{(2 \sin^2 \theta + \sin \theta \cos \theta + 4 \sin \theta \cos \theta + 2 \cos^2 \theta)}{\cos \theta} \\ &= \frac{2 (\sin^2 \theta + \cos^2 \theta) + 5 \sin \theta \cos \theta}{\cos \theta} \\ &= \frac{2 + 5 \sin \theta \cos \theta}{\cos \theta} \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= \frac{2}{\cos \theta} + \frac{5 \sin \theta \cos \theta}{\cos \theta} \\ &= 2 \sec \theta + 5 \sin \theta \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q17

$$\frac{2 \sin \theta}{1 + \cos \theta + \sin \theta} = x$$

$$\rightarrow \frac{2 \sin \theta(1 - \cos \theta + \sin \theta)}{(1 + \cos \theta - \sin \theta)(1 - \cos \theta + \sin \theta)} = x \quad [\text{Rationalizing the denominator}]$$

$$\Rightarrow \frac{2 \sin \theta(1 - \cos \theta + \sin \theta)}{(1 - \sin \theta)^2 - \cos^2 \theta} = x$$

$$\rightarrow \frac{2 \sin \theta - 2 \sin \theta \cos \theta + 2 \sin^2 \theta}{1 + \sin^2 \theta + 2 \sin \theta - \cos^2 \theta} = x$$

$$\rightarrow \frac{2 \sin \theta(1 + \cos \theta - \sin \theta)}{2 \sin^2 \theta + 2 \sin \theta} = x$$

$$\rightarrow \frac{2 \sin \theta(1 + \cos \theta - \sin \theta)}{2 \sin \theta(1 + \sin \theta)} = x$$

$$\Rightarrow \frac{1 + \cos \theta - \sin \theta}{1 + \sin \theta} = x \quad [\text{Cancelling the } 2 \sin \theta \text{ in both Numerator and Denominator}]$$

Hence Proved

Q18

$$\text{Now, } \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$= \sqrt{\frac{1 - (a^2 - b^2)^2}{(a^2 + b^2)^2}} \quad \left[\because \sin \theta = \frac{a^2 - b^2}{a^2 + b^2} \right]$$

$$= \sqrt{\frac{(a^2 + b^2)^2 - (a^2 - b^2)^2}{(a^2 + b^2)^2}}$$

$$= \sqrt{\frac{(a^2 + b^2 + a^2 - b^2)(a^2 + b^2 - a^2 + b^2)}{a^2 + b^2}} \quad (\text{Using } x^2 - y^2 = (x - y)(x + y))$$

$$= \sqrt{\frac{2a^2 \times 2b^2}{a^2 + b^2}}$$

$$= \frac{2ab}{a^2 + b^2} \dots \dots \dots (ii)$$

$$\text{Now } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{a^2 - b^2}{\frac{a^2 + b^2}{\frac{2ab}{a^2 + b^2}}}$$

$$= \frac{a^2 - b^2}{2ab}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{a^2 + b^2}{2ab} \quad (\text{from (ii)})$$

$$\text{and } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{a^2 + b^2}{a^2 - b^2} \quad (\text{from (i)})$$

Q19

$$\begin{aligned} & \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \\ &= \sqrt{\frac{\frac{a}{b} + 1}{\frac{a}{b} - 1}} + \sqrt{\frac{\frac{a}{b} - 1}{\frac{a}{b} + 1}} \quad [\text{Dividing both Numerator and denominator by } b] \\ &= \sqrt{\frac{\tan \theta + 1}{\tan \theta - 1}} + \sqrt{\frac{\tan \theta - 1}{\tan \theta + 1}} \\ &= \sqrt{\frac{\frac{\sin \theta}{\cos \theta} + 1}{\frac{\sin \theta}{\cos \theta} - 1}} + \sqrt{\frac{\frac{\sin \theta}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta} + 1}} \\ &= \sqrt{\frac{\frac{\sin \theta + \cos \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\cos \theta}}} + \sqrt{\frac{\frac{\sin \theta - \cos \theta}{\cos \theta}}{\frac{\sin \theta + \cos \theta}{\cos \theta}}} \\ &= \sqrt{\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}} + \sqrt{\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}} \\ &= \frac{\sin \theta + \cos \theta + \sin \theta - \cos \theta}{\sqrt{\sin^2 \theta - \cos^2 \theta}} \\ &= \frac{2 \sin \theta}{\sqrt{\sin^2 \theta - \cos^2 \theta}} \end{aligned}$$

Q20

$$\text{Given} = \tan \theta = \frac{a}{b}$$

$$\text{To show: } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\text{Since, } \tan \theta = \frac{a}{b}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{a}{b}$$

$$\Rightarrow b \sin \theta = a \cos \theta = \lambda \text{ (say)}$$

$$\Rightarrow \sin \theta = \frac{\lambda}{b} \text{ and } \cos \theta = \frac{\lambda}{a}$$

$$\text{how } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{\frac{a.\lambda}{b} - \frac{b.\lambda}{a}}{\frac{a.\lambda}{b} + \frac{b.\lambda}{a}}$$

$$= \frac{\lambda \left(\frac{a}{b} - \frac{b}{a} \right)}{\lambda \left(\frac{a}{b} + \frac{b}{a} \right)}$$

$$= \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{b} + \frac{b}{a}}$$

$$= \frac{\frac{a^2 - b^2}{ab}}{\frac{a^2 + b^2}{ab}}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Proved

Q21

Given, $\operatorname{cosec} \theta - \sin \theta = a^3$, $\sec \theta - \cos \theta = b^3$

To show: $a^2 b^2 (a^2 + b^2) = 1$

Since, $\operatorname{cosec} \theta - \sin \theta = a^3$

$$\Rightarrow \frac{1}{\sin \theta} - \sin \theta = a^3 \quad \left(\because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right)$$

$$\Rightarrow \frac{1 - \sin^2 \theta}{\sin \theta} = a^3$$

$$\Rightarrow \frac{\cos^2 \theta}{\sin \theta} = a^3 \quad \left(\because 1 - \sin^2 \theta = \cos^2 \theta \right)$$

$$\Rightarrow a = \frac{\cos^{2/3} \theta}{\sin^{1/3} \theta}$$

Since, $\frac{1}{\cos \theta} - \cos \theta = b^3 \quad \left(\because \sec \theta = \frac{1}{\cos \theta} \right)$

$$\Rightarrow \frac{1 - \cos^2 \theta}{\cos \theta} = b^3$$

$$\Rightarrow \frac{\sin^2 \theta}{\cos \theta} = b^3 \quad \left(\because 1 - \cos^2 \theta = \sin^2 \theta \right)$$

$$\Rightarrow b = \frac{\sin^{2/3} \theta}{\cos^{1/3} \theta}$$

$$\text{Now, } a^2 b^2 (a^2 + b^2) = \frac{\cos^{4/3} \theta}{\sin^{2/3} \theta} \times \frac{\sin^{4/3} \theta}{\cos^{2/3} \theta} \left(\frac{\cos^{4/3} \theta}{\sin^{2/3} \theta} + \frac{\sin^{4/3} \theta}{\cos^{2/3} \theta} \right)$$

$$= \cos^{2/3} \theta \times \sin^{2/3} \theta \frac{(\cos^{6/3} \theta + \sin^{6/3} \theta)}{\sin^{2/3} \theta \cos^{2/3} \theta}$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

Proved

Q22

Let,

$$\cot \theta (1 + \sin \theta) = 4m \quad \text{---(i)}$$

$$\text{and, } \cot \theta (1 - \sin \theta) = 4n \quad \text{---(ii)}$$

To show: $(m^2 - n^2)^2 = mn$

From (i) and(ii), we get

$$m = \frac{\cot \theta (1 + \sin \theta)}{4} \quad \& \quad n = \frac{\cot \theta (1 - \sin \theta)}{4}$$

$$\begin{aligned} \text{LHS} &= (m^2 - n^2)^2 \\ &= ((m+n)(m-n))^2 \\ &= (m+n)^2 (m-n)^2 \\ &= \left(\frac{\cot \theta (1 + \sin \theta) + \cot \theta (1 - \sin \theta)}{4} \right)^2 \left(\frac{\cot \theta (1 + \sin \theta) - \cot \theta (1 - \sin \theta)}{4} \right)^2 \\ &= \left(\frac{\cot \theta (1 + \sin \theta + 1 - \sin \theta)}{4} \right)^2 \times \left(\frac{\cot \theta (1 + \sin \theta - 1 + \sin \theta)}{4} \right)^2 \\ &= \frac{\cot^2 \theta}{16} \times 4 \times \frac{\cot^2 \theta}{16} \times 4 \sin^2 \theta \\ &= \frac{\cot^2 \theta}{16} \times \frac{\cos^2 \theta}{\sin^2 \theta} \sin^2 \theta && \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\ &= \frac{\cot \theta}{4} \times \frac{\cot \theta}{4} \times (1 - \sin^2 \theta) && \left[\because \cos^2 \theta = 1 - \sin^2 \theta \right] \\ &= \frac{\cot \theta (1 + \sin \theta)}{4} \times \frac{\cot \theta (1 - \sin \theta)}{4} \\ &= mn \end{aligned}$$

Q23

To show: $\sin^6 \theta + \cos^6 \theta = \frac{4 - 3(m^2 - 1)^2}{4}$, where $m^2 \leq 2$

Since, $\sin \theta + \cos \theta = m$... (i)

$$\Rightarrow (\sin \theta + \cos \theta)^2 = m^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = m^2$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = m^2 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\Rightarrow 2 \sin \theta \cos \theta = m^2 - 1$$

$$\Rightarrow \sin \theta \cos \theta = \frac{m^2 - 1}{2} \quad \dots \text{(ii)}$$

$$\therefore \text{LHS} = \sin^6 \theta + \cos^6 \theta$$

$$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta$$

$$= 1 \cdot \left((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta \right)$$

(adding and subtracting $2 \sin^2 \theta \cos^2 \theta$)

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3(\sin \theta \cos \theta)^2$$

$$= 1 - 3 \frac{(m^2 - 1)^2}{4} \quad (\text{from (ii)})$$

$$= \frac{4 - 3(m^2 - 1)^2}{4}, \text{ where } m^2 \leq 2$$

= RHS

Proved

Q24

$$\begin{aligned}
 \text{LHS} &= a^2 - a - b + 1 \\
 &= (\sec \theta - \tan \theta)(\sec \theta + \cot \theta) + \sec \theta - \tan \theta - \csc \theta - \cot \theta + 1 \\
 &= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right) \left(\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right) + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1 \\
 &= \frac{1}{\sin \theta \cos \theta} - \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} - \tan \theta \times \cot \theta + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1 \\
 &= \frac{1}{\sin \theta \cos \theta} - \frac{1}{\sin \theta} - \frac{1}{\cos \theta} - 1 + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1 \\
 &= \frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \frac{1 - \sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{1 - (\cos^2 \theta + \sin^2 \theta)}{\sin \theta \cos \theta} \\
 &= \frac{1 - 1}{\sin \theta \cos \theta} = 0 = \text{RHS. Hence Proved}
 \end{aligned}$$

Q25

$$\begin{aligned}
 \text{LHS} &= \left| \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} + \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} \right| \\
 &= \left| \frac{(\sqrt{1 - \sin \theta})^2 + (\sqrt{1 + \sin \theta})^2}{\sqrt{(1 + \sin \theta)(1 - \sin \theta)}} \right| \\
 &= \left| \frac{1 - \sin \theta + 1 + \sin \theta}{\sqrt{1 - \sin^2 \theta}} \right| \\
 &= \left| \frac{2}{\cos \theta} \right| \quad \left(\because 1 - \sin^2 \theta = \cos^2 \theta \Rightarrow \sqrt{1 - \sin^2 \theta} = \cos \theta \right) \\
 &= \frac{-2}{\cos \theta} \quad \left(\because \frac{\pi}{2} < \theta < \pi \Rightarrow \cos \theta < 0 \right) \\
 &= \text{RHS}
 \end{aligned}$$

Q26

We have,

$$T_n = \sin^n \theta + \cos^n \theta \quad (i)$$

To show: $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$

$$\begin{aligned} \text{LHS} &= \frac{T_3 - T_5}{T_1} \\ &= \frac{(\sin^3 \theta + \cos^3 \theta) - (\sin^5 \theta + \cos^5 \theta)}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta - \sin^5 \theta + \cos^3 \theta - \cos^5 \theta}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta (1 - \sin^2 \theta) + \cos^3 \theta (1 - \cos^2 \theta)}{\sin \theta + \cos \theta} \\ &= \frac{\sin^3 \theta \cos^2 \theta + \cos^3 \theta \sin^2 \theta}{\sin \theta + \cos \theta} \\ &= \frac{\sin^2 \theta \cos^2 \theta + (\sin \theta + \cos \theta)}{\sin \theta + \cos \theta} \\ &= \sin^2 \theta \cos^2 \theta \end{aligned}$$

[Substituting the values of T_3 , T_5 and T_1 from (i)]

[$\because 1 - \sin^2 \theta = \cos^2 \theta$
and $1 - \cos^2 \theta = \sin^2 \theta$]

$$\begin{aligned} \text{RHS} &= \frac{\sin^5 \theta + \cos^5 \theta - (\sin^7 \theta + \cos^7 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta - \sin^7 \theta + \cos^5 \theta - \cos^7 \theta}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta (1 - \sin^2 \theta) + \cos^5 \theta (1 - \cos^2 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^5 \theta \cos^2 \theta + \cos^5 \theta \sin^2 \theta}{\sin^3 \theta + \cos^3 \theta} \\ &= \frac{\sin^2 \theta \cos^2 \theta (\sin^3 \theta + \cos^3 \theta)}{\sin^3 \theta + \cos^3 \theta} \\ &= \sin^2 \theta \cos^2 \theta \end{aligned}$$

\therefore LHS = RHS Proved.

$$\text{LHS} = 27_6 - 37_4 + 1$$

$$= 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$= 2\left((\sin^2 \theta)^3 + (\cos^2 \theta)^3 - 3(\sin^2 \theta)^2 + (\cos^2 \theta)^2\right) + 1$$

$$= 2\left((\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta)^2 - (\cos^2 \theta)^2 - (\sin^2 \theta \cos^2 \theta)\right) -$$

$$3\left((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta\right) + 1$$

$$\left[\begin{array}{l} \text{Using } a^3 + b^3 = (a+b)(a^2 - b^2 - ab) \text{ and adding and subtracting} \\ 2 \sin^2 \theta \cos^2 \theta \end{array} \right]$$

$$= 2\left((\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta\right) - 3(1 - 2 \sin^2 \theta \cos^2 \theta) + 1$$

$$= 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3 + 6 \sin^2 \theta \cos^2 \theta + 1$$

$$= 2 - 6 \sin^2 \theta \cos^2 \theta - 2 + 6 \sin^2 \theta \cos^2 \theta$$

$$= 0$$

$$= \text{RHS Proved.}$$

$$\text{LHS} = 67_{10} - 157_8 + 107_6 - 1$$

$$= 6(\sin^{10}\theta + \cos^{10}\theta) - 15(\sin^8\theta + \cos^8\theta) + 10(\sin^6\theta + \cos^6\theta) - 1$$

$$= 6\sin^{10}\theta - 15\sin^8\theta + 10\sin^6\theta + 6\cos^{10}\theta - 15\cos^8\theta + 10\cos^6\theta - 1$$

$$= \sin^6\theta(6\sin^4\theta - 15\sin^2\theta + 10) + \cos^6\theta(6\cos^4\theta - 15\cos^2\theta + 10) - (\sin^2\theta + \cos^2\theta)^3$$

$$[\because 1 = \sin^2\theta + \cos^2\theta]$$

$$= \sin^6\theta(6\sin^4\theta - 15\sin^2\theta + 10) + \cos^6\theta(6\cos^4\theta - 15\cos^2\theta + 10)$$

$$- (\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta))$$

$$[\text{Using } (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$= \sin^6\theta(6\sin^4\theta - 15\sin^2\theta + 10 - 1) + \cos^6\theta(6\cos^4\theta - 15\cos^2\theta + 10 - 1) - 3\sin^2\theta\cos^2\theta \times 1$$

$$[\because \cos^2\theta + \sin^2\theta = 1]$$

$$= \sin^6\theta(6\sin^4\theta - 14\sin^2\theta + 9) + \cos^6\theta(6\cos^4\theta - 14\cos^2\theta + 9) - 3\sin^2\theta\cos^2\theta$$

$$[\text{On splitting the middle term}]$$

$$= \sin^6\theta(3\sin^2\theta(2\sin^2\theta - 3) - 3(2\sin^2\theta - 3)) + \cos^6\theta(3\cos^2\theta(2\cos^2\theta - 3) - 3(2\cos^2\theta - 3)) - 3\sin^2\theta\cos^2\theta$$

$$= \sin^6\theta(2\sin^2\theta - 3)(3\sin^2\theta - 3) + \cos^6\theta(2\cos^2\theta - 3)(3\cos^2\theta - 3) - 3\sin^2\theta\cos^2\theta$$

$$= \sin^6\theta \times (-3)(2\sin^2\theta - 3)(1 - \sin^2\theta) + \cos^6\theta \times (-3)(2\cos^2\theta - 3)(1 - \cos^2\theta) - 3\sin^2\theta\cos^2\theta$$

$$= -3\sin^6\theta(2\sin^2\theta - 3)\cos^2\theta - 3\cos^6\theta(2\cos^2\theta - 3)\sin^2\theta - 3\sin^2\theta\cos^2\theta$$

$$= 6\sin^0\theta + \cos^2\theta + 6\sin^6\theta\cos^2\theta - 6\cos^6\theta\sin^2\theta + 9\cos^6\theta + \sin^2\theta - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta + \cos^2\theta(\sin^6\theta - \cos^6\theta) + 9\sin^2\theta\cos^2\theta(\sin^4\theta + \cos^4\theta) - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta\cos^2\theta((\sin^2\theta)^3 + (\cos^2\theta)^3) + 9\sin^2\theta\cos^2\theta((\sin^2\theta)^2 + (\cos^2\theta)^2) - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$$

$$+ 9\sin^2\theta\cos^2\theta(\sin^4\theta + \cos^4\theta) - 3\sin^2\theta\cos^2\theta$$

$$[\text{Using } a^3 - b^3 = (a+b)(a^2 + b^2 - ab)]$$

$$= -6\sin^2\theta\cos^2\theta(\sin^4\theta\cos^4\theta - \sin^2\theta\cos^2\theta) + 9\sin^2\theta\cos^2\theta(\sin^4\theta + \cos^4\theta)$$

$$- 3\sin^2\theta\cos^2\theta \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

$$\begin{aligned}
&= -6 \sin^2 \theta \cos^2 \theta (\sin^4 \theta + \cos^4 \theta) - 6 \sin^4 \theta \cos^4 \theta + 9 \sin^2 \theta \cos^2 \theta (\sin^4 \theta + \cos^4 \theta) - 3 \sin^2 \theta \cos^2 \theta \\
&= 3 \sin^2 \theta \cos^2 \theta (\sin^4 \theta + \cos^4 \theta) + 6 \sin^4 \theta \cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta \\
&= 3 \sin^2 \theta \cos^2 \theta \left((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta \right) \\
&\quad + 6 \sin^4 \theta \cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta \quad \left(\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \right) \\
&= 3 \sin^2 \theta \cos^2 \theta \left((\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta \right) + 6 \sin^4 \theta \cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta \\
&= 3 \sin^2 \theta \cos^2 \theta (1 - 2 \sin^2 \theta \cos^2 \theta) + 6 \sin^4 \theta \cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta \\
&= 3 \sin^2 \theta \cos^2 \theta - 6 \sin^4 \theta \cos^4 \theta + 6 \sin^4 \theta \cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta \\
&= 0 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Ex 5.3

Q1

We have,

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\Rightarrow \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$\Rightarrow \operatorname{cosec} \theta = \pm \sqrt{1 + \cot^2 \theta}$$

In the third quadrant $\operatorname{cosec} \theta$ is negative

$$\begin{aligned} \therefore \operatorname{cosec} \theta &= -\sqrt{1 + \cot^2 \theta} \\ &= -\sqrt{1 + \left(\frac{12}{5}\right)^2} \quad \left[\because \cot \theta = \frac{12}{5} \right] \\ &= -\sqrt{1 + \frac{144}{25}} \\ &= -\sqrt{\frac{169}{25}} \\ &= -\frac{13}{5} \end{aligned}$$

$$\therefore \operatorname{cosec} \theta = -\frac{13}{5}$$

$$\text{Now, } \tan \theta = \frac{1}{\cot \theta}$$

$$= \frac{1}{\frac{12}{5}}$$

$$= \frac{5}{12}$$

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

In the 2nd quadrant $\sin \theta$ is positive and $\tan \theta$ is negative

$$\begin{aligned} \therefore \sin \theta &= \sqrt{1 - \cos^2 \theta} \\ &= \sqrt{1 - \left(-\frac{1}{2}\right)^2} \quad \left[\because \cos \theta = -\frac{1}{2} \right] \\ &= \sqrt{1 - \frac{1}{4}} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\text{Now, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{-\frac{1}{2}} = -2$$

$$\text{and } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

$$\text{Hence, } \sin \theta = \frac{\sqrt{3}}{2}, \quad \tan \theta = -\sqrt{3},$$

$$\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}, \quad \sec \theta = -2 \quad \text{and} \quad \cot \theta = \frac{-1}{\sqrt{3}}$$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\begin{aligned}\therefore \operatorname{cosec}\theta &= -\sqrt{1+\cot^2\theta} \\ &= -\sqrt{1+\left(\frac{4}{3}\right)^2} \\ &= -\sqrt{1+\frac{16}{9}} \\ &= -\sqrt{\frac{25}{9}} \\ &= -\frac{5}{3}\end{aligned}$$

$$\text{Now, } \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{-\frac{5}{3}} = \frac{-3}{5}$$

$$\text{and, } \cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{4}{5}} = \frac{-5}{4}$$

$$\text{Hence, } \sin\theta = \frac{-3}{5}, \quad \cos\theta = \frac{-4}{5},$$

$$\operatorname{cosec}\theta = -\frac{5}{3}, \quad \sec\theta = \frac{-5}{4} \text{ and } \cot\theta = \frac{4}{3}$$

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

In the 1st quadrant $\cos \theta$ is positive and $\tan \theta$ is also positive

$$\begin{aligned} \therefore \cos \theta &= \sqrt{1 - \sin^2 \theta} \\ &= \sqrt{1 - \left(\frac{3}{5}\right)^2} && \left[\because \sin \theta = \frac{3}{5} \right] \\ &= \sqrt{1 - \frac{9}{25}} \\ &= \sqrt{\frac{16}{25}} \\ &= \frac{4}{5} \end{aligned}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\text{Now, } \csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\text{and, } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\text{Hence, } \cos \theta = \frac{4}{5}, \quad \csc \theta = \frac{5}{3}, \quad \tan \theta = \frac{3}{4},$$

$$\sec \theta = \frac{5}{4}, \quad \text{and } \cot \theta = \frac{4}{3}$$

Q2

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

In the 2nd quadrant $\cos \theta$ is negative and $\tan \theta$ is also negative

$$\begin{aligned} \therefore \cos \theta &= -\sqrt{1 - \sin^2 \theta} \\ &= -\sqrt{1 - \left(\frac{12}{13}\right)^2} \quad \left[\because \sin \theta = \frac{12}{13} \right] \\ &= -\sqrt{1 - \frac{144}{169}} \\ &= -\sqrt{\frac{25}{169}} \\ &= -\frac{5}{13} \end{aligned}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

$$\text{Now, } \sec \theta = \frac{1}{\cos \theta} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5}$$

$$\begin{aligned} \therefore \sec \theta + \tan \theta &= -\frac{13}{5} - \frac{12}{5} \\ &= \frac{-13 - 12}{5} \\ &= -\frac{25}{5} \end{aligned}$$

$$= -5$$

$$\Rightarrow \sec \theta + \tan \theta = -5$$

Q3

We have,

$$\sin \theta = \frac{3}{5}, \quad \tan \phi = \frac{1}{2} \quad \text{and} \quad \frac{\pi}{2} < \theta < \pi < \frac{3\pi}{2}$$

\Rightarrow θ lies in the second quadrant and ϕ lies in the third quadrant.

$$\text{Now, } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

In the 2nd quadrant $\cos \theta$ is negative and $\tan \theta$ is also negative

$$\begin{aligned} \therefore \cos \theta &= -\sqrt{1 - \sin^2 \theta} \\ &= -\sqrt{1 - \left(\frac{3}{5}\right)^2} \\ &= -\sqrt{1 - \frac{9}{25}} \\ &= -\sqrt{\frac{16}{25}} \\ &= -\frac{4}{5} \end{aligned}$$

$$\Rightarrow \cos \theta = -\frac{4}{5}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4} \quad \text{----- (1)}$$

$$\text{Now, } \sec^2 \phi - \tan^2 \phi = 1$$

$$\Rightarrow \sec^2 \phi = 1 + \tan^2 \phi$$

$$\Rightarrow \sec \phi = \pm \sqrt{1 + \tan^2 \phi}$$

In the third quadrant $\sec \phi$ is negative

$$\begin{aligned}\therefore \sec \phi &= -\sqrt{1 + \left(\frac{1}{2}\right)^2} \\ &= -\sqrt{1 + \frac{1}{4}} \\ &= -\sqrt{\frac{5}{4}} \\ \Rightarrow \sec \phi &= -\frac{\sqrt{5}}{2} \text{----- (ii)}\end{aligned}$$

$$\begin{aligned}\therefore 8 \tan \theta - \sqrt{5} \sec \phi & \\ &= 8 \times \left(\frac{-3}{4}\right) - \sqrt{5} \times \left(-\frac{\sqrt{5}}{2}\right) \quad [\text{by equations (i) and (ii)}] \\ &= -2 \times 3 + \frac{5}{2} \\ &= -6 + \frac{5}{2} \\ &= \frac{-12 + 5}{2} \\ &= \frac{-7}{2}\end{aligned}$$

$$\therefore 8 \tan \theta - \sqrt{5} \sec \phi = -\frac{7}{2}$$

Q4

We have,

$$\sin\theta + \cos\theta = 0$$

$$\Rightarrow \sin\theta = -\cos\theta \text{ ----- (i)}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = -1$$

$$\Rightarrow \tan\theta = -1$$

We know that,

$$\sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$$

$$\Rightarrow \sec\theta = \pm\sqrt{1 + \tan^2\theta}$$

In the 4th quadrant $\sec\theta$ is positive.

$$\begin{aligned}\therefore \sec\theta &= \sqrt{1 + \tan^2\theta} \\ &= \sqrt{1 + (-1)^2} \\ &= \sqrt{1 + 1} \\ &= \sqrt{2}\end{aligned}$$

$$\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{2}}$$

putting $\cos\theta = \frac{1}{\sqrt{2}}$ in equation (i), we get,

$$\sin\theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

Hence, $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\cos\theta = \frac{1}{\sqrt{2}}$.

Q5

We have,

$$\cos \theta = -\frac{3}{5}, \quad \text{and } \pi < \theta < \frac{3\pi}{2}$$

\Rightarrow θ lies in the 3rd quadrant

We know that,

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

In the 3rd quadrant $\sin \theta$ is negative and $\tan \theta$ is positive.

$$\begin{aligned} \therefore \sin \theta &= -\sqrt{1 - \cos^2 \theta} \\ &= -\sqrt{1 - \left(-\frac{3}{5}\right)^2} \quad \left[\because \cos \theta = -\frac{3}{5} \right] \\ &= -\sqrt{1 - \frac{9}{25}} \\ &= -\sqrt{\frac{16}{25}} \\ &= -\frac{4}{5} \end{aligned}$$

$$\Rightarrow \sin \theta = -\frac{4}{5}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

$$\text{Now, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

$$\text{and, } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\begin{aligned}
 \therefore \frac{\cos \theta + \tan \theta}{\sec \theta - \tan \theta} &= \frac{-\frac{5}{4} + \frac{3}{4}}{-\frac{5}{3} - \frac{4}{3}} \\
 &= \frac{-\frac{5+3}{4}}{\frac{-5-4}{3}} \\
 &= \frac{-\frac{2}{4}}{-\frac{9}{3}} \\
 &= \frac{2}{4} \times \frac{3}{9} \\
 &= \frac{1}{6}
 \end{aligned}$$

$$\therefore \frac{\cos \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{1}{6}$$

Ex 5.3

Q1(i)

$$\begin{aligned}\sin \frac{5\pi}{3} &= \sin \left(2\pi - \frac{\pi}{3} \right) \\ &= -\sin \frac{\pi}{3} \quad (\because \sin(2\pi - \theta) = -\sin \theta) \\ &= \frac{-\sqrt{3}}{2}\end{aligned}$$

Q1(ii)

$$\begin{aligned}3060^\circ &= 17\pi & (\because \pi = 180^\circ) \\ \therefore \sin 3060^\circ &= \sin 17\pi \\ &= 0 & (\because \sin n\pi = 0 \text{ for all } n \in \mathbb{Z})\end{aligned}$$

Q1(iii)

$$\begin{aligned}\tan \frac{11\pi}{6} &= \tan \left(2\pi - \frac{\pi}{6} \right) \\ &= -\tan \frac{\pi}{6} & (\because \tan(2\pi - \theta) = -\tan \theta) \\ &= \frac{-1}{\sqrt{3}}\end{aligned}$$

Q1(iv)

$$\begin{aligned}1125^\circ &= 6\pi + \frac{\pi}{4} \quad (\pi = 180^\circ) \\ \cos(-1125^\circ) &= \cos \left(-\left(6\pi + \frac{\pi}{4} \right) \right) \\ &= \cos \left(6\pi + \frac{\pi}{4} \right) & (\because \cos(-\theta) = \cos \theta) \\ &= \cos \left(2 \times 3\pi + \frac{\pi}{4} \right) \\ &= \cos \frac{\pi}{4} & (\because \cos(2k\pi + \theta) = \cos \theta, k \in \mathbb{N}) \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Q1(v)

$$\begin{aligned}
 \tan 315^\circ &= \tan\left(2\pi - \frac{\pi}{4}\right) \\
 &= -\tan\frac{\pi}{4} && (\because \tan(2\pi - \theta) = -\tan\theta) \\
 &= -1
 \end{aligned}$$

Q1(iv)

$$\begin{aligned}
 \sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\
 &= \sin\frac{\pi}{6} && (\because 3\pi - \frac{\pi}{6} \text{ lies in second quadrant}) \\
 &= \frac{1}{2}
 \end{aligned}$$

Alternative solution

$$\begin{aligned}
 \sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\
 &= \sin\left(2\pi + \left(\pi - \frac{\pi}{6}\right)\right) \\
 &= \sin\left(\pi - \frac{\pi}{6}\right) && (\because \sin(2\pi + \theta) = \sin\theta, \text{ as sine is periodic with period } 2\pi) \\
 &= \sin\frac{\pi}{6} && (\because \sin(\pi - \theta) = \sin\theta) \\
 &= \frac{1}{2}
 \end{aligned}$$

Q1(vii)

$$\begin{aligned}
 \cos 570^\circ &= \cos\left(3\pi + \frac{\pi}{6}\right) \\
 &= \cos\left(2\pi + \left(\pi + \frac{\pi}{6}\right)\right) \\
 &= \cos\left(\pi + \frac{\pi}{6}\right) && (\because \cos(2\pi + \theta) = \cos\theta, \text{ as cosine} \\
 &&& \text{is periodic with period } 2\pi) \\
 &= -\cos\frac{\pi}{6} && (\because \cos(\pi + \theta) = -\cos\theta) \\
 &= \frac{-\sqrt{3}}{2}
 \end{aligned}$$

Q1(viii)

$$\begin{aligned}
\sin(-330^\circ) &= \sin\left(-\left(2\pi - \frac{\pi}{6}\right)\right) \\
&= \sin\left(2\pi - \frac{\pi}{6}\right) && (\because \sin(-\theta) = -\sin\theta) \\
&= -\left(-\sin\frac{\pi}{6}\right) && (\because \sin(2\pi - \theta) = -\sin\theta) \\
&= \sin\frac{\pi}{6} \\
&= \frac{1}{2}
\end{aligned}$$

Q1(ix)

$$\begin{aligned}
\operatorname{cosec}(-1200^\circ) &= \operatorname{cosec}\left(-\left(7\pi - \frac{\pi}{3}\right)\right) \\
&= \operatorname{cosec}\left(7\pi - \frac{\pi}{3}\right) && (\because \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta) \\
&= -\operatorname{cosec}\left(2 \times 3\pi + \left(\pi - \frac{\pi}{3}\right)\right) \\
&= -\operatorname{cosec}\left(\pi - \frac{\pi}{3}\right) && \left. \begin{array}{l} (\because \operatorname{cosec} \text{ is periodic of period } 2\pi, \\ \therefore \operatorname{cosec}(2\pi + \theta) = \operatorname{cosec}(2n\pi + \theta) \\ = \operatorname{cosec}\theta \text{ for all } n \in \mathbb{N} \end{array} \right) \\
&= -\operatorname{cosec}\frac{\pi}{3} && (\because \operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta) \\
&= \frac{-2}{\sqrt{3}}
\end{aligned}$$

Q1(x)

$$\begin{aligned}
\tan(-585^\circ) &= -\tan(585^\circ) && (\because \tan(-\theta) = -\tan\theta) \\
&= -\tan\left(3\pi + \frac{\pi}{4}\right) \\
&= -\tan\left(2\pi + \left(\pi + \frac{\pi}{4}\right)\right) && (\because \tan(2\pi + \theta) = \tan\theta) \\
&= -\tan\frac{\pi}{4} && (\because \tan(\pi + \theta) = \tan\theta) \\
&= -1
\end{aligned}$$

Q1(xi)

$$\begin{aligned}\cos(855^\circ) &= \cos\left(5\pi - \frac{\pi}{4}\right) \\ &= \cos\left(2 \times 2\pi + \left(\pi - \frac{\pi}{4}\right)\right) \\ &= \cos\left(\pi - \frac{\pi}{4}\right) && (\because \cos(2k\pi + \theta) = \cos \theta \text{ for all } k \in \mathbb{N}) \\ &= -\cos \frac{\pi}{4} && (\because \cos(\pi - \theta) = -\cos \theta) \\ &= \frac{-1}{\sqrt{2}}\end{aligned}$$

Q1(xii)

$$\begin{aligned}\sin 1845^\circ &= \sin\left(10\pi + \frac{\pi}{4}\right) \\ &= \sin\left(2 \times 5\pi + \frac{\pi}{4}\right) \\ &= \sin \pi && (\because \sin(2k\pi + \theta) = \sin \theta, \text{ for all } k \in \mathbb{N}) \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Q1(xiii)

$$\begin{aligned}\cos 1755^\circ &= \cos\left(10\pi - \frac{\pi}{4}\right) \\ &= \cos\left(2 \times 5\pi - \frac{\pi}{4}\right) \\ &= \cos \frac{\pi}{4} && (\because \cos(2k\pi - \theta) = \cos \theta, k \in \mathbb{N}) \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Q1(xiv)

$$\begin{aligned}4530^\circ &= \left(25\pi + \frac{\pi}{6}\right) \\ \therefore \sin 4530 &= \sin \left(25\pi + \frac{\pi}{6}\right) \\ &= \sin \left(2 \times 12\pi + \left(\pi + \frac{\pi}{6}\right)\right) \\ &= \sin \left(\pi + \frac{\pi}{6}\right) \quad (\because \sin(2k\pi + \theta) = \sin \theta, k \in \mathbb{N}) \\ &= -\sin \frac{\pi}{6} \quad (\because \sin(\pi + \theta) = -\sin \theta) \\ &= \frac{-1}{2}\end{aligned}$$

Q2(i)

$$\begin{aligned}\text{LHS} &= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ \\ &= \tan \left(\pi + \frac{\pi}{4}\right) \cot \left(2\pi + \frac{\pi}{4}\right) + \tan \left(4\pi + \frac{\pi}{4}\right) \cot \left(4\pi - \frac{\pi}{4}\right) \\ &= \tan \frac{\pi}{4} \cdot \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \times \left(-\cot \frac{\pi}{4}\right) \quad \left(\because \cot \left(4\pi - \frac{\pi}{4}\right) = -\cot \frac{\pi}{4}\right) \\ &= 1 \cdot 1 + 1 \cdot (-1) \\ &= 0 \\ &= \text{RHS} \\ &\quad \text{Proved}\end{aligned}$$

Q2(ii)

$$\begin{aligned}
\text{LHS} &= \sin \frac{8\pi}{3} \cos \frac{23\pi}{6} + \cos \frac{13\pi}{3} \sin \frac{35\pi}{6} \\
&= \sin \left(3\pi - \frac{\pi}{3} \right) \cos \left(4\pi - \frac{\pi}{6} \right) + \cos \left(4\pi + \frac{\pi}{3} \right) \sin \left(6\pi - \frac{\pi}{6} \right) \\
&= \sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3} \left(-\sin \frac{\pi}{6} \right) && (\because \sin(6\pi - \theta) = -\sin \theta) \\
&= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \left(-\frac{1}{2} \right) \\
&= \frac{3}{4} - \frac{1}{4} \\
&= \frac{2}{4} \\
&= \frac{1}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Q2(iii)

$$\begin{aligned}
\text{LHS} &= \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ \\
&= \cos 24^\circ + \cos 204^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ \\
&= \cos 24^\circ + \cos (\pi + 24^\circ) + \cos 55^\circ + \cos (\pi - 55^\circ) + \cos \left(2\pi - \frac{\pi}{3} \right) \\
&= \cos 24^\circ - \cos 24^\circ + \cos 55^\circ - \cos 55^\circ + \cos \frac{\pi}{3} \\
&= \cos \frac{\pi}{3} \\
&= \frac{1}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Q2(iv)

$$\text{LHS} = \tan(-225^\circ) \cot(-405^\circ) - \tan(-765^\circ) \cot(675^\circ)$$

$$= -\tan 225^\circ (-\cot 405^\circ) + \tan 765^\circ \cot 765^\circ$$

$$\left(\begin{array}{l} \because \tan(-\theta) = -\tan \theta \\ \& \cot(-\theta) = -\cot \theta \end{array} \right)$$

$$= \tan\left(\pi + \frac{\pi}{4}\right) \cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right) \cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan \frac{\pi}{4} \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \times \left(-\cot \frac{\pi}{4}\right)$$

$$\left(\because \cot(4\pi - \theta) = -\cot \theta \right)$$

$$= 1 \cdot 1 + 1(-1)$$

$$= 1 - 1$$

$$= 0$$

$$= \text{RHS}$$

Proved

Q2(v)

$$\text{LHS} = \cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ)$$

$$= \cos\left(3\pi + \frac{\pi}{6}\right) \sin\left(3\pi - \frac{\pi}{6}\right) - \sin 330^\circ \cos 390^\circ$$

$$\left(\begin{array}{l} \because \sin(-\theta) = -\sin \theta \text{ and} \\ \cos(-\theta) = \cos \theta \end{array} \right)$$

$$= -\cos \frac{\pi}{6} \sin \frac{\pi}{6} - \sin\left(2\pi - \frac{\pi}{6}\right) \cos\left(2\pi + \frac{\pi}{6}\right)$$

$$= -\sin \frac{\pi}{6} \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cdot \cos \frac{\pi}{6}$$

$$\left(\because \sin(2\pi - \theta) = -\sin \theta \right)$$

$$= 0$$

$$= \text{RHS}$$

Proved

Q2(vi)

$$\begin{aligned}
\text{LHS} &= \tan \frac{11\pi}{3} - 2 \sin \frac{4\pi}{6} - \frac{3}{4} \operatorname{cosec}^2 \frac{\pi}{4} + 4 \cos^2 \frac{17\pi}{6} \\
&= \tan \left(4\pi - \frac{\pi}{3} \right) - 2 \sin \frac{2\pi}{3} - \frac{3}{4} \times (\sqrt{2})^2 + 4 \cos^2 \left(3\pi - \frac{\pi}{6} \right) \\
&= -\tan \frac{\pi}{3} - 2 \sin \left(\pi - \frac{\pi}{3} \right) - \frac{3}{4} \times 2 + 4 \cos^2 \frac{\pi}{6} \\
&\quad \left(\because \tan \left(4\pi - \frac{\pi}{3} \right) = -\tan \frac{\pi}{3}, \cos \left(3\pi - \frac{\pi}{6} \right) = -\cos \frac{\pi}{6} \right) \\
&= -\sqrt{3} - 2 \sin \frac{\pi}{3} - \frac{3}{2} + 4 \times \left(\frac{\sqrt{3}}{2} \right)^2 \\
&= -\sqrt{3} - 2 \times \frac{\sqrt{3}}{2} - \frac{3}{2} + 4 \times \frac{3}{4} \\
&= -\sqrt{3} - \sqrt{3} - \frac{3}{2} + 3 \\
&= -2\sqrt{3} - \frac{3+6}{2} \\
&= -2\sqrt{3} + \frac{3}{2} \\
&= \frac{3 - 4\sqrt{3}}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Q2(vii)

$$\begin{aligned}
\text{LHS} &= 3 \sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} \\
&= 3 \times \frac{1}{2} \times 2 - 4 \sin \left(\pi - \frac{\pi}{6} \right) \times 1 \\
&= 3 - 4 \sin \frac{\pi}{6} && \left(\because \sin (\pi - \theta) = \sin \theta \right) \\
&= 3 - 4 \times \frac{1}{2} \\
&= 3 - 2 \\
&= 1 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Q3(i)

$$\begin{aligned} \text{LHS} &= \frac{\cos(2\pi + \theta) \operatorname{cosec}(2\pi + \theta) \tan\left(\frac{\pi}{2} + \theta\right)}{\sec\left(\frac{\pi}{2} + \theta\right) \cos \theta \cot(\pi + \theta)} \\ &= \frac{\cos \theta \times \operatorname{cosec} \theta (-\cot \theta)}{-\operatorname{cosec} \theta \cdot \cos \theta \cot \theta} \quad \left(\begin{array}{l} \because \tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta \\ \& \sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta \end{array} \right) \\ &= 1 \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q3(ii)

$$\begin{aligned} \text{LHS} &= \frac{\operatorname{cosec}(90^\circ + \theta) + \cot(450^\circ + \theta)}{\operatorname{cosec}(90^\circ - \theta) + \tan(180^\circ - \theta)} + \frac{\tan(180^\circ + \theta) + \sec(180^\circ - \theta)}{\tan(360^\circ + \theta) - \sec(-\theta)} \\ &= \frac{\sec \theta + \cot\left(2\pi + \frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta} \\ &\quad \left(\because \operatorname{cosec}(90^\circ + \theta) = \sec \theta, \operatorname{cosec}(90^\circ - \theta) = \sec \theta, \tan(180^\circ - \theta) = -\tan \theta, \sec(-\theta) = \sec \theta \right) \\ &= \frac{\sec \theta + \cot\left(\frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + 1 \quad \left(\because \cot(2\pi + \theta) = \cot \theta \right) \\ &= \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} + 1 \quad \left(\because \cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta \right) \\ &= 1 + 1 \\ &= 2 \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q3(iii)

$$\begin{aligned} \text{LHS} &= \frac{\sin(180^\circ + \theta) \cos(90^\circ + \theta) \tan(270^\circ - \theta) \cot(360^\circ - \theta)}{\sin(360^\circ - \theta) \cos(360^\circ + \theta) \operatorname{cosec}(-\theta) \sin(270^\circ + \theta)} \\ &= \frac{\sin \theta (-\sin \theta) \cot \theta (-\cot \theta)}{-\sin \theta \cos \theta (-\operatorname{cosec} \theta) (-\cos \theta)} \quad \left(\begin{array}{l} \because \tan(270^\circ - \theta) = \cot \theta \\ \& \sin(270^\circ + \theta) = -\cos \theta \end{array} \right) \\ &= \frac{-\sin \theta \times \sin \theta \times \cos \theta \times \cos \theta \times \sin \theta}{-\sin \theta \times \cos \theta \times \sin \theta \times \sin \theta \times \cos \theta} \quad \left(\begin{array}{l} \because \cot \theta = \frac{\cos \theta}{\sin \theta} \\ \& \operatorname{cosec} \theta = \frac{1}{\sin \theta} \end{array} \right) \\ &= 1 \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q3(iv)

$$\begin{aligned} \text{LHS} &= \left\{ 1 + \cot \theta - \sec\left(\frac{\pi}{2} + \theta\right) \right\} \left\{ 1 + \cot \theta + \sec\left(\frac{\pi}{2} + \theta\right) \right\} \\ &= \{1 + \cot \theta - (-\operatorname{cosec} \theta)\} \{1 + \cot \theta - \operatorname{cosec} \theta\} \\ &\quad \left(\because \sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta \right) \\ &= \{(1 + \cot \theta) + \operatorname{cosec} \theta\} \{(1 + \cot \theta) - \operatorname{cosec} \theta\} \\ &= (1 + \cot \theta)^2 - \operatorname{cosec}^2 \theta \\ &= 1 + \cot^2 \theta + 2 \cot \theta - \operatorname{cosec}^2 \theta \\ &= \operatorname{cosec}^2 \theta + 2 \cot \theta - \operatorname{cosec}^2 \theta \quad \left(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \right) \\ &= 2 \cot \theta \\ &= \text{RHS} \\ &\quad \text{Proved} \end{aligned}$$

Q3(v)

$$\begin{aligned}\text{LHS} &= \frac{\tan(90^\circ - \theta) \sec(180^\circ - \theta) \sin(-\theta)}{\sin(180^\circ + \theta) \cot(360^\circ - \theta) \operatorname{cosec}(90^\circ - \theta)} \\ &= \frac{\cot \theta \times (-\sec \theta) \times (-\sin \theta)}{-\sin \theta \times (-\cot \theta) \times \sec \theta} \\ &= 1 \\ &= \text{RHS} \\ &\text{Proved}\end{aligned}$$

Q4

$$\begin{aligned}\text{LHS} &= \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} \\ &= \sin^2 \frac{\pi}{18} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} \\ &= \sin^2 \left(\frac{\pi}{2} - \frac{4\pi}{9} \right) + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{9} \right) \\ &= \cos^2 \frac{4\pi}{9} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9} \\ &= 1 + 1 \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= 2 \\ &= \text{RHS} \\ &\text{Proved}\end{aligned}$$

$$\begin{aligned}&\left(\because \frac{\pi}{18} - \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} - \frac{\pi}{2} - \frac{\pi}{9} \right) \\ &\left(\because \sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \right)\end{aligned}$$

Q6(ii)

We have $A + B + C = \pi$ (\because sum of 3 angles of a triangle is $\pi = 180^\circ$)

$$\Rightarrow A + B = \pi - C$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\Rightarrow = \sin\frac{C}{2} \quad \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$$

$$\text{Hence } \cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$$

Proved

Q6(iii)

We have $A + B + C = \pi$ (\because sum of 3 angles of a triangle is $\pi = 180^\circ$)

$$\Rightarrow A + B = \pi - C$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$= \cot\frac{C}{2} \quad \left(\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right)$$

$$\text{Hence } \tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$$

Proved

Q7

$\therefore A, B, C, D$ are the angles of a cyclic quadrilateral in order,

$$\therefore A + C = \pi \text{ \& } B + D = \pi$$

$$\Rightarrow \pi - A = C \text{ \& } \pi - D = B$$

$$\Rightarrow \cos(\pi - A) = \cos C \text{ (i)}$$

$$\text{\& } \cos(\pi - D) = \cos B \text{ (ii)}$$

$$\text{Now, } \cos(180^\circ - A) + \cos(180^\circ + B) + (180^\circ + C) - \sin(90^\circ + D)$$

$$= \cos C + (-\cos B) - \cos C - \cos D$$

$$\left(\because \cos(180^\circ + B) = -\cos B, \cos(180^\circ + C) = -\cos C \text{ \& using (i)} \right)$$

$$= -\cos B - \cos D$$

$$= -\cos B - (-\cos B) \text{ (using (ii))}$$

$$= -\cos B + \cos B$$

$$= 0$$

Proved

Q8(i)

$$\operatorname{cosec}(90^\circ + \theta) + x \cos \theta \cot(90^\circ + \theta) = \sin(90^\circ + \theta)$$

$$\Rightarrow \sec \theta + x \cos \theta \times (-\tan \theta) = \cos \theta$$

$$\Rightarrow \frac{1}{\cos \theta} + x \cos \theta \times \frac{(-\sin \theta)}{\cos \theta} = \cos \theta$$

$$\Rightarrow \frac{1}{\cos \theta} - x \sin \theta = \cos \theta$$

$$\Rightarrow \frac{1 - x \sin \theta \cos \theta}{\cos \theta} = \cos \theta$$

$$\Rightarrow 1 - x \sin \theta \cos \theta = \cos^2 \theta$$

$$\Rightarrow 1 - \cos^2 \theta = x \sin \theta \cos \theta$$

$$\Rightarrow \sin^2 \theta = x \sin \theta \cos \theta$$

$$\Rightarrow \sin \theta = x \cos \theta$$

$$\Rightarrow x = \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

Hence $x = \tan \theta$

Q8(ii)

We have $x \cot(90^\circ + \theta) + \tan(90^\circ + \theta) \sin \theta + \operatorname{cosec}(90^\circ + \theta) = 0$

$$\Rightarrow x(-\tan \theta) - \cot \theta \times \sin \theta + \sec \theta = 0$$

$$\Rightarrow -x \tan \theta - \frac{\cos \theta}{\sin \theta} \times \sin \theta + \frac{1}{\cos \theta} = 0$$

$$\Rightarrow -x \frac{\sin \theta}{\cos \theta} - \cos \theta + \frac{1}{\cos \theta} = 0$$

$$\Rightarrow \frac{-x \sin \theta - \cos^2 \theta + 1}{\cos \theta} = 0$$

$$\Rightarrow -x \sin \theta + 1 - \cos^2 \theta = 0$$

$$\Rightarrow -x \sin \theta + \sin^2 \theta = 0$$

$$\Rightarrow x \sin \theta = \sin^2 \theta$$

$$\Rightarrow x = \frac{\sin^2 \theta}{\sin \theta}$$

$$\Rightarrow x = \sin \theta$$

Q9(i)

$$\text{LHS} = \tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ$$

$$= \tan 4\pi - \cos\left(\frac{3\pi}{2}\right) - \sin\left(\pi \frac{\pi}{6}\right) \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \left(\because \pi = 180^\circ\right)$$

$$= 0 - 0 - \sin \frac{\pi}{6} \left(-\sin \frac{\pi}{6}\right) \left(\because \tan n\pi = 0 \text{ for all } n \in \mathbb{Z} \text{ \& } \cos \frac{3\pi}{2} = 0\right)$$

$$= \sin^2 \frac{\pi}{6}$$

$$= \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4}$$

$$= \text{RHS}$$

Proved

Q9(ii)

$$\text{LHS} = \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ$$

$$= \sin \left(4\pi + \frac{\pi}{3} \right) \sin \left(3\pi - \frac{\pi}{3} \right) + \cos \left(\frac{\pi}{2} + \frac{\pi}{6} \right) \sin \left(\pi - \frac{\pi}{6} \right)$$

$$(\because \pi = 180^\circ)$$

$$= \sin \frac{\pi}{3} \times \sin \frac{\pi}{3} + \left(-\sin \frac{\pi}{6} \right) \sin \frac{\pi}{6}$$

$$\left(\begin{array}{l} \because \sin \left(4\pi + \frac{\pi}{3} \right) = \sin \frac{\pi}{3} \\ \& \sin \left(3\pi - \frac{\pi}{3} \right) = \sin \frac{\pi}{3} \end{array} \right)$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$= \text{RHS}$$

Proved

Q9(iii)

$$\text{LHS} = \sin 780^\circ \sin 120^\circ + \cos 240^\circ \sin 390^\circ$$

$$= \sin \left(4\pi + \frac{\pi}{3} \right) \sin \left(\frac{\pi}{2} + \frac{\pi}{6} \right) + \cos \left(\pi + \frac{\pi}{6} \right) \sin \left(2\pi + \frac{\pi}{6} \right)$$

$$= \sin \frac{\pi}{3} \times \cos \frac{\pi}{6} - \cos \frac{\pi}{3} \times \left(+\sin \frac{\pi}{6} \right)$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$= \text{RHS}$$

Proved

Q9(iv)

$$\begin{aligned}\text{LHS} &= \sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ \\ &= \sin \left(3\pi + \frac{\pi}{3} \right) \cos \left(2\pi + \frac{\pi}{6} \right) + \cos \left(3\pi - \frac{\pi}{3} \right) \sin \left(\pi - \frac{\pi}{6} \right) \\ &= -\sin \frac{\pi}{3} \cos \frac{\pi}{6} - \cos \frac{\pi}{3} - \sin \frac{\pi}{6} \quad \left(\because \sin \left(3\pi + \frac{\pi}{3} \right) = -\sin \frac{\pi}{3} \text{ \& } \cos \left(3\pi - \frac{\pi}{3} \right) = -\cos \frac{\pi}{3} \right) \\ &= \frac{-\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{-3}{4} - \frac{1}{4} \\ &= \frac{-4}{4} \\ &= -1 \\ &= \text{RHS} \\ &\quad \text{Proved}\end{aligned}$$

Q9(v)

$$\begin{aligned}\text{LHS} &= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ \\ &= \tan \left(\pi + \frac{\pi}{4} \right) \cot \left(2\pi + \frac{\pi}{4} \right) + \tan \left(4\pi + \frac{\pi}{4} \right) \cot \left(4\pi - \frac{\pi}{4} \right) \\ &= \tan \frac{\pi}{4} \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \left(-\cot \frac{\pi}{4} \right) \\ &= 1 \cdot 1 + 1 \cdot (-1) \\ &= 1 - 1 \\ &= 0 \\ &= \text{RHS} \\ &\quad \text{Proved}\end{aligned}$$